OUTCOME 1 - TUTORIAL 2

1 Heat transfer rates

Interfaces: conduction (Fourier's law, thermal conductivity, thermal resistance, temperature gradient, composite plane walls and thick cylinders);
convection (description of forced and natural convection, convective heat transfer coefficient, film and overall coefficient)

Radiation: nature of radiation; Stefan-Boltzman law; black and grey body radiation; emissivity; absorptivity; correction for overall heat transfer coefficient

Lagging: material types; conductivity; energy costs; economic lagging

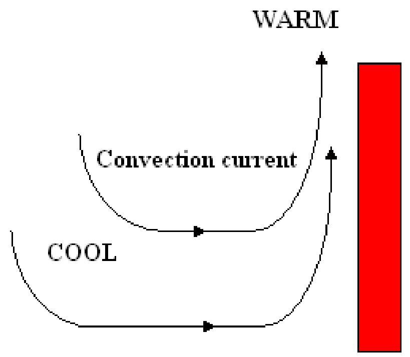
You should judge your progress by completing the self assessment exercises. On completion of this tutorial the student should be able to do the following.

- Explain natural and forced convection.
- Explain the use of the surface heat transfer coefficient.
- Explain the use of the overall heat transfer coefficient.
- Combine convection and conduction theory to solve problems involving flat, cylindrical and spherical surfaces.
- Explain the basic theory behind radiated heat transfer.
- Explain the affect of the emissivity and shape of the surface.
- Calculate effective surface heat transfer coefficient.
- Solve basic problems involving convection and radiation.

CONVECTION

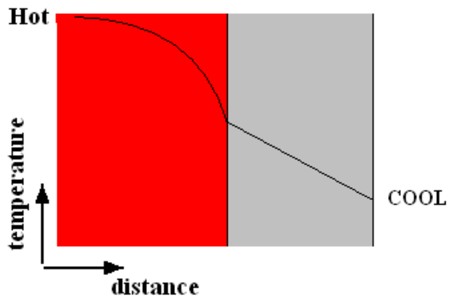
Convection is the study of heat transfer between a fluid and a solid body. Natural convection occurs when there is no forced flow of the fluid. Forced convection occurs when the fluid is forced to flow over the object.

Consider a hot vertical surface placed in a cool fluid. The molecules in contact with the surface will receive heat transfer through the process of conduction. The fluid in contact with the surface will become hotter and less dense. If the fluid is a liquid that evaporates, the vapour will be less dense than the liquid. Because the fluid is less dense than the bulk fluid, it will rise and cool fluid will replace it. Natural convection currents are set up.

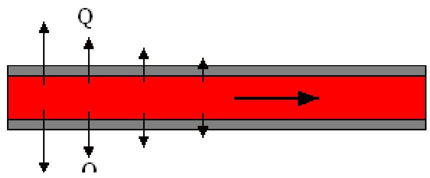


Clearly the rate of heat transfer depends on the thermal conductivity of the fluid in contact with the surface and the volumetric expansion properties of the fluid. The flow of fluids over a surface is also a major topic in fluid mechanics and the work on boundary layers covered in other tutorials is important for a deep understanding of the topic.

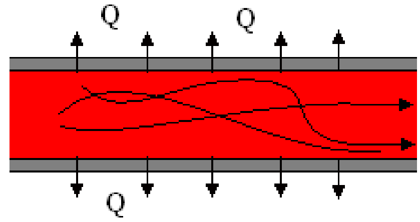
When a fluid is in contact with a solid surface, the temperature of the fluid will vary in the region close to the surface. The diagram shows how the temperature might vary in a hot fluid in contact with a cooler solid surface. Clearly if we can make the temperature at the interface greater, the heat transfer will be increased.



Consider a hot fluid flowing through a long pipe with heat transfer required into the wall of the pipe. The fluid in contact with the surface will reach the same temperature as the pipe at some point and further contact will not increase the transfer. The heat transfer will decrease with distance as shown.



