

# EDEXCEL NATIONAL CERTIFICATE/DIPLOMA

## UNIT 67 - FURTHER ELECTRICAL PRINCIPLES

### NQF LEVEL 3

#### OUTCOME 1

#### TUTORIAL 3 - DIRECT CURRENT MOTORS

##### **Unit content**

**1 Be able to apply direct current (DC) circuit analysis methods and consider the types, construction and characteristics of a DC motor and generator**

**Direct current (DC) circuit theorems:** Thévenin's theorem e.g. application of theorem to a parallel circuit having two sources of electromotive force (EMF) and three resistors; Norton's theorem e.g. application of theorem to a parallel circuit having two sources of EMF and three resistors; maximum power transfer theorem e.g. application of theorem to a series circuit with a source of EMF, internal resistance and a load resistor; application to a more complex circuit where Thévenin needs to be applied first

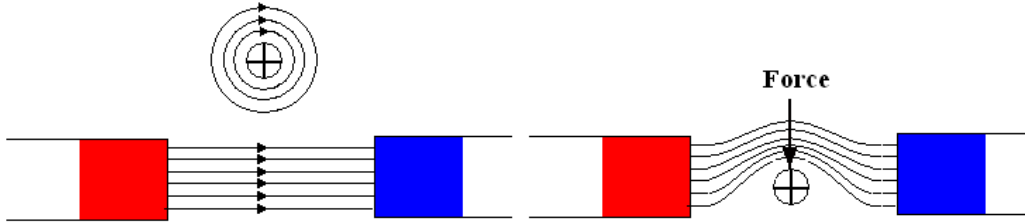
**Direct current (DC) motor:** type e.g. shunt, series, compound; construction e.g. windings, motor starter circuits, speed control (series resistance in the armature circuit); characteristics e.g. EMF generated, torque, back EMF, speed and power, efficiency

**Direct current (DC) generator:** type e.g. separately-excited, shunt, series compound; construction e.g. main frame or yolk, commutator, brushes, pole pieces, armature, field windings; characteristics e.g. generated voltage/field current (open circuit characteristics), terminal voltage/load current (load characteristic),  $V = E - I_a R_a$

You should have studied the basic principles of motors in the basic principles module.

## THE GENERAL PRINCIPLES OF D.C. MOTORS

A major discovery leading to the invention of the electric motor was that a conductor placed in a magnetic field experiences a force when current flows in it. Consider a conductor placed in a gap between the poles of a magnet. When current passes through the conductor, we have two magnetic fields, the circular lines around the conductor and the parallel lines between the poles. The lines of magnetism between the north and south poles would rather pass over the top of the conductor because both lines are in the same direction on top. The lines behave like elastic bands and force the conductor down. If the direction of either the current or the magnetic field is reversed, the force will act up.

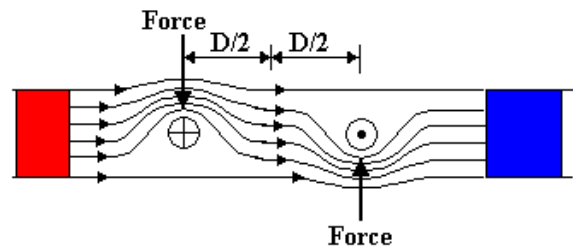


The force on the conductor is directly proportional to the current 'I', the magnetic flux density 'B' and the length 'l' of the conductor within the flux. This is the important equation for the force on a conductor.

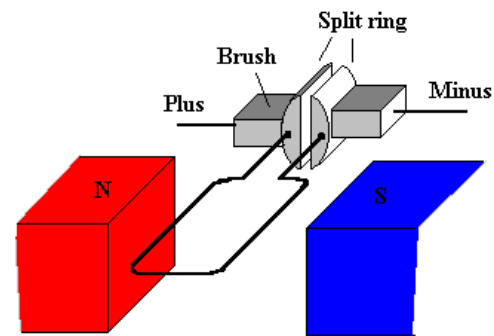
$$F = B l I$$

### TORQUE ON A LOOP

This principle may be used to produce a simple electric motor. Consider a single loop coil of diameter D placed in the magnetic field as shown. The current flows into one side and out of the other. This produces a downwards force on one side and an upwards force on the other at a radius D/2. This produces a torque on the coil of  $T = F D/2$ . This will make the loop rotate and but when it turns 90° the radius is zero and the torque is zero so it will stop.

