EDEXCEL NATIONAL CERTIFICATE/DIPLOMA

UNIT 67 - FURTHER ELECTRICAL PRINCIPLES
NQF LEVEL 3

OUTCOME 2

TUTORIAL 1 - TRANSIENTS

Unit content

2 **Understand the transient behaviour of resistor-capacitor (RC) and resistor-inductor (RL) DC circuits**

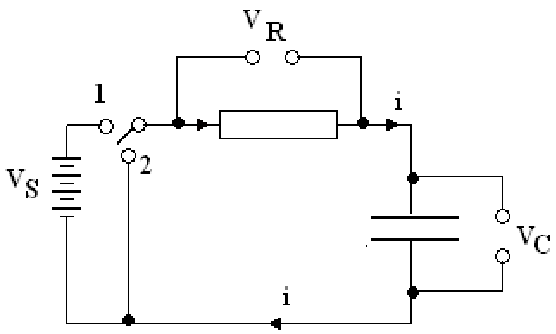
Transient behaviour of RC circuit: variation of current and voltage with time when charging/discharging; time constant; graphical determination of growth and decay of voltage and current when charging/discharging; practical RC circuit to demonstrate transient behaviour; demonstrate the effect of the circuit time constant on a rectangular waveform e.g. integrator and differentiator circuits; calculations e.g. time constant, growth of capacitor voltage, initial and steady state values of current, decay of resistor voltage

Transient behaviour of RL circuit: variation of current and voltage with time when connected/disconnected to a DC voltage source; time constant; graphical determination of growth and decay of current and voltage when connected/disconnected to a DC voltage source; practical RL circuit to demonstrate transient behaviour; calculations e.g. time constant, growth of current, decay of induced voltage, current decay

It is assumed that students have already studied electrical principles.

You will find more useful information on capacitor charging and discharging at this web site:
http://www.electronics-tutorials.ws/rc/rc_1.html

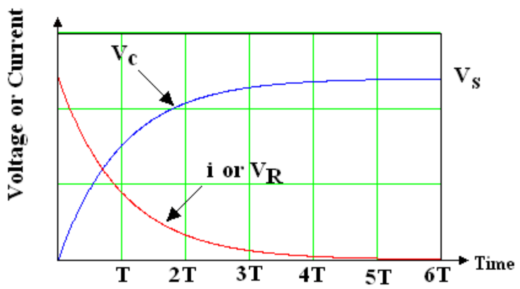
1. CHARGING AND DISCHARGING OF A CAPACITOR



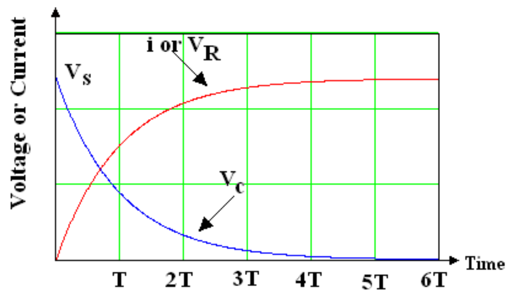
The circuit shows a capacitor in series with a resistor and a DC source. When the switch is in position 1, the battery will send current through the resistor and charge the capacitor. The voltage V_C is initially zero and the voltage V_R is the same as V_S so the capacitor charges at a fast rate. As the capacitor charges, the voltage V_C increases and V_R decreases and the current reduces until the $V_C = V_S$ and the current is zero. The capacitor is then charged to V_S.

If the switch is moved to position 2, the charge rushes out of the capacitor through the resistor, dissipating all the energy as heat until no current flows and $V_C = V_R = 0$.

If we recorded the voltage V_c and V_R and I against time for charging and discharging we get graphs like this.

