FLUID MECHANICS

TUTORIAL No.8B

CENTRIFUGAL PUMPS

When you have completed this tutorial you should be able to

- Derive the dimensionless parameters of a pump
  - Flow Coefficient
  - Head Coefficient
  - Power Coefficient
  - Specific Speed.
- Explain how to match a pump to system requirements.
- Explain the general principles of Centrifugal Pumps.
- Construct blade vector diagrams for Centrifugal Pumps.
- Deduce formulae for power and efficiency and Head.
- Solve numerical problems for Centrifugal Pumps.

1. DIMENSIONAL ANALYSIS

The power 'P' of any rotary hydraulic pump depends upon the density 'ρ' , the speed 'N', the characteristic diameter 'D', the head change 'ΔH', the volume flow rate 'Q' and the gravitational constant 'g'. The general equation is:

$$P = f(\rho, N, D, \Delta H, Q, g)$$

It is normal to consider gΔH as one quantity. $P = f\{\rho, N, D, (g\Delta H), Q\}$

There are 6 quantities and 3 dimensions so there are three dimensionless groups Π_1, Π_2 and Π_3. First form a group with P and ρND.

$$P = \varphi(\rho ND) = \Pi_1 \rho^a N^b D^c$$

$$M^1L^2T^{-3} = (ML^{-3})^a (T^{-1})^b (D^1)^c$$

Mass $1 = a$

Time $-3 = -b \qquad b = 3$

Length $2 = -3a + c = -3 + c \quad c = 5$

$$P = \Pi_1 \rho^1 N^3 D^5 \qquad \Pi_1 = \frac{P}{\rho \ N^3 D^5} = \text{ Power Coefficient}$$

Next repeat the process between Q and ρND

$$Q = \varphi(\rho ND) = \Pi_2 \rho^a N^b D^c$$

$$M^3T^{-1} = (ML^{-3})^a (T^{-1})^b (D^1)^c$$

Time $-1 = -b \qquad b = 1$

Mass $0 = a$

Length $3 = -3a + c \quad c = 3$

$$Q = \Pi_2 \rho^0 N^1 D^3 \qquad \Pi_2 = \frac{Q}{ND^3} = \text{ Flow Coefficient}$$

Next repeat the process between gΔH and ρND

$$(g\,\Delta H) = \varphi(\rho ND) = \Pi_3 \rho^a N^b D^c$$

$$M^0L^2T^{-2} = (ML^{-3})^a (T^{-1})^b (D^1)^c$$

Mass $0 = a$ Time $-2 = -b \qquad b = 2$ Length $2 = -3a + c \quad c = 2$

$$Q = \Pi_3 \rho^0 N^2 D^2 \qquad \Pi_3 = \frac{Q}{N^2D^2} = \text{ Head Coefficient}$$

Finally the complete equation is $\dfrac{P}{\rho N^3 D^5} = \varphi\left(\dfrac{Q}{ND^3}\right)\left(\dfrac{g\,\Delta H}{N^2D^2}\right)$

SPECIFIC SPEED Ns

The specific speed is a parameter used for pumps and turbines to determine the best design to match a given pumped system. The formula may be derived from consideration of the pump geometry or by dimensional analysis. The latter will be used here.

$$\frac{P}{\rho N^3 D^5} = \varphi\left(\frac{Q}{ND^2}\right)\left(\frac{g\,\Delta H}{N^2 D^2}\right)$$

The three dimensionless numbers represent the power coefficient, the flow coefficient and the head coefficient respectively. Now consider a family of geometrically similar machines operating at dynamically similar conditions. For this to be the case the coefficients must have the same values for each size. Let the 3 coefficients be Π_1, Π_2 and Π_3 such that

$$\Pi_1 = \frac{P}{\rho N^3 D^5} \qquad \Pi_2 = \frac{Q}{ND^3} \quad D = \left(\frac{Q}{N\Pi_2}\right)^{\frac{1}{3}} \qquad \Pi_3 = \frac{g\,\Delta H}{N^2 D^2} \quad D = \left(\frac{g\,\Delta H}{N^2 \Pi_3}\right)^{\frac{1}{2}}$$