To improve the heat transfer, it is necessary to promote turbulent flow so that the fluid in the core is moved to the edges and comes in contact with the wall.



BASIC CONVECTION LAW

The heat transfer rate between a fluid and a solid surface by convection is usually given as

$$\Phi = -h\,A\,\Delta\theta = h\,A\,(\theta_h - \theta_c)$$

h is called the surface heat transfer coefficient and has units of $W/m^2\,K$.
A is the surface area.
The thermal resistance is $R = 1/hA$ and this may be used for compound problems.

The values of h depend on all the points raised previously and have largely been determined by empirical methods for specific conditions. For example, the value would be different for a flat vertical surface and a flat horizontal surface even if all other conditions are the same. Advanced studies will reveal formulae for finding h under a variety of conditions but at this stage we will simply use the values given.

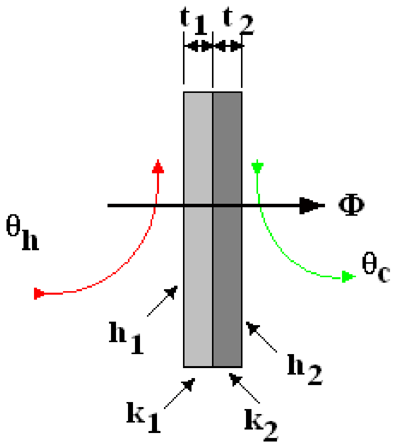
WORKED EXAMPLE No. 1

Calculate the heat transfer per square meter between a fluid with a bulk temperature of 66°C with a wall with a surface temperature of 25°C given h = 5 W/m^2 K.

SOLUTION

$\Phi = h\ A\ (\theta_h - \theta_c) = 5 \times 1\ (66 - 25) = 205$ W

COMPOUND LAYERS



We can now solve problems involving conduction and convection. Consider the case of the heat transfer from a hot fluid to cold fluid through a wall made from two layers.

We have four thermal resistances.

$R_1 = 1/h_1A \qquad R_2 = t_1/k_1A \qquad R_3 = t_2/k_2A \qquad R_4 = 1/h_2A$

$R = R_1 + R_2 + R_3 + R_4$

$\Phi = (\theta_h - \theta_c)/R$

OVERALL HEAT TRANSFER COEFFICIENT

First let's look at the overall heat transfer coefficient for conduction and convection. Consider heat being convected to a surface, then conducted through a wall and convected to a fluid on the other side. There are three thermal resistances

$R_A = \dfrac{1}{h_2\ A} \qquad R_B = \dfrac{x}{k\ A} \qquad R_C = \dfrac{1}{h_2\ A}$

In terms of the overall heat transfer coefficient $\Phi = U\ A\ \Delta\theta$

In terms of thermal resistance $\Phi = \dfrac{\Delta\theta}{(R_A + R_B + R_C)} = \dfrac{A\ \Delta\theta}{\left(\dfrac{1}{h_1} + \dfrac{x}{k} + \dfrac{1}{h_2}\right)}$

Equate and $\dfrac{A\ \Delta\theta}{\left(\dfrac{1}{h_1} + \dfrac{x}{k} + \dfrac{1}{h_2}\right)} = U\ A\ \Delta\theta$

It is apparent that $\dfrac{1}{U} = \dfrac{1}{h_1} + \dfrac{1}{h_2} + \dfrac{x}{k}$

This maybe extended to any number of layers in series.

WORKED EXAMPLE No. 2

Calculate the heat transfer per square meter between a fluid with a bulk temperature of 160°C and another with a bulk temperature of 15°C with a wall between them made of two layers A and B both 50 mm thick. The surface heat transfer coefficient for the hot fluid is 5 W/m^2 K and for the cold fluid 3 W/m^2 K. The thermal conductivity of layer A is 20 W/m K and for B it is 0.5 W/m K. Also calculate the overall heat transfer coefficient.