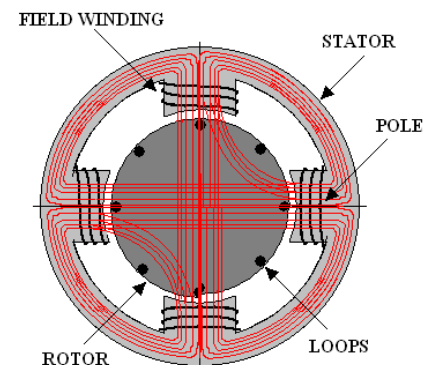


If we reverse the current as it passes the 90° position the torque will continue to make it rotate. Switching the direction of the current every half rotation will produce continuous rotation. This can be done with a split ring as shown.

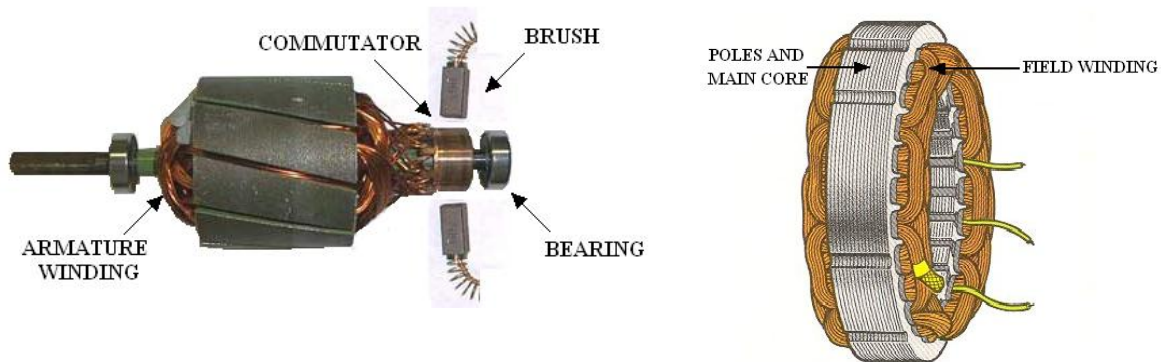


The design is further improved by using several loops and more magnets and switching the current to the loops within the flux. The split ring becomes a commutator with many segments. Each pair of segments is connected to a loop and each pair in turn becomes connected to the brushes as it rotates.

A practical D.C. motor has two main parts, the **STATOR** and **ROTOR**. Normally, many conductors are wound lengthwise in slots on the rotor as shown in the cross section. The magnets or poles are placed in the stator. When current is supplied to the loops, the magnetic field produced interacts with the field from the poles producing torque and rotation. In order to keep the process going, the current must be switched to the loops moving into position under the poles. This may be done with a **COMMUTATOR** and **BRUSHES** but it can be done electronically.



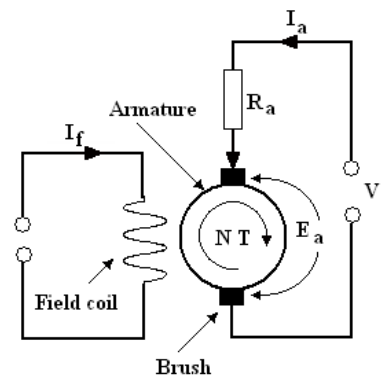
The pictures below show the main parts of a motor where the rotor is the armature.



The magnetic poles may be permanent magnets but in more powerful motors they are electro-magnets with coils wound on the poles called the **FIELD WINDINGS**. The stator and rotor form a magnetic circuit and the flux crosses the loops on the armature as shown. The strength of the magnetic flux can be changed by changing the current. The rotor in the diagram would be called the **ARMATURE**.

You can see the typical construction and explanation at this web location.  
[http://www.wisc-online.com/objects/index\\_tj.asp?objID=IAU9508](http://www.wisc-online.com/objects/index_tj.asp?objID=IAU9508)

The next diagram shows a typical schematic representation of a D.C. motor. Current has to be supplied to the field windings to produce the magnetic flux. Current must be supplied to the armature to produce the torque. Note the brushes used to contact armature.