Equating $\left(\dfrac{Q}{N\Pi_2}\right)^{\frac{1}{3}} = \left(\dfrac{g\,\Delta H}{N^2\Pi_3}\right)^{\frac{1}{2}}$

$$\frac{1}{N}\left(\frac{g\,\Delta H}{\Pi_3}\right)^{\frac{1}{2}} = \frac{Q^{\frac{1}{3}}}{\Pi_2^{\frac{1}{3}} N^{\frac{1}{3}}}$$

$$\frac{(\Delta H)^{\frac{1}{2}}}{Q^{\frac{1}{3}} N^{\frac{2}{3}}} = \frac{\Pi_3^{\frac{1}{2}}}{\Pi_2^{\frac{1}{3}} g^{\frac{1}{2}}} = \text{constant}$$

$$\frac{(\Delta H)^{\frac{1}{2}}}{KQ^{\frac{1}{3}}} = N^{\frac{2}{3}}$$

$$\left[\frac{(\Delta H)^{\frac{1}{2}}}{KQ^{\frac{1}{3}}}\right]^{\frac{3}{2}} = N = \frac{(\Delta H)^{\frac{3}{4}}}{K^{\frac{1}{2}} Q^{\frac{1}{2}}} \qquad \frac{NQ^{\frac{1}{2}}}{(\Delta H)^{\frac{3}{4}}} = K^{-\frac{1}{2}} = \text{constant}$$

This constant is called the Specific Speed $Ns = \dfrac{NQ^{\frac{1}{2}}}{(\Delta H)^{\frac{3}{4}}}$

Ns is a dimensionless parameter that and the units used are normally rev/min for speed, m^3/s for flow rate and metres for head. Other units are often used and care should be taken when quoting Ns values.

It follows that for a given speed, the specific speed is large for large flows and low heads and small for small flows and large heads. The important value is the one that corresponds to the conditions that produce the greatest efficiency.

2. MATCHING PUMPS TO SYSTEM REQUIREMENTS

The diagram shows a typical relationship between the head and flow of a given CF pump at a given speed.

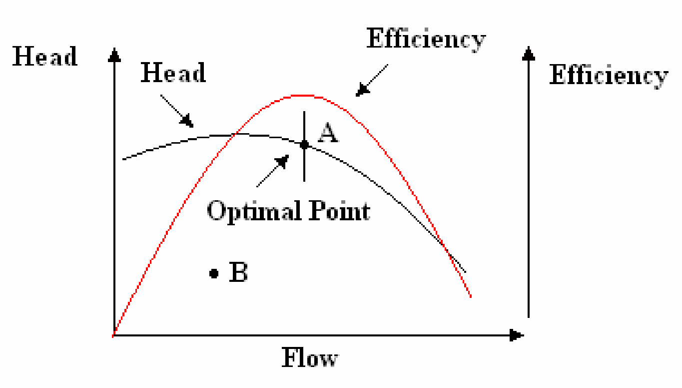


Figure 1

The Ns value may be calculated using the flow and head corresponding to the maximum efficiency at point A.

SELECTING PUMP SIZE

The problem is that the optimal point of any given pump is unlikely to correspond to the system requirements for example at point B. What we should do ideally is find a geometrically similar pump that will produce the required head and flow at the optimal point.

The geometrically similar pump will run under dynamically similar conditions so it follows that the specific speed Ns is the same for both pumps at the optimal point. The procedure is to first calculate the specific speed of the pump using the flow and head at the optimal conditions.

$$N_s = \frac{N_A Q_A^{\frac{1}{2}}}{H_A^{\frac{3}{4}}}$$

Suppose point B is the required operating point defined by the system.

$N_s = \dfrac{N_B Q_B^{\frac{1}{2}}}{H_B^{\frac{3}{4}}}$ Equating, we can calculate N_B, the speed of the geometrically similar pump.