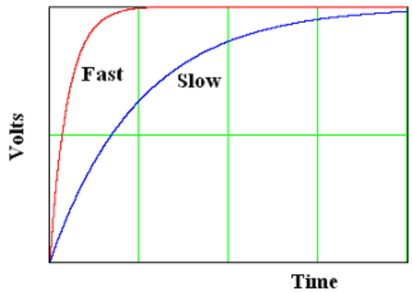


CHARGING



DISCHARGING

If the resistance and capacitance are both large the charging will take longer like filling a large container through a tap that is nearly closed. If they are small, the charging is fast like filling a small container with the tap fully open. The diagram shows this affect on charging.



TIME CONSTANT

We call the product RC the time constant and denote it 'T' or sometime 'τ'. If you refer to the absolute basic units of Farads and Ohms you will see that the units are indeed seconds. When the voltage and current are changing, we say they are in a **TRANSIENT STAGE**. When they reach a constant value we say they are in a **STEADY STATE**. It is widely accepted that it takes a time of 5T or 5RC to reach a steady state. We need mathematics to derive the equations of these curves but you don't have to follow the derivation in order to complete this tutorial (although it helps). The following derivation uses straight forward algebra and basic calculus.

CHARGING

The current heat flows from the source a rate i amps through the resistor and into the capacitor.

Current is the rate of charge with time so $i = \frac{\Delta Q}{\Delta t}$

The voltage across the resistor is $V_R = i\,R$ $\quad i = \frac{V_R}{R}$. Equating $\frac{V_R}{R} = \frac{\Delta Q}{\Delta t}$

Over an infinitesimally small change in time this becomes the calculus form $\frac{V_R}{R} = \frac{dQ}{dt}$

Hence $V_R = R\frac{dQ}{dt}$ ....(A)

For the capacitor, the we know $Q = CV_C$

For a small change $\delta Q = C\delta V_C$

For an infinitesimally small change $dQ = C\, dV_C$ ….(B)

Substitute (B) into (A) $V_R = RC\frac{dV_C}{dt}$

$V_C = V_S - V_R$ hence $(V_S - V_C) = RC\frac{dV_C}{dt} = T\frac{dV_C}{dt}$

This equation tells us how V_C varies with time t but in order to use it, we must solve it.

In order to find T we must solve the equation $(V_S - V_C) = T\frac{dV_C}{dt}$

Let $V_S - V_C = x$ Differentiate and since V_S is a constant we find $-dV_C = dx$

The equation becomes $x = -T\frac{dx}{dt}\, x$ Rearranging $\frac{dt}{T} = -\frac{dx}{x}$

Integrating $-\frac{1}{T}\int_0^t dt = \int\frac{dx}{x} = [\ln x]$

The limits become clear when we substitute $x = V_S - V_C$

$$-\frac{1}{T}\int_0^t dt = \int\frac{dx}{x} = [\ln(V_S - V_C)]_0^{V_C} \qquad -\frac{t}{T} = [\ln(V_S - V_C) - \ln(V_S)]$$

$$-\frac{t}{T} = \ln\left(\frac{V_S - V_C}{V_S}\right) = \ln\left(1 - \frac{V}{V_S}\right)$$ Take antilogs and $$e^{-\frac{t}{T}} = 1 - \frac{V_C}{V_S}$$

Rearrange $$V_C = V_S\left(1 - e^{-\frac{t}{T}}\right)$$

This relates the voltage over the capacitor to time from the moment when the switch is thrown to position 1.

DISCHARGING

When the switch in the circuit is thrown to position 2 the voltage over the capacitor is V_S at t = 0 and if the derivation is repeated with the correct limits of integration we find the relationship is:

$$V_C = V_S e^{-\frac{t}{T}}$$

The graph below shows the result for T = 2 seconds and V_S = 10 V.
Note the voltage across the resistor is $V_S - V_C$

THE MEANING OF THE TIME CONSTANT

Consider the charging curve. $$V_C = V_S\left(1 - e^{-\frac{t}{T}}\right)$$

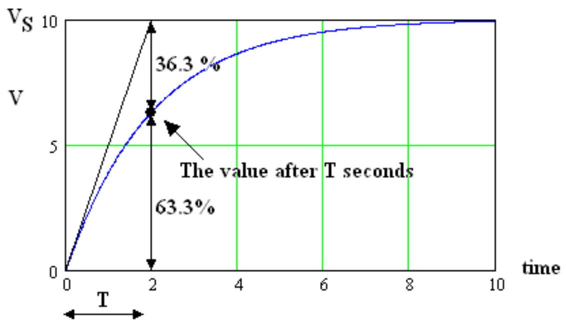
The rate of change of voltage at any time is the gradient of the curve and simply obtained by differentiating.

