EDEXCEL NATIONAL CERTIFICATE/DIPLOMA

UNIT 67 - FURTHER ELECTRICAL PRINCIPLES
NQF LEVEL 3

OUTCOME 1

TUTORIAL 1 - DIRECT CURRENT CIRCUIT THEOREMS

Unit content

1 **Be able to apply direct current (DC) circuit analysis methods and consider the types**, **construction and characteristics of a DC motor and generator**

Direct current (DC) circuit theorems: Thévenin's theorem e.g. application of theorem to a parallel circuit having two sources of electromotive force (EMF) and three resistors; Norton’s theorem e.g. application of theorem to a parallel circuit having two sources of EMF and three resistors; maximum power transfer theorem e.g. application of theorem to a series circuit with a source of EMF, internal resistance and a load resistor; application to a more complex circuit where Thevenin needs to be applied first

Direct current (DC) motor**:** type e.g. shunt, series, compound; construction e.g. windings, motor starter circuits, speed control (series resistance in the armature circuit); characteristics e.g. EMF generated, torque, back EMF, speed and power, efficiency

Direct current (DC) generator**:** type e.g. separately-excited, shunt, series compound; construction e.g. main frame or yolk, commutator, brushes, pole pieces, armature, field windings; characteristics e.g. generated voltage/field current (open circuit characteristics), terminal voltage/load current (load characteristic), $V = E - I_a R_a$

A full derivation of the equations used here may be found at the following web address

http://netlecturer.com/NTOnLine/T08_THEOREMS/p04NortonST.htm

DIRECT CURRENT (DC) CIRCUIT THEOREMS

CURRENT SOURCES AND EMF

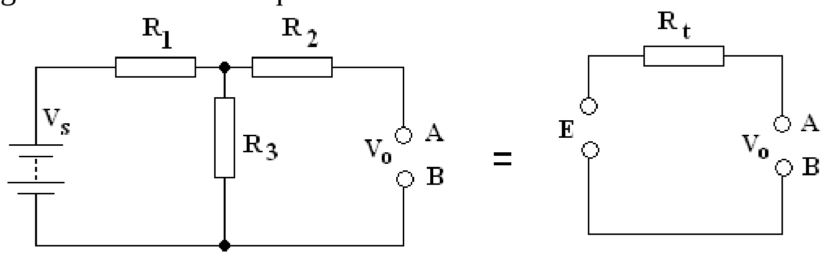
It's a good idea to make sure you understand what these are as they are vital in the following work.

An EMF (denoted E or e) is an ideal voltage source with no internal resistance so the voltage does not change with the current drawn from it. The current depends only on the load connected to it.

A current source is a place in a circuit where a current appears and this is fixed no matter what load the current flows into so the voltage depends on the load connected to it.

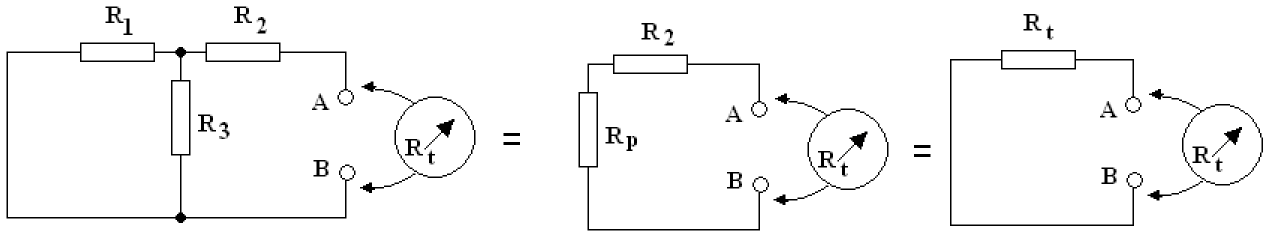
THÉVENIN'S THEOREM

Thévenin's theorem (also known as Helmholtz's thorem) is used to simplify a resistance network with voltage and current sources within it. We replace the network with a single emf 'E' and a resistance R_t in series with it. This is like replacing the network with a battery that contains internal reistance. The diagram illustrates the equivalent circuit.