We still don't know the size of the pump that will produce the head and flow at B. Since the head and flow coefficients are the same then:-

Equating Flow Coefficients we get $D_B = D_A\left(\dfrac{Q_B N_A}{Q_A N_B}\right)^{1/3}$

Equating head coefficients we get we get $D_B = \dfrac{N_A}{N_B}\sqrt{\dfrac{H_B}{H_A}}$

If the forgoing is correct then both will give the same answer.

WORKED EXAMPLE No. 1

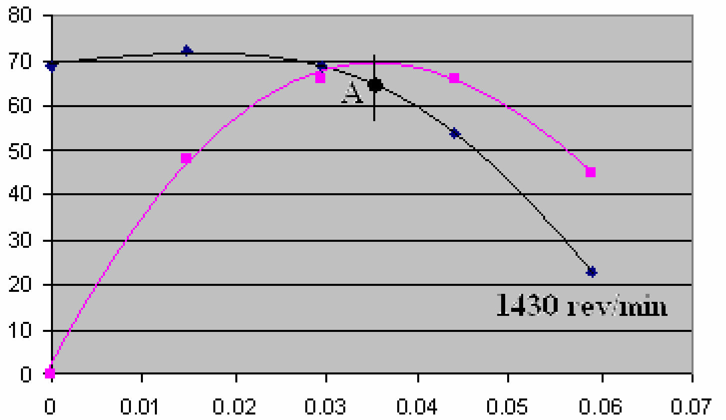
A centrifugal pump is required to produce a flow of water at a rate of 0.0160 m^3/s against a total head of 30.5 m. The operating characteristic of a pump at a speed of 1430 rev/min and a rotor diameter of 125 mm is as follows.

Efficiency	0	48	66	66	45	%
Q_A	0	0.0148	0.0295	0.0441	0.059	m^3/s
H_A	68.6	72	68.6	53.4	22.8	m

Determine the correct size of pump and its speed to produce the required head and flow.

SOLUTION

Plot the data for the pump and determine that the optimal head and flow are 65 m and 0.036m^3/s



Calculate Ns at point A $N_s = \frac{N_A Q_A^{\frac{1}{2}}}{H_A^{\frac{3}{4}}} = \frac{1430 \text{ x } 0.036^{1/2}}{65^{3/4}} = 11.85$

Calculate the Speed for a geometrically similar pump at the required conditions.

$$N_B = Ns\frac{H_B^{\frac{3}{4}}}{Q_B^{\frac{1}{2}}} = \frac{11.85 \text{ x } 30.5^{3/4}}{0.016^{1/2}} = 1216 \text{ rev/min}$$

Next calculate the diameter of this pump.

$$D_B = D_A\left(\frac{Q_B N_A}{Q_A N_B}\right)^{1/3} = 125\left(\frac{0.016 \text{ x } 1430}{0.036 \text{ x } 1216}\right)^{1/3} = 101 \text{ mm}$$

or $D_B = D_A \frac{N_A}{N_B}\sqrt{\frac{H_B}{H_A}} = 125 \text{ x } \frac{1430}{1216}\sqrt{\frac{30.5}{65}} = 101 \text{ mm}$

Answer:- we need a pump 101 mm diameter running at 1216 rev/min.

RUNNING WITH THE WRONG SIZE

In reality we are unlikely to find a pump exactly the right size so we are forced to use the nearest we can get and adjust the speed to obtain the required flow and head. Let B be the required operating point and A the optimal point for the wrong size pump. We make the flow and head coefficients the same for B and some other point C on the operating curve. The diameters cancel because they are the same.

$$\frac{Q_B}{N_B D_B^3} = \frac{Q_C}{N_C D_C^3} \qquad Q_B = Q_C \frac{N_B}{N_C} \qquad \frac{g\,H_C}{N_C^2 D_C^2} = \frac{g\,H_A}{N_C^2 D_C^2} \qquad H_B = H_C \frac{N_B^2}{N_C^2}$$

Substitute $\frac{N_B}{N_A} = \frac{Q_B}{Q_A}$ to eliminate the speed

$$H_C = H_B \left(\frac{Q_C}{Q_B}\right)^2$$

This is a family of parabolic curves starting at the origin. If we take the operating point B we can determine point C as the point where it intersects the operating curve at speed A.
The important point is that the efficiency curve is unaffected so at point B the efficiency is not optimal.