$$\frac{dV_C}{dt} = \left(\frac{V_S}{T}\right)\left(e^{\frac{-t}{T}}\right)$$

Consider that at t = 0 (the start of the process)

$$\frac{dV_C}{dt} = \left(\frac{V_S}{T}\right)$$ (Remember that $e^0 = 1$)

If the change continued at this rate, the voltage would become V_S after T seconds as illustrated on the graph.



We could define T as the time taken to reach the maximum value if the initial rate of change is maintained. In other words it is where the initial gradient intercepts the final value as shown.

Another meaning is obtained by examining the value after T seconds. Putting t = T we have

$V_C = V_S\left(1 - e^{-\frac{t}{T}}\right) = V_S\left(1 - e^{-1}\right) = 0.633\, V_S$ So the time constant is the time taken to change by 63.3% of the final value. This is useful when finding T from a graph.

If calculate the voltage at t = 4T we find V = 98.2%V_S and this is taken as another definition of T. It is widely accepted that when t = 5T the voltage has reached its maximum.

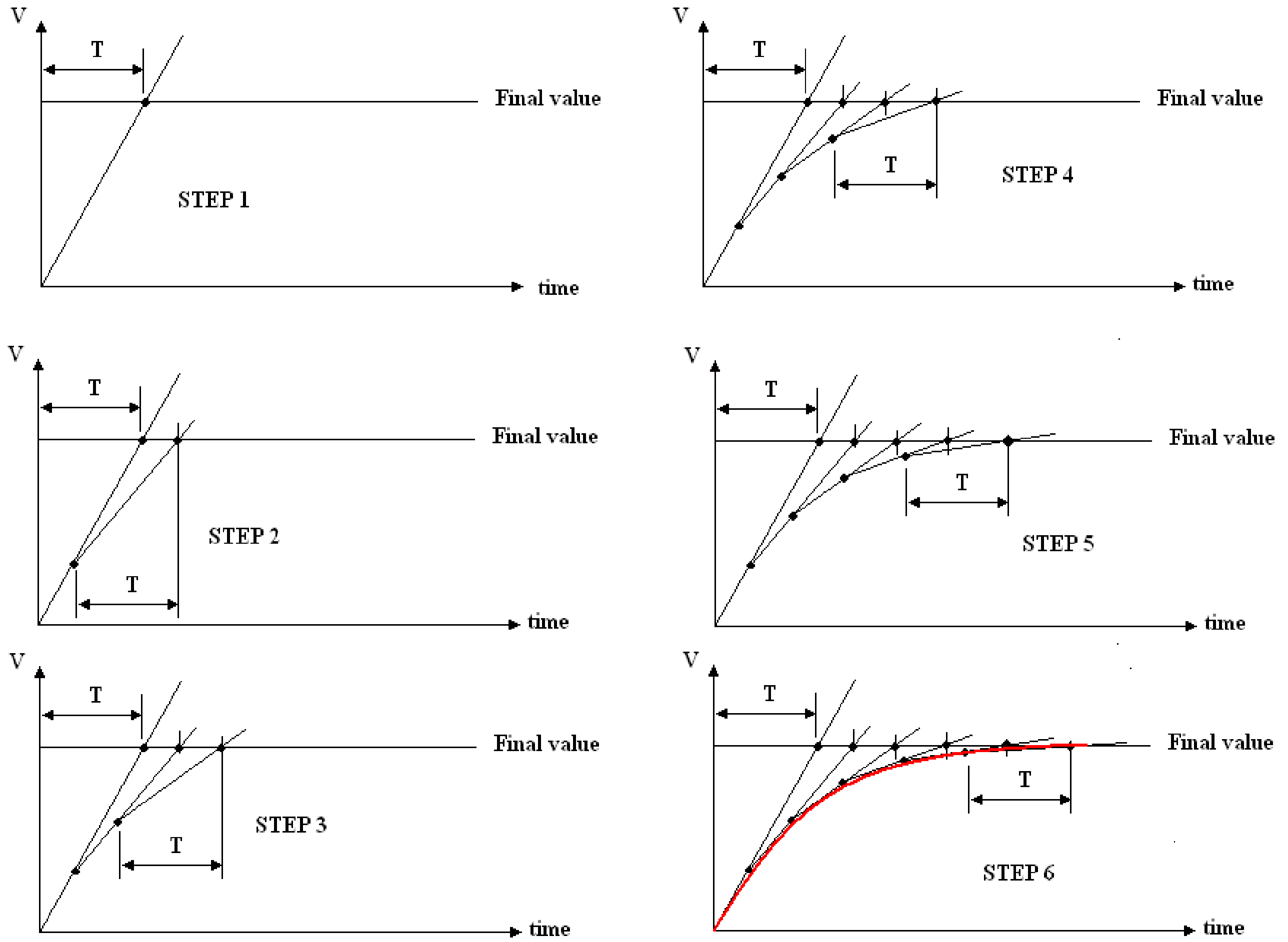
GRAPHICAL CONSTRUCTION METHOD

If we know the time constant we can construct the charge or discharge curves with a graphical method as follows.

Step 1 - Draw the final value line and mark off the time constant. Connect the origin to this point.

Step 2 - Choose a point on the last line drawn and mark off another time constant from this point. Project to the final value line and connect the two points.