**TERMINOLOGY USED**

- $I_a$  = armature Current
- $V$  = Supply voltage
- $E_a$  = EMF at the armature.
- $R_a$  = resistance of the armature coils.
- $E_a = V - I_a R_a$
- If the armature windings were perfect conductors then  $E_a = V$
- $I_f$  = Field current
- $T$  = Torque
- $N$  = rotor speed in rev/s

**THEORY**

The conductors in a motor pass through the flux from the poles and so an e.m.f. is generated in them that opposes the flow of current from the source. The voltage applied at the terminals to make the motor rotate is  $V$  and this must be larger than  $E$  to overcome the resistance of the armature and brushes so it follows that  $V = E_a + I_a R_a$

If the conductor is part of a loop of radius  $D/2$  (metres) then the torque produced is  $T = F D/2$  (N m)

The force is  $F = B \ell v$  and  $B = \phi/A$  where  $\phi$  is the flux in Weber and  $A$  the area of the flux path.

Substitute for  $F$  and  $B$  and  $T = \frac{\phi \ell I D}{2A}$

For a given motor the only variables are the flux and the armature current so the equation reduces to

$$T = k_1 \phi I_a \quad \dots\dots\dots (1)$$

The electric power consumed by the armature is  $P = E_a I_a$

The mechanical power produced is  $P = \omega T = 2\pi NT$

Where  $\omega$  is the speed in radians/s and  $N$  is the speed in rev/s. Note that  $\omega = 2\pi N$

Equating mechanical and electric power we have  $2\pi NT = I_a E_a$   
 $E_a = 2\pi NT / I_a$  .....(2a)  
 or  $T = I_a E_a / 2\pi N$  .....(2b)

Since  $T = k_1 \phi I_a$  we may substitute into (2a)  $E_a = k_1 \phi 2\pi N = \mathbf{K_2 \phi N}$  .....(2)

**LOSSES DUE TO ARMATURE RESISTANCE**

The armature circuit has a resistance mainly due to the resistance of the copper windings and brushes. This is denoted  $R_a$ . The emf ( $E_a$ ) of the armature is the terminal voltage  $V$  minus the resistance loss, hence:

$E_a = V - I_a R_a$  .....(3)

Substitute (2) for  $E_a$  and  $N = (V - I_a R_a) / 2\pi k_1 \phi$  .....(4)

Often the term  $I_a R_a$  is small in comparison to  $V$  and a close approximation is  $N = \frac{V}{k\phi}$

These equations are the basis of the following work.

It was shown in tutorial 2 that the e.m.f. of the armature depends on the number of poles and the number and type of windings on the armature. It was shown that :

$E = \frac{Z P N \phi}{60}$  for wave winding.  $E = \frac{Z N \phi}{60}$  for lap winding. For both, the only variables for

a given generator are the speed  $N$  and the flux  $\phi$ . We could say  $E = k N \phi$

$P$  is the number of pole pairs and  $Z$  is the total number of conductors.

$Z =$  number of slots x number of conductors per slot.

**WORKED EXAMPLE No. 1**

A d.c. motor is connected to a 400V supply and takes 40 Amps. The armature resistance is 0.3  $\Omega$ . The motor has 4 poles wave wound with 444 conductors. The useful flux per pole is 0.025 Wb. Calculate the e.m.f. on the armature and the motor speed.

$E = V - I_a R_a = 400 - (40 \times 0.3) = 388V$   
 $E = \frac{Z P N \phi}{60}$   $388 = \frac{444 \times 2 \times N \times 0.025}{60}$   $N = 1049 \text{ rev/min}$

**WORKED EXAMPLE No. 2**

A d.c. motor runs at 600 rev/min and is supplied at 300 V. Calculate the approximate speed when the supply is reduced to 200 V and the flux is reduced by 30%.

**SOLUTION**

$N_1 = 600 = \frac{V_1}{k\phi_1} = \frac{300}{k\phi_1}$   $k\phi_1 = \frac{300}{600}$   
 $N_2 = \frac{V_2}{k\phi_2} = \frac{200}{k\phi_1 \times 0.7} = \frac{200}{\frac{300}{600} \times 0.7} = 571 \text{ rev/min}$

Note the solution is unaffected by using rev/min

## MOTOR FIELD WINDING CONFIGURATIONS

There are various ways of arranging the field windings and armature but the two most common ways are in **SERIES**, **SHUNT** and a combination called **COMPOUND**. You need to study this carefully.