Network   Equivalent Circuit

To determine R_t we evaluate the resistance as seen at the terminals A B with the voltage source replaced by a short circuit. (Note current sources are replaced with open circuit)

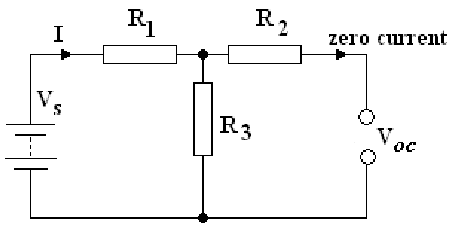


You should be able to deduce the resistance of this network.

First the parallel resistors $R_p = \dfrac{R_1 R_3}{R_1 + R_3}$

Then the total resistance $R_t = R_2 + \dfrac{R_1 R_3}{R_1 + R_3}$

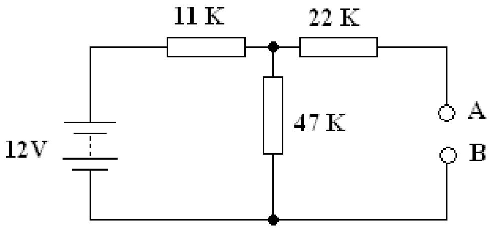
The EMF is the open circuit voltage. With no current at terminals A B the voltage is the voltage across R_3.



$$I = \frac{V_s}{R_1 + R_3} \qquad E = V_{oc} = IR_3 \qquad E = \frac{R_3 V_s}{R_1 + R_3}$$

WORKED EXAMPLE No. 1

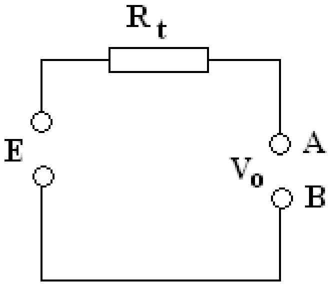
For the circuit shown, find the values in Thevenin's equivalent circuit. If a load of 10 kΩ is attached to terminals AB, what is the current drawn and voltage.



SOLUTION

$$R_t = R_2 + \frac{R_1R_3}{R_1 + R_3} = 22 + \frac{11 \text{ x } 47}{11 + 47} = 30.91\,\text{k}\Omega$$

$$E = \frac{R_3V_s}{R_1 + R_3} = \frac{47 \text{ x } 12}{11 + 47} = 9.724\text{ V}$$



When a 10 KΩ load is placed across AB the total resistance is 30.91 + 10 = 40.91 kΩ.

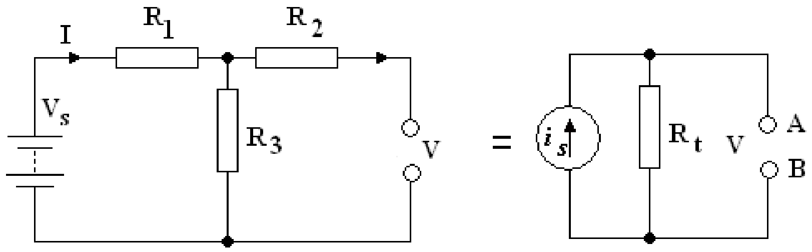
The current is I = 9.724/40.91 = 0.238 mA

The voltage is V = 0.238 mA x 10 kΩ = 2.38 V

You can check out solutions at this web address:
http://hyperphysics.phy-astr.gsu.edu/hbase/electric/Thevenin.html
You will find an animated explanation at this address.
http://www.wisc-online.com/objects/index_tj.asp?objID=DCE5803

NORTON'S THEOREM

In this theorem the network is replaced with a current source i_s in parallel with a resistor R_t. Consider the same circuit as before.