Repeat this process as shown and you will have a series of tangents to the charging curve. This may be drawn in as shown in red. The closer you make your points the more accurate the graph will be.



The construction of a discharge curve is the same but inverted.

WORKED EXAMPLE No. 1

A capacitor of value 50 μF is charged from zero to 100 V through a 5 MΩ resistor. Calculate the time constant and the time taken for the voltage to rise to 50 V.

SOLUTION

$T = RC = 50 \times 10^{-6} \times 5 \times 10^{6} = 250$

$$V_C = V_S\left(1 - e^{-\frac{t}{T}}\right) \qquad 50 = 100\left(1 - e^{\frac{-t}{250}}\right) \qquad 0.5 = \left(1 - e^{\frac{-t}{250}}\right) \qquad e^{\frac{-t}{250}} = 1 - 0.5 = 0.5$$

$$\frac{-t}{250} = \ln(0.5) = -0.6931 \qquad t = 250 \times 0.6931 = 173.2 \text{ seconds}$$

WORKED EXAMPLE No. 2

A Capacitance of 200 μF is connected in series with a resistor of 20 kΩ. The voltage across the network is suddenly changed from 0V to 10V.

Calculate the time constant T and deduce the time taken for the voltage on the capacitor to rise to 5V.

SOLUTION

$T = RC = 20 \times 10^{3} \times 200 \times 10^{-6} = 4$ seconds

$V = 5 = 10(1 - e^{-t/4})$

$0.5 = (1 - e^{-t/4})$

$0.5 = e^{-t/4}$

$\ln(0.5) = -t/4$

$-0.693 = -t/4$

$t = 2.77$ s when $V = 5$ V

WORKED EXAMPLE No. 3

A Capacitance of 10 μF is charged up to 50V. The capacitor is discharged by connecting a 2 MΩ resistor across the terminal.

Calculate the time constant T and deduce the time taken for the voltage on the capacitor to fall to 1V.