### SERIES MOTOR

In this case, the field winding is in series with the armature. The same current flows through the armature and the field winding.

The mechanical power of any rotor is  $P = 2\pi NT$

The electric power of any rotor is ideally  $P = E_a I_a$

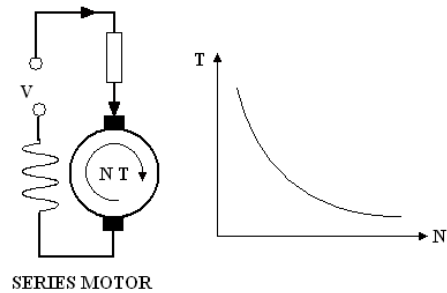
Equating and rearranging we see that  $T = E_a I_a / 2\pi N$

If the electric power is constant,  $E_a I_a$  are constant so  $T = \text{Constant} / N$

This means that for a constant electrical power the speed would increase as the load is removed and decrease as the load increases as shown by the graph.

**DISADVANTAGE** - At low torque (no load conditions) the motor is liable to over speed and become damaged.

**ADVANTAGE** - At low speed there is a high torque (starting torque) which is ideal for servo applications.



### WORKED EXAMPLE No. 3

A series wound motor produces 30 kW of mechanical power and is connected to a 250 V supply. The motor runs at 800 rev/min when under load. The load is reduced to 200 N m and a resistance of 0.5  $\Omega$  is connected in series with the motor. Assuming no energy losses, calculate the speed.

### SOLUTION

Initial conditions.

Motor current  $I = \text{Power/Volts} = 30\,000/250 = 120 \text{ A}$

Mechanical power  $= 2\pi NT/60 = 30\,000 \text{ W}$

$T = (30\,000 \times 60)/(2\pi \times 800) = 358 \text{ N m.}$

$T = K_1 \phi I_a$  Since flux is proportional to current and the field current is the armature current

then  $T = K I_a^2$

$358 = K \times 120^2$   $K = 0.02486$

Final conditions. Use the same value for K.  $T = 200 = 0.02486 I_a^2$

$I_a = 89.7 \text{ Amps}$  Input Power  $= V I_a = 250 \times 89.7 = 22\,423 \text{ Watts}$

Loss in 0.5  $\Omega$  resistance is  $I_a^2 R = 89.7^2 \times 0.5 = 4\,023 \text{ W}$

Useful Power  $= 22\,423 - 4\,023 = 18\,400 \text{ Watts} = 2\pi NT/60$

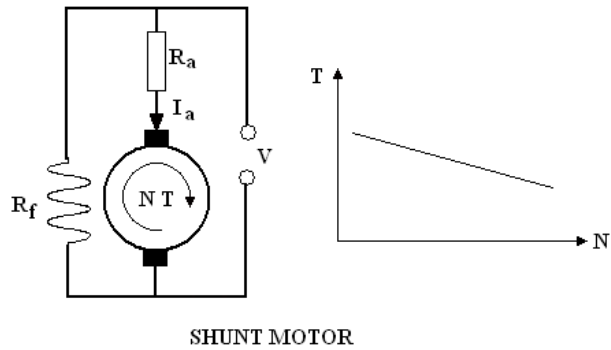
$N = (18\,400 \times 60)/(2\pi \times 200) = 878.5 \text{ rev/min}$