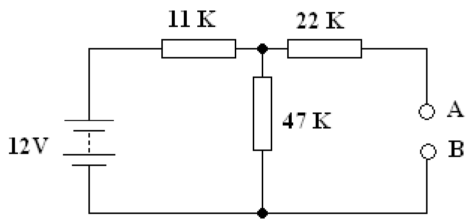
The resistance R_t is calculated as before. $R_t = R_2 + \frac{R_1 R_3}{R_1 + R_3}$

The current source $\boldsymbol{i_s}$ is found by determining the open circuit voltage at the terminals AB and dividing it by R_t.

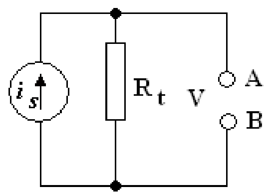
As before $V(\text{open circuit}) = \frac{R_3 V_s}{R_1 + R_3}$ hence $i_s = \frac{V(\text{o.c.})}{R_t}$ Note this stays constant what ever the current drawn at AB.

WORKED EXAMPLE No. 2

Determine the current source for the Norton equivalent circuit of the same network (example 1) and show the voltage and current at the terminals is the same as before when a 10 kΩ load is connected across AB.



SOLUTION



$$R_t = R_2 + \frac{R_1 R_3}{R_1 + R_3} = 22 + \frac{11 \text{ x } 47}{11 + 47} = 30.91 \text{ k}\Omega$$

$$V(o.c.) = \frac{R_3 V_s}{R_1 + R_3} = \frac{47 \text{ x } 12}{11 + 47} = 9.724 \text{ V}$$

$$i_s = \frac{V(\text{o.c.})}{R} = \frac{9.724}{30.91} = 0.315 \text{ mA}$$

$I_1 + I_2 = 0.315$ mA

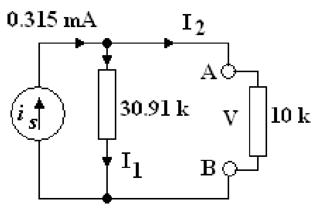
$V = I_1 \text{ x } 30.91 = I_2 \text{ x } 10$

$(0.315 - I_2) \text{x } 30.91 = I_2 \text{ x } 10$

$0.315 = 1.325\, I_2$

$I_2 = 0.238$ mA

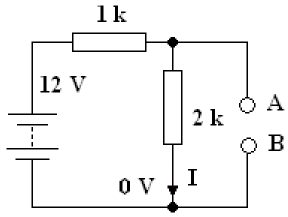
V = 10 kΩ x0.238 mA = 2.38 V



You can check out solutions at this web address:
http://hyperphysics.phy-astr.gsu.edu/hbase/electric/Norton.html

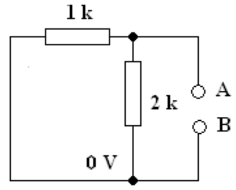
You can find an animated explanation of Norton's theorem at this web address
http://www.wisc-online.com/objects/index_tj.asp?objID=DCE10004

WORKED EXAMPLE No. 3



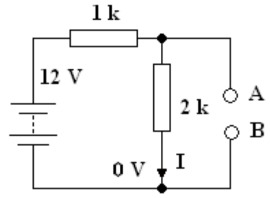
Use Thévenin's theory to simplify the network shown.
A load resistor of 0.933 kΩ is placed across AB.
Determine the terminal voltage and current drawn.

SOLUTION



To find the Thévenin resistance R we make the voltage source a closed circuit and calculate R at A-B.

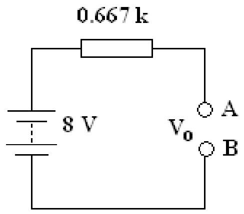
$$R_t = \frac{1 \text{ x } 2}{1+2} = 0.667 \text{ k}\Omega$$



To find the E calculate the open circuit voltage.

$$I = \frac{12}{2+1} = 4 \text{ mA} \quad E = 4 \text{ mA x } 2 \text{ k}\Omega = 8 \text{ V}$$

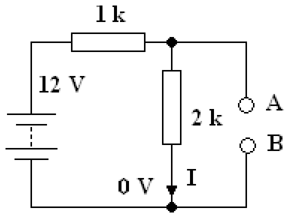
The equivalent circuit is then as shown.