SOLUTION

$T = RC = 2 \times 10^{6} \times 10 \times 10^{-6} = 20$ seconds

$V = 1 = 50e^{-t/20}$

$1/50 = e^{-t/20}$

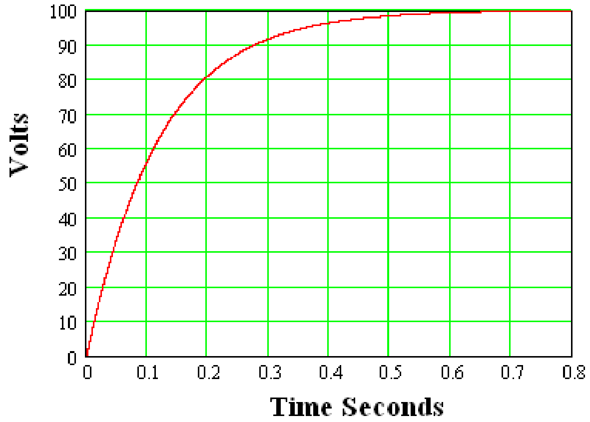
$\ln(1/50) = -t/20$

$-3.912 = -t/20$

$t = 78.24$ s when $V = 1$ V

SELF ASSESSMENT EXERCISE No. 1

1. The graph shows a charging curve for a capacitor and resistance. Work out the time constant T and determine the capacitance if the resistor value is 6 kΩ.



(Answers 0.12 s and 20 μF)

2. Calculate the time constant for an RC circuit with a resistance of 220 Ω and capacitance of 470 nF in series. (103 μs)

   How long dies it take for the voltage to become a steady value?

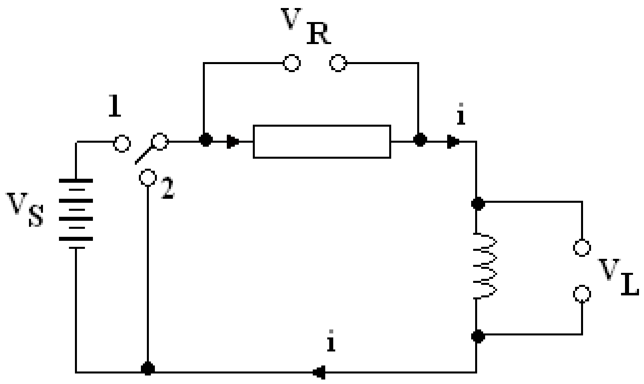
3. A capacitor of 2000 μF is charged to 12 V and then a resistor of 5 Ω is connected across it.
   Calculate the charge stored. (0.024 Coulomb)
   Calculate the energy stored. (0.144 J)
   Calculate time taken to discharge to 1 V (0.025 s)

4. A capacitance of 1500 μF is charged to 10 V through a resistance of 2 MΩ. Calculate the time constant. Construct the charging curve and determine the time taken to charge to 5 V. How long does it take to reach a steady value? (2100 s and 15000 s approx)

2 CHARGE – DISCHARGE OF AN INDUCTOR

When a resistor is connected in series with an inductor we get a similar charging and discharging effect. When the switch is thrown to position 1 there is a rush of current. At this moment the rate of change of current di/dt is a maximum so we get maximum back emf on the inductor. After a time the current settles down to a maximum value of $I = V_s/R$ so $V_s = IR$

Let the current at any time be i.



$$V_R = iR$$
$$V_L = L\,di/dt$$
$$V_S = iR + L\frac{di}{dt}$$
$$IR = iR + L\frac{di}{dt}$$
$$R(I-i) = L\frac{di}{dt}$$
$$(I-i) = \frac{L}{R}\frac{di}{dt}$$

If we examined the units of L/R we would find that this is seconds and L/R is the time constant T for the circuit. T = L/R

$$(I-i) = T\frac{di}{dt} \qquad dt = T\frac{di}{I-i}$$

If we substitute x = I – i then dx = -di

$$dt = -T\frac{dx}{x} \qquad \int_0^t dt = -T\int\frac{dx}{x} \qquad t = -T[\ln x] = -T[\ln(I-i)]_0^i$$

$$-\frac{t}{T} = \ln(I-i) - \ln(I) = \ln\left(\frac{I-i}{I}\right) = \ln\left(1-\frac{i}{I}\right)$$