SOLUTION

For the hot fluid to the wall $R_1 = 1/h_1A = 1/(5 \times 1) = 0.2$ K/W

For Layer A $R_2 = t_1/ k_1A = 0.05/(20 \times 1) = 0.0025$ K/W

For Layer B $R_3 = t_2/ k_2A = 0.05/(0.5 \times 1) = 0.1$ K/W

For the wall to the cold fluid $R_4 = 1/h_2A = 1/(3 \times 1) = 0.333$ K/W

The total thermal resistance is $R = 0.2 + 0.0025 + 0.1 + 0.333 = 0.6355$ K/W

$\Phi = (\theta_h - \theta_c)/R = (160 - 15)/ 0.6355 = 228.2$ W

$\Phi = U A (\theta_h - \theta_c)$

$U = 228.2/(1 \times 145) = 1.573$ W/m^2 K or

$$\frac{1}{U} = \frac{1}{h_1} + \frac{x_1}{k_1} + \frac{x_2}{k_2} + \frac{1}{h_2} = \frac{1}{5} + \frac{0.05}{20} + \frac{0.05}{0.5} + \frac{1}{3} = 0.636 \qquad U = 1/0.636 = 1.573 \text{ W/m}^2\text{ K}$$

SELF ASSESSMENT EXERCISE No.1

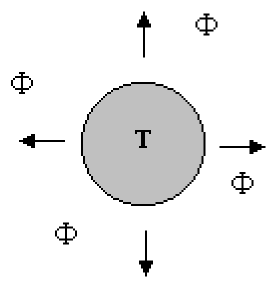
1. Calculate the heat transfer through a steel plate with water on one side at 90°C and air on the other at 15°C. The plate has an area of 1.5 m^2 and it is 20 mm thick. The thermal conductivity is 60 W/m K. The surface heat transfer coefficients for the air and water respectively are 0.006 W/m^2 K and 0.08 W/m^2 K. Calculate the heat loss and the overall heat transfer coefficient.
(Answer 0.628 W and 5.581 x 10^{-3} W/m^2 K)

2. A wall with an area of 25 m^2 is made up of four layers. On the inside is plaster 15 mm thick, then there is brick 100 mm thick, then insulation 60 mm thick and finally brick 100 mm thick. The thermal conductivity of plaster is 0.1 W/m K.
The thermal conductivity brick is 0.6 W/m K
The thermal conductivity the insulation is is 0.08 W/m K
The one side of the wall is in contact with air at 22°C and the other with air at -5°C. The surface heat transfer coefficient for both surfaces is 0.006 W/m^2 K.
Calculate the heat loss and the overall heat transfer coefficient.
(Answer 2.02 W and 3 x 10^{-3} W/m^2 K)

3. A steam pipe 8m long has an external diameter of 100 mm and it is covered by lagging 50 mm thick. The pipe contains steam at 198 °C and the temperature of the atmosphere surrounding the pipe is 18°C. The thermal conductivity of the lagging is 0.15 W/m K.
The surface heat transfer coefficient is 10 W/m^2 K.
Assuming the pipe has the same uniform temperature as the steam; calculate the heat loss and the surface temperature of the lagging.
(Answer 1610 W and 50°C)

4. A spherical tank 2 m diameter contains liquefied fuel gas. It is covered in insulation 120 mm thick with a thermal conductivity of 0.025 W/m K. The surface heat transfer coefficient between the lagging and the surrounding air is 30 W/m^2 K. The air is at 25°C and the liquid is -125°C. Assume the inner surface temperature is the same as the liquid. Calculate the heat transfer rate required to keep the liquid at a constant temperature and surface temperature of the insulation.
(Answer 437 W and 24.1°C)

RADIATION



A hot body radiates energy in the form of electromagnetic radiation. It is found that the energy radiated depends upon the absolute temperature to the power of 4.

A **BLACK BODY** is one that can absorb all the radiated energy falling on it.