When 0.933 kΩ is placed across AB the load current drawn is I_L = 8/(0.667 + 0.933) = 5 mA

The voltage drop over the resistor is 5 mA x 0.933 kΩ = 4.67 V

WORKED EXAMPLE No. 4



Repeat the last example using Norton's theorem.

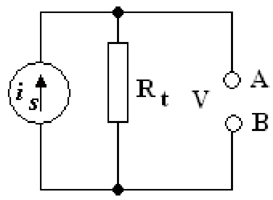
SOLUTION

As before $R_t = \frac{1 \text{ x } 2}{1+2} = 0.667 \text{ k}\Omega$

Next find V(oc) With an open circuit the current flowing in both resistors is 12/3 = 4 mA

V(oc) = 2 kΩ x 4 mA = 8 V

Now find i_s $\quad i_s = \frac{V(oc)}{R_t} = \frac{8}{0.667} = 12 \text{ mA}$



The equivalent circuit is then as shown.
The current source delivers 12 mA into a parallel resistor network.

$$R_p = \frac{0.667 \text{ x } 0.933}{0.667 + 0.933} = 0.3889 \text{ k}\Omega$$

The voltage at AB is hence i_s x R_p = 12 x 0.3889 = 4.67 V

The current into the load is I_L = 4.667/0.933 = 5 mA

In the last example we could have calculated the load current as follows. The voltage across the current source is :

$$V_s = I_L R_L = IR_t = (i_s - I_L)R_t$$

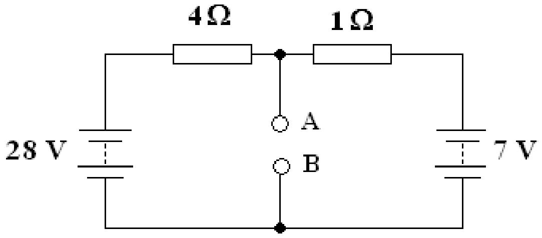
$$I_L R_L = (R_t i_s - R_t I_L) \qquad I_L R_L + R_t I_L = R_t i_s$$

$$I_L(R_L + R_t) = R_t i_s \qquad I_L = \frac{R_t i_s}{R_L + R_t}$$

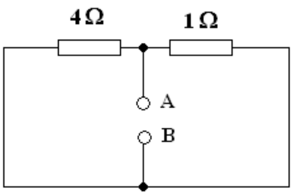
Hence in the example $I_L = \frac{0.667 \text{ x } 12}{0.933 + 0.667} = 5 \text{ mA}$

<u>WORKED EXAMPLE No. 5</u>

Using Thévenin's therem, calculate the voltage and current at A-B when a load resistor of 1Ω is placed across them.



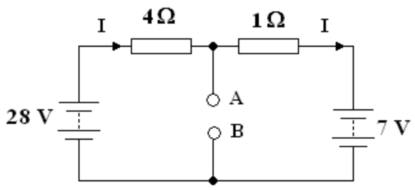
<u>SOLUTION</u>



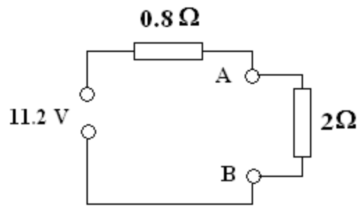
<u>Step 1</u> Find R_t by replacing the voltage sources with a short circuit and finding the resistance at A-B.

$$R_t = \frac{4 \text{ x } 1}{4+1} = 0.8\,\Omega \quad \text{(Parallel resistors)}$$

<u>Step 2</u> Find the open circuit voltage at AB



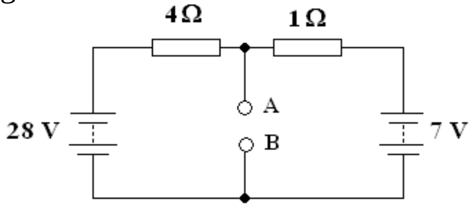
The voltage difference across the resistors is 28 - 7 - 21 V
I = 21/5 = 4.2 A
The voltage at A-B is 28 - (4.2 x 4) = 11.2 V
Or
7 + (4.2 x 1) = 11.2 V



The equivalent circuit with a 2Ω load resistor is shown.
The current drawn is 11.2/2.8 = 4 A
The terminal voltage at A-B is 4 x 2 = 8 V

WORKED EXAMPLE No. 6

Repeat the last problem using Norton's theorem.