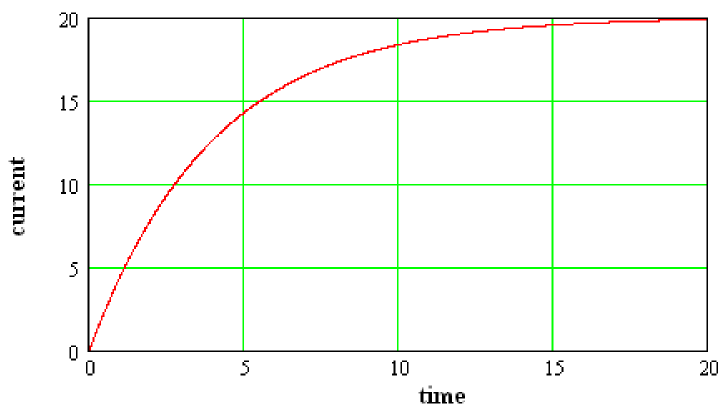
Take antilogs

$$e^{-\frac{t}{T}} = 1-\frac{i}{I} \qquad \frac{i}{I} = 1-e^{-\frac{t}{T}} \qquad i = I\left(1-e^{-\frac{t}{T}}\right) = \frac{V}{R}\left(1-e^{-\frac{t}{T}}\right)$$

This relates the current flowing in the circuit to time and the result is a current that rises fast and then levels off at the constant value.

The graph shows the result for V = 10 V, R = 0.5 Ω and L = 2H giving T = 4 seconds (The values are not practical and only used to illustrate the result)

The time constant may be defined as the time taken to reach 63.3% of the maximum current. Pure inductance is impossible and inductors always have some resistance in the conductor. Circuits usually show a practical inductor as a pure inductance in series with a resistance. In items like transformers, the resistance of the coil can be quite high.



WORKED EXAMPLE No. 3

An Inductance of 4 mH also has a resistance of 0.3 Ω.
Calculate the time constant.
What is the steady state current when 20 V is applied across it?
What is the current 0.02 s after the voltage is suddenly changed from zero to 20 V?

SOLUTION

T = L/R = 0.004/0.3 = 0.013 s
I = V/R = 20/0.3 = 66.7 A

$$i = I\left(1 - e^{-\frac{t}{T}}\right) = 66.7\left(1 - e^{-\frac{0.02}{0.013}}\right) = 51.8\ \text{A}$$

SELF ASSESSMENT EXERCISE No. 2

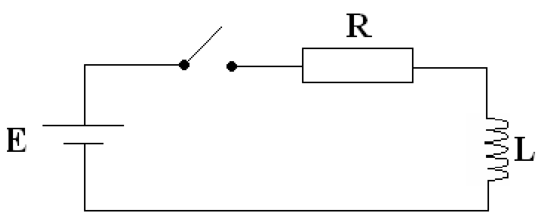
1. Calculate the time constant for a series R – L circuit with an inductance of 6 μH and resistance 0.02 Ω. (3 ms)

2. An inductor with inductance 60 mH and resistance 0.7 Ω suddenly has 2V connected across it.
Calculate the steady state current. (2.857A)
Calculate the energy stored and power dissipated. (0.245 J and 5.714 W)
Calculate the time taken for the current to rise to 0.5 A. (16 ms)

3. The voltage across the inductor shown is initially zero. Show that when the switch is closed that the voltage across the inductor will be

$v = E\,e^{-t/\tau}$

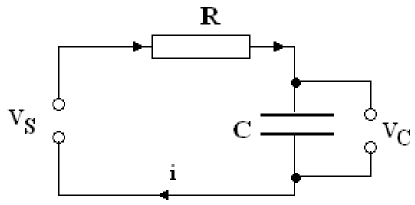
where t is the time elapsed after the switch is closed. Determine an expression for the time constant τ in terms of R and L.