A black body will radiate energy according to the law $\Phi = \sigma A T^4$

σ is a constant called the Stefan-Boltzmann constant and has a value of 56.7×10^{-9} W/m^2 k^4

If two identical black bodies at temperatures T_1 and T_2 radiate heat to each other, the net heat transfer is $\Phi = \sigma A(T_1^4 - T_2^4)$

For bodies other than black, the heat radiated depends upon the type of surface and we introduce a property ε called the emissivity to correct the calculation.

$$\Phi = \varepsilon \sigma A (T_1^4 - T_2^4)$$

For two identical bodies with different emissivities ε_1 and ε_2 it can be shown that

$$\Phi = \frac{\sigma A (T_1^4 - T_2^4)}{1/\varepsilon_1 + 1/\varepsilon_2}$$

These equations take no account of the shape and orientation of the bodies with respect to each other and should be used with caution.

TYPICAL VALUES OF ε

These values depend on the temperature and are given as a rough guide only.

Material	Aluminium Polished	Aluminium Oxidised	Copper Polished	Copper Oxidised	Tungsten Polished	Tungsten Oxidised
ε	0.04	0.12	0.02	0.6	0.02	0.06

Material	Steel Polished	Steel Oxidised	Brick	White Paint	Black Paint
ε	0.1	0.8	0.9	0.9	0.97

WORKED EXAMPLE No.3

Two identical bodies radiate heat to each other. One body is at 30°C and the other at 250 °C. The emissivity of both is 0.7. Calculate the net heat transfer per square meter.

SOLUTION

$$\Phi = \varepsilon \sigma A (T_1^4 - T_2^4) = 0.7 \times 56.7 \times 10^{-9} \times 1 \times \left(523^4 - 303^4\right) = 2635 \text{ W}$$

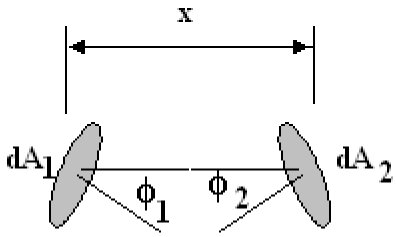
EFFECTIVE RADIATION SURFACE HEAT TRANSFER COEFFICIENT

Sometimes we wish to calculate the radiated heat by a formula similar to the convection formula so that $\Phi r = h_r\, A\, \Delta T$

h_r is the radiated surface heat transfer coefficient. (Note $\Delta T \equiv \Delta\theta$)

Equate $\Phi = \dfrac{\sigma A\,(T_1^4 - T_2^4)}{1/\varepsilon_1 + 1/\varepsilon_2} = h_r A(T_1 - T_2)$ $\quad h_r = \dfrac{\sigma A\,(T_1^4 - T_2^4)}{(1/\varepsilon_1 + 1/\varepsilon_2)A(T_1 - T_2)} = \dfrac{\sigma\ (T_1 + T_2)(T_1^2 + T_2^2)}{(1/\varepsilon_1 + 1/\varepsilon_2)}$

SHAPE FACTOR



Consider two small black areas radiating energy to each other as shown. The normal to the area makes an angle ϕ with the line joining them. It can be shown that the net heat transfer is:

$$d\Phi = \frac{\sigma \cos\phi_1 \cos\phi_2\, dA_1\, dA_2}{\pi\, x^2}\left(T_2^4 - T_1^4\right)$$

Solving for complete areas involves two integrations.

The shape factor is given by $F = \int_A \dfrac{\sigma \cos\phi_1 \cos\phi_2\, dA}{\pi\, x^2}$ This has one value when solved with respect to area 1 and another when solved with respect to area 2.

$F(1\text{-}2) = \int_A \dfrac{\sigma \cos\phi_1 \cos\phi_2\, dA_2}{\pi\, x^2}$ is the factor for area 1 with respect to area 2.