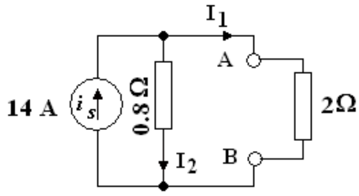
SOLUTION

Step 1 Find R_t as before $R_t = \frac{4 \times 1}{4+1} = 0.8\,\Omega$

Step 2 Find the open circuit voltage at AB as before is 11.2 V

Step 2 Find the source current i_s = 11.2/0.8 = 14 A
Note this is the current at A-B when the terminals are shorted and may be found as:

i_s = 28/4 + 7/1 = 14 A

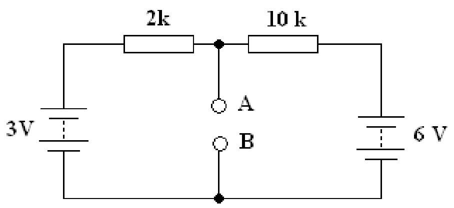


The equivalent circuit with the load resistor is shown.
The terminal voltage is $V = 0.8\ I_2 = 2\ I_1$
$I_1 + I_2 = 14$ $\qquad I_2 = 14 - I_1$
$0.8\ (14 - I_1) = 2\ I_1$
$11.2 = 2.8\ I_1$
$I_1 = 4$ A
V = 4A x 2Ω = 8 V

Or we could solve as follows
$R_p = \frac{0.8 \times 2}{0.8+2} = 0.571\Omega$ $\quad V_{AB}$ = 0.571 x 14 = 8 V $\quad I_1$ = 8/2 = 4 A

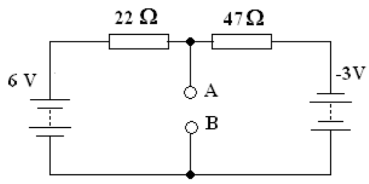
SELF ASSESSMENT EXERCISE No.1

1. Using both Thévenin's and Norton's theorem, determine the load current and terminal voltage when a resistor of 4.7 kΩ is placed across A-B in the circuit shown.



(Ans. 0.55 mA and 2.58 V)

2. Using both Thévenin's and Norton's theorem, determine the load current and terminal voltage when a resistor of 10 Ω is placed across A-B in the circuit shown.

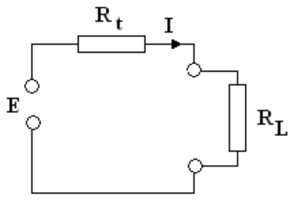


(Ans. 125.3 mA and 1.253 V)

MAXIMUM POWER TRANSFER THEOREM

In the context of this work, this states that the maximum power that can be obtained from a source occurs when the load resistance is the same as the source.

Suppose the source is reduced to an emf and series resistance as shown in the previous work using Thévenin's theorem.



$I = \frac{E}{R_t + R_L}$ $P = I^2 R_L$ (Dissipated in the load resistor)

$$P = \left(\frac{E}{R_t + R_L}\right)^2 R_L = \frac{ER_L}{R_t^2 + R_L^2 + 2R_tR_L} = \frac{E}{\frac{R_t^2}{R_L} + R_L + 2R_t}$$

For the next section you need to know about maxima and minima theory.