### **SELF ASSESSMENT EXERCISE No.1**

1. A series motor is connected to a 500 V supply. The motor power output is 10 kW at 1200 rev/min. The efficiency of the motor is 92 %.  
Determine the current and torque. **(21.74 A and 79.58 N m)**
2. A series motor has an armature resistance of 0.5 ohms and a field resistance of 1 ohm. It is connected to a 300 V supply and draws 50 Amperes from it. The motor runs at 1000 rev/min. The flux per pole is 0.03 Wb. The current is dropped to 25 Amperes and the flux per pole drops to 0.02 Wb. Determine the speed of the motor under these conditions. **(1 750rev/min)**
3. A 120 V, 20 kW series motor runs at 1200 rev/min on full load. A resistance of 0.6  $\Omega$  is connected in series with the motor and an output torque of 110 Nm is produced. Calculate the speed at this condition. It may be assumed that there are no energy losses and that the field flux is directly proportional to current. **(Ans. 443 rev/min)**
4. The armature of a 200 V d.c. motor has a resistance of 0.4  $\Omega$ . The no load armature current is 2.0 A. When a torque is applied, the armature current increases to 50 A, and the speed is 1200 rev/min. Find the no load speed stating any assumptions made. **(Ans.33200 rev/min)**

## SHUNT MOTOR

In this case the field winding is connected in parallel with the armature as shown. The field current is constant so flux  $\phi$  cannot be changed except by changing the supply voltage and so is regarded as constant.



$$E_a = V - I_a R_a$$

$$T = k_1 \phi I_a \quad \text{so} \quad I_a = T / \phi k_1$$

$$E_a = V - T R_a / \phi k_1 = k N \phi$$

$\phi$  is constant so everything is a constant except T and N.

For constant electrical power it follows that

$$T = C_1 - C_2 N$$

This shows that at zero speed the starting torque is  $C_1$  and as speed increases, the torque drops off. The ideal Torque - Speed characteristic is as shown. In reality the line is curved down due to other effects not considered.

### WORKED EXAMPLE No. 4

A D.C. motor is shunt wound and is supplied with 400 V. The armature resistance is  $1.2 \Omega$  and the field winding has a resistance of  $300 \Omega$ . When running with a torque of 70 Nm, the motor takes 19 Amps. Determine the speed of the motor assuming the electrical power is converted into mechanical power with 100% efficiency.

### SOLUTION

The field is connected across the supply. The current taken by the field winding is

$$I_f = 400/300 = 1.333 \text{ amp}$$

The current taken by the armature is hence  $19 - 1.333 = 17.666 \text{ A}$

$$E_a = V - I_a R_a = 400 - 17.666 \times 1.2 = 378.8 \text{ V}$$

$$\text{Electric power converted into mechanical power} = E_a I_a = 378.8 \times 17.666 = 6692 \text{ Watts}$$

Assuming the conversion process is 100% the Mechanical power = 6692 W

$$2\pi N T = 6692$$

$$N = 6692 / (2\pi T) = 6692 / (2\pi \times 70) = 15.21 \text{ rev/s or } 912.9 \text{ rev/min}$$

### WORKED EXAMPLE No. 5

It is observed that the same motor as in the previous example has a torque of 105 Nm when it stalls. Assuming the torque speed relationship is linear of the form  $T = C_1 - C_2 N$  determine the constants  $C_1$  and  $C_2$ .

### SOLUTION

$$\text{When } N = 0, \quad T = C_1 = 105 \text{ Nm}$$

$$\text{When } N = 15.21 \text{ rev/s, } T = 70 = 105 - C_2(15.21) \text{ hence } C_2 = 2.3 \text{ Nm s/rev}$$

### **WORKED EXAMPLE No. 6**

Part 1. A D.C. motor is shunt wound and is supplied with 500 V. The armature resistance is  $1.0 \Omega$  and the field winding has a resistance of  $500 \Omega$ . When running with a torque of 100 Nm, the motor takes 21 Amps. Determine the speed of the motor.

#### **SOLUTION**

The field is connected across the supply. The current taken by the field winding is

$$I_f = 500/500 = 1 \text{ amp}$$

The current taken by the armature is hence  $21 - 1 = 20$  amps.

$$E_a = V - I_a R_a = 500 - 20 \times 1 = 480 \text{ V}$$

$$\text{Electric power converted into mechanical power} = E_a I_a = 480 \times 20 = 9600 \text{ Watts}$$

Assuming the conversion process is 100% the Mechanical power = 9600

$$2\pi NT = 9600$$

$$N = 9600/(2\pi T) = 9600/(2\pi \times 100) = 15.27 \text{ rev/s or } 916.6 \text{ rev/min}$$

Part 2. The field current remains unchanged and the torque is increased to 120 Nm. Determine the new speed.