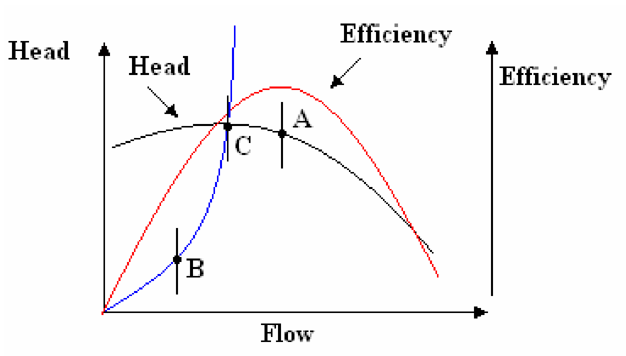


Figure 2

WORKED EXAMPLE No. 2

If only the 125 mm pump in WE 1 is available, what speed must it be run at to obtain the required head and flow? What is the efficiency and input power to the pump?

SOLUTION

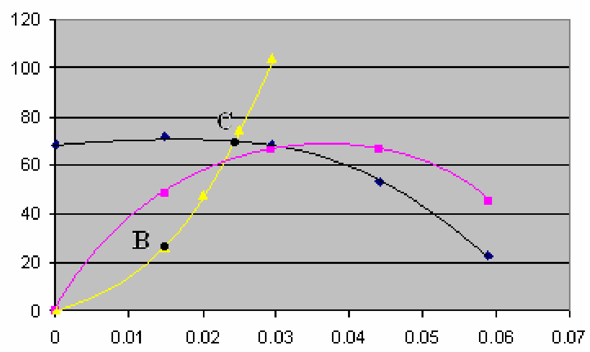
B is the operating point so we must calculate H_C and Q_C

$$H_C = H_B \left(\frac{Q_C}{Q_B}\right)^2 = 30.5\left(\frac{Q_C}{0.016}\right)^2 = 119141 Q_C^2$$

This must be plotted to determine Q_C
From the plot H_C = 74 m
Q_C = 0.025 m^3/s
Equate flow coefficients to find the speed at B

$$\frac{Q_B}{N_B D_B^3} = \frac{Q_C}{N_C D_C^3} \qquad \frac{0.016}{N_B} = \frac{0.025}{1430}$$

N_B = 915 rev/min



Check by repeating the process with the head coefficient.

$$\frac{g\,H_B}{N_B^2 D_B^2} = \frac{g\,H_A}{N_A^2 D_A^2} \qquad N_B = N_A\sqrt{\frac{H_B}{H_A}} = 1430\sqrt{\frac{30.5}{74}} = 918 \text{ rev/min}$$

The efficiency at this point is 63% Water Power = mgH = 16 x 9.81 x 30.5 = 4787 W

Power Input = WP/η = 4787/0.63 =7598 W

WORKED EXAMPLE No.3

A pump draws water from a tank and delivers it to another with the surface 8 m above that of the lower tank. The delivery pipe is 30 m long, 100 bore diameter and has a friction coefficient of 0.003. The pump impeller is 500 mm diameter and revolves at 600 rev/min. The pump is geometrically similar to another pump with an impeller 550 mm diameter which gave the data below when running at 900 rev/min.

ΔH (m)	37	41	44	45	42	36	28
$Q(m^3/s)$	0	0.016	0.32	0.048	0.064	0.08	0.096

Determine the flow rate and developed head for the pump used.