P is a maximum when $\frac{R_t^2}{R_L} + R_L + 2R_t$ is a minimum

Let $X = R_t^2 R_L^{-1} + R_L + 2R_t$

Differentiate with respect to R_L

$$\frac{dX}{dR_L} = -R_t^2 R_L^{-2} + 1$$

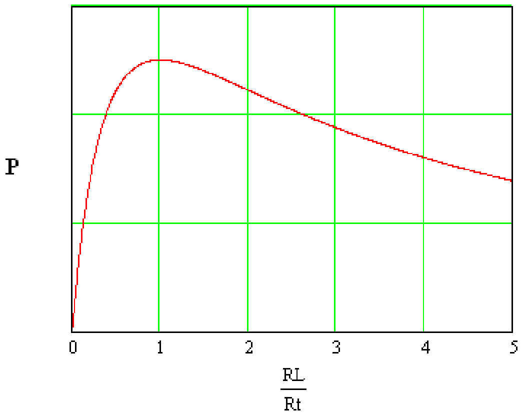
Equate to zero and X is a minima when $R_t^2 R_L^{-2} = 1$

Hence $R_t^2 = R_L^2$

$R_t = R_L$

For a more complete proof we would have to demonstrate that this gives a minima and not a maxima. It follows then that when $R_t = R_L$ the power is a maximum.

The theorem can be demonstrated graphically by plotting P against R_L/R_t for any value of E.



It can be seen that the max occurs when $R_L/R_t = 1$

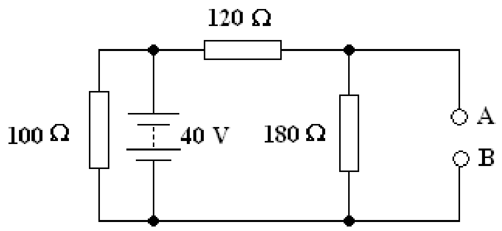
The power dissipated by the source resistance R_t is equal to P so it follows that at maximum power the efficiency is 50%.

You will find a good demonstration of this at the following web address

http://www.wisc-online.com/objects/index_tj.asp?objID=DCE9904

WORKED EXAMPLE No. 7

Find the value of R_L in the circuit below that will dissipate most power. What is the maximum power dissipated in R_L?

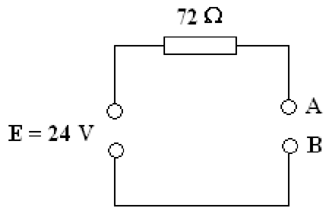


SOLUTION

To find Thévenin's resistance find the resistance at AB with the voltage source replaced by a short circuit. In this case the 100Ω resistor has no affect and the other two are in parallel.

$$R_t = \frac{120 \text{ x } 180}{120 + 180} = 72\Omega$$

Maximum power will be dissipated in the load when $R_L = R_t = 72\ \Omega$
The open circuit voltage at AB is found as follows.
I = 40/(120 + 180) = 0.1333 A
E = V(oc) = 0.1333 x 180 = 24 V

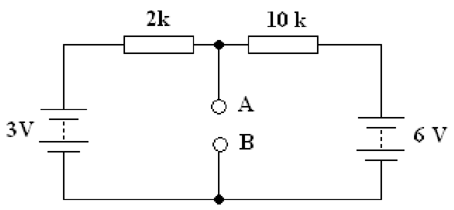


When the load resistor is connected the current flowing through it is 24/(72 + 72) = 0.167 A
$P = I^2R = (0.167)^2 \text{ x } 72 = 2$ W
The same power is dissipated in R_t.

SELF ASSESSMENT EXERCISE No.2

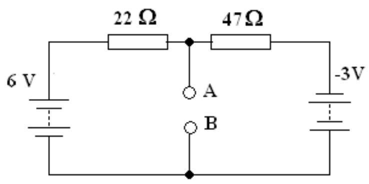
Note these are the same circuits as in exercise No. 1 so you have already done most of the work.

1. Determine the load resistor to be used at AB in order to draw maximum power from the source. What is the power?



(Ans. 1667 Ω and 1.834 x10^{-3} W)

2. Determine the load resistor to be used at AB in order to draw maximum power from the source. What is the power?



(Ans. 15 Ω and 0.163 W)