#### **SOLUTION**

Torque =  $K_1 \phi I_a$  Since the flux is constant then  $T = KI_a$

From the first set of data we have  $100 = K \times 20$  so  $K = 5$

For the second set of data we have  $120 = kI_a = 5 \times I_a$

$$I_a = 24 \text{ amps. } E = 500 - 24 \times 1 = 476 \text{ V}$$

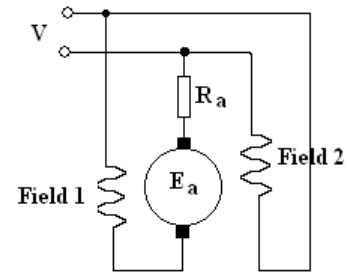
$$\text{Electric Power} = 476 \times 24 = 11\,424 \text{ Watts}$$

Equating to mechanical power  $11\,424 = 2\pi NT = 2\pi N \times 120$

$$N = 11\,424/(2\pi \times 120) = 15.15 \text{ rev/s or } 909 \text{ rev/min}$$

## COMPOUND MOTOR

The compound motor is a cross between the other two with both a parallel and series field winding. For constant electric power, the Torque - speed characteristic is between that of the other two.



### SELF ASSESSMENT EXERCISE No. 2

1. A 230 V d.c. motor has a separate field coil with a constant current of 2.2 A. The armature resistance is  $0.15 \Omega$ . The motor was tested with no load applied to the shaft. The armature current was found to vary with armature voltage as follows.

Current	4.9 A	4.5 A	4.0 A	3.6 A
Volts	230	180	130	80

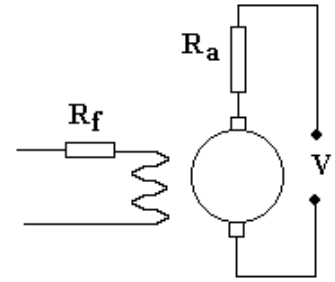
The speed at 230 V was 1150 rev/min. Determine the speeds at the other voltages.

(Ans. 890, 650, 400 rev/min)

2. The armature resistance of a 250 V d.c. shunt wound motor is  $0.7 \Omega$  and the armature current is 2.0 A when operating under no load conditions. A torque is then applied to the shaft and the armature current rises to 60 A and the speed falls to 1000 rev/min. Determine the no load speed.  
(Ans. 1200 rev/min)
3. Sketch the Speed - Torque characteristic for a series motor and shunt motor. Explain why the series motor is more suited to traction and winching operations.

## STARTING LARGE D.C. MOTORS

Consider a basic D.C Motor. The terminal voltage is  $V$ . The back e.m.f on the armature is  $E$ . The armature resistance is  $R_a$ . The field resistance is  $R_f$ . The flux per pole is  $\Phi$ .  $N$  is the motor speed.



From earlier work it was shown that  $V = E_a + I_a R_a$

$$E_a = K_1 N \Phi$$

$$T = K_2 \Phi I_f$$

When the motor is started, the speed is zero so there is no back e.m.f.

It follows that

$$V = I_a R_a$$

The starting current without protection would be  $V/R_a$  and this would be very large. In addition to this, there will be a load with inertia connected to the motor and a large current is needed to provide the torque.

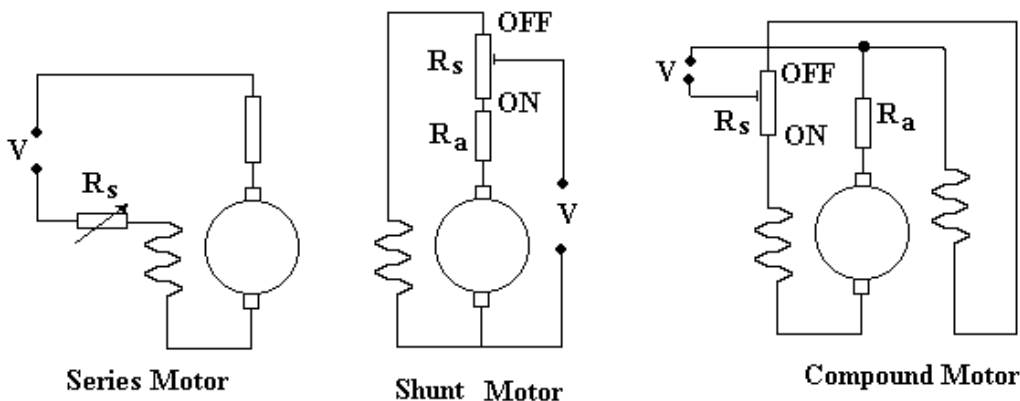
### WORKED EXAMPLE No. 7

A motor with a separate field coil has a terminal voltage  $V = 400 \text{ V}$  and an armature resistance of  $0.2 \Omega$ . Calculate the starting current in the armature.