3. CAPACITORS FOR INTEGRATING AND DIFFERENTIATING

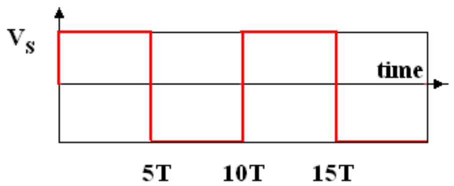
The basic relationship between voltage and current flowing into a capacitor is $i = C\frac{dV_C}{dt}$ so a capacitor can be used integrate and differentiate a voltage.

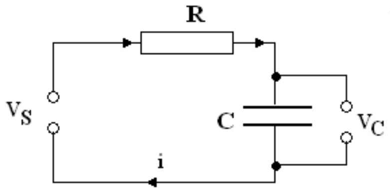
INTEGRATOR CIRCUIT

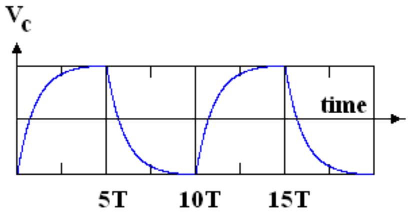


If a voltage V_s is applied to the circuit shown, the voltage across the capacitor is $V_c = \frac{1}{C}\int_0^t i\,dt$

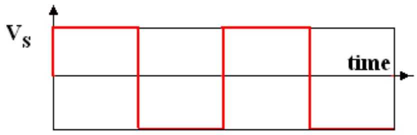
Suppose that a square wave form is applied to the input with a frequency just sufficient to allow the capacitor to charge and discharge on each cycle. The output V_c will be a series a charging and discharging curves.

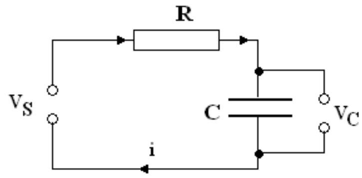


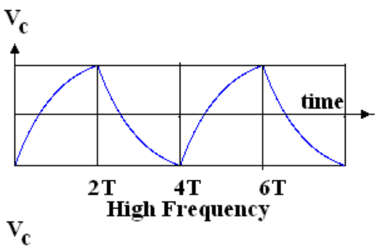


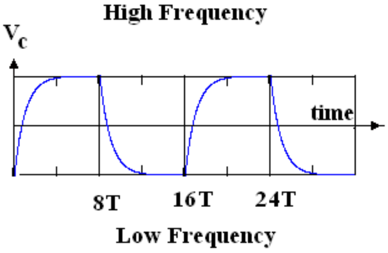


If the frequency is slower the capacitor will fully charge and dwell before discharging. If the frequency is faster, the capacitor will not fully charge or discharge and the waveforms will look like this. If the constants are so arranged, a triangular wave form can be produced.



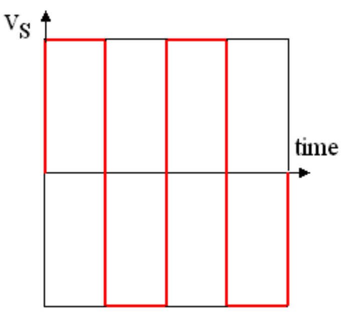


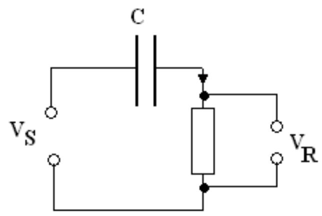


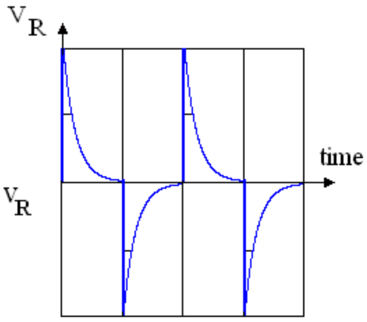


DIFFERENTIATOR CIRCUIT

If a square wave is applied to this circuit, the voltage V_R will be the inverse of the previous outputs







SELF ASSESSMENT EXERCISE No. 3

1. Sketch the output of an integrator and differentiator circuit when a triangular wave form is applied to the input. Explain your reasoning.