$F(2\text{-}1) = \int_A \dfrac{\sigma \cos\phi_1 \cos\phi_2\, dA_1}{\pi\, x^2}$ is the factor for area 2 with respect to area 1.

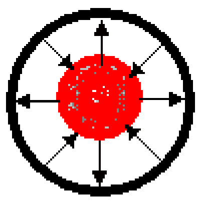
The net heat transfer from between both areas is $\Phi = \sigma\left(T_2^4 - T_1^4\right)A_1\, F(1-2) = \sigma\left(T_2^4 - T_1^4\right)A_2\, F(2-1)$

It follows that $A_1\, F(1-2) = A_2\, F(2-1)$

When we have two surfaces with different emissivity it can be shown that:

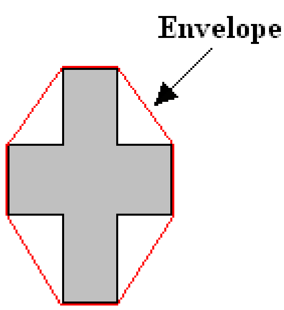
$$\Phi = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\left\{\dfrac{(1-\varepsilon_1)}{\varepsilon_1 A_1}\right\} + \left\{\dfrac{(1-\varepsilon_2)}{\varepsilon_2 A_2}\right\} + \left\{\dfrac{1}{A_1 F(1-2)}\right\}}$$

TOTALLY ENCLOSED BODY



Consider a body totally surrounded by another body. Heat is exchanged between the two. It can be shown that the shape factor is 1.0 so the net heat transfer is $\Phi = \varepsilon\, \sigma\, A\,(T_1^4 - T_2^4)$

A is the envelope area of the enclosed body. For difficult shapes like that shown, it should be regarded as the area that would be obtained by stretching an elastic membrane over it.



WORKED EXAMPLE No.4

A radiator may be treated as a black body with a true surface area of 12 m^2 and an envelope area of 5 m^2. It has a surface temperature of 55°C and is situated in a dark room at 15°C. The surface heat transfer coefficient is 4.5 W/m^2 K. Calculate the radiated heat transfer and the convected heat transfer rate. Calculate the radiated surface heat transfer coefficient and obtain the same answers using it.

SOLUTION

Radiated heat - $\Phi = \varepsilon\,\sigma\,A_e\,(T_1^4 - T_2^4) = 1 \times 56.7 \times 10^{-9} \times 5 \times \left(328^4 - 288^4\right) = 1331\,W$

Convected heat transfer $\Phi = hA\,(T_1 - T_2) = 4.5 \times 12 \times (328 - 288) = 2160W$

Total 1331 + 2160 = 3491 W

Effective radiated surface heat transfer coefficient

$h_r = \varepsilon\,\sigma\,(T_1 + T_2)(T_1^2 + T_2^2) = 1 \times 56.7 \times 10^{-9} \times (328 + 288)\left(328^2 + 288^2\right)$

$h_r =$ 6.65 $W/m^2\,k^4$

$\Phi = h_r\,A_e\,\Delta\theta + h\,A\,\Delta\theta = (6.65 \times 5 \times 40) + (4.5 \times 12 \times 40) = 1331 + 2160 = 3491$ W

SELF ASSESSMENT EXERCISE No.2

1. A space vehicle is in a totally dark vacuum at absolute zero. The envelope area is 20 m^2. The heat loss into space must not exceed 200W. The emissivity of the surface is 0.2. Calculate the surface temperature of the vehicle.
(173.3 K)

2. A radiator may be treated as a black body with a true surface area of 6 m^2 and an envelope area of 4 m^2. It has a surface temperature of 40°C and is situated in a dark room at 20°C. The surface heat transfer coefficient is 4 W/m^2 K. Calculate the radiated heat transfer and the convected heat transfer rate. Calculate the radiated surface heat transfer coefficient and obtain the same answers using it.
(985 W and 6.32 $W/m^2\,k^4$)