### SOLUTION

$$I_a = V / R_a = 400 / 0.2 = 2000 \text{ Amps.}$$

In order to limit the current, it is normal to insert a variable resistance in series with the armature which is gradually reduced as the motor speeds up and then latched in place in the zero resistance position. In the event of an interruption to the power, the starting resistance is unlatched and springs back to the starting position. We should consider how the starting resistance is used with different motor field configurations. The three types or configurations are shown below.



### SERIES MOTOR

In the case of a series wound motor, the starting resistance is placed in series as shown.

### SHUNT MOTOR

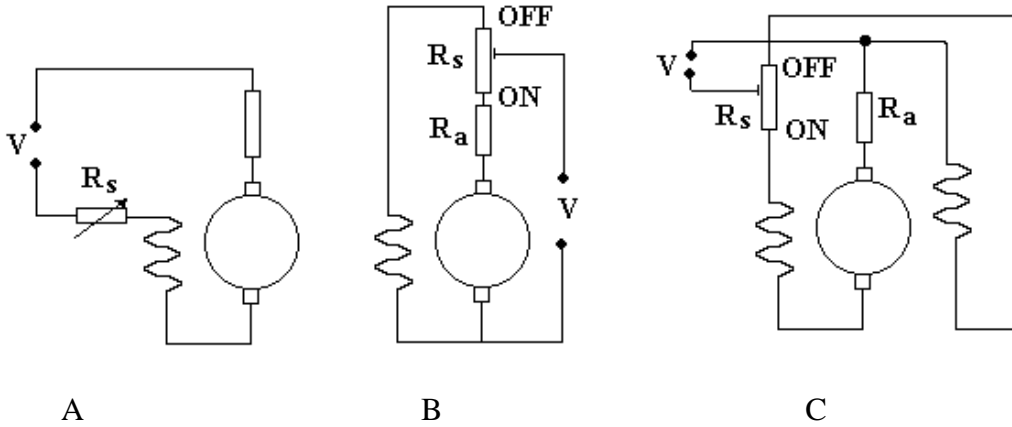
In the case of a shunt wound motor, the starting resistance is placed as shown so that the field is initially connected to the supply and the armature is in series with it. As the motor speeds up the field resistance is gradually increased and the resistance in series with the armature is reduced.

### COMPOUND MOTOR

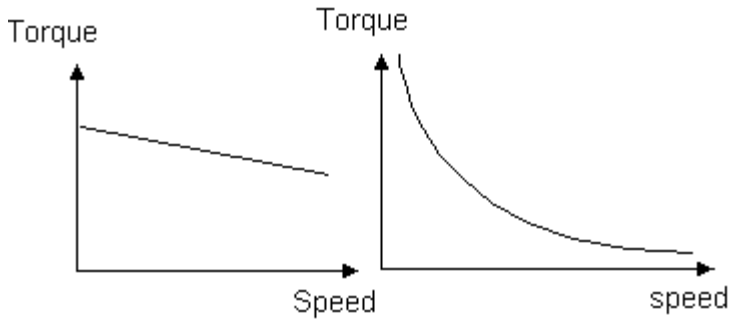
Compound motor has a starting resistance as shown.

**SELF ASSESSMENT EXERCISE No. 3**

1. Identify which of the diagrams shows a series, shunt and compound wound motor.



2. Identify which torque - speed graph represents series and shunt wound motors.



3. A 500V shunt wound d.c. motor has an armature resistance of  $0.5\Omega$  and a field coil resistance of  $250\Omega$ . Calculate the total current taken from the source at start up. What value resistance must be placed in series with the armature to reduce the starting current to 50A? (9.917  $\Omega$ )