SOLUTION

First determine the head flow characteristic for the system.
ΔH = developed head of the pump = $8 + 4fLu^2/2gd$ + minor losses
No details are provided about minor losses so only the loss at exit may be found.
$h_L = 4fLu^2/2gd + u^2/2g$
$\Delta H = = 8 + 4fLu^2/2gd + u^2/2g$

$u = 4Q/\pi d^2 = 127.3\ Q$
$\Delta H = 8 + 4x\ 0.003 \text{ x } 30\ 000(127.3Q)^2/(2g \text{ x } 0.1) + (127.3Q)^2/2g$
$\Delta H = 8 + 3800Q^2$

Produce a table and plot ΔH against Q for the system.

ΔH (m)	8	8.38	14.08	32.3	46
$Q(m^3/s)$	0	0.01	0.04	0.08	0.1

Plot the system head and pump head against flow and find the matching point.
This is at H = 34.5 and Q = 0.084 m^3/s

Next determine the head - flow characteristic for the pump actually used by assuming dynamic and geometric similarity.

Flow Coefficient Q/ND^3 = constant
$Q_2 = Q_2\ (N_1/N_2)(D_1{}^3/D_2{}^3)$ $\qquad$ $Q_2 = (600/900)\ (500/550)^3 = 0.5\ Q_1$
$\Delta H/(ND)^2$ = constant
$\Delta H_2 = \Delta H_1(N_2D_2/N_1D_1)^2$ $\qquad$ $\Delta H_2 = \Delta H_1\{600 \text{ x } 500/900 \text{ x } 550\}^2 = 0.367\ \Delta H_1$

Produce a table for the pump using the coefficients and data for the first pump.

ΔH_2 (m)	13.58	15.05	16.15	16.51	15.41	13.21	10.28
$Q_2(m^3/s)$	0	0.08	0.016	0.024	0.032	0.04	0.048

Plot this graph along with the system graph and pick off the matching point.

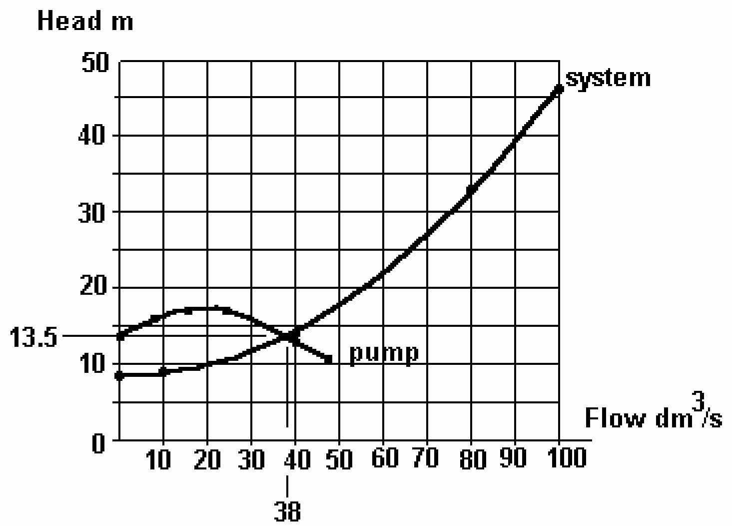


Figure 3

Ans. 13.5 m head and 38 dm^3/s flow rate.

SELF ASSESSMENT EXERCISE No.1

1. A centrifugal pump must produce a head of 15 m with a flow rate of 40 dm^3/s and shaft speed of 725 rev/min. The pump must be geometrically similar to either pump A or pump B whose characteristics are shown in the table below.

   Which of the two designs will give the highest efficiency and what impeller diameter should be used ?

   Pump A D = 0.25 m N = 1 000 rev/min

Q (dm^3/s)	8	11	15	19
H (m)	8.1	7.9	7.3	6.1
η%	48	55	62	56

   Pump B D = 0.55 m N = 900 rev/min

Q (dm^3/s)	6	8	9	11
H (m)	42	36	33	27
η%	55	65	66	58

   Answer Pump B with D= 0.455 m

2. Define the Head and flow Coefficients for a pump.

   Oil is pumped through a pipe 750 m long and 0.15 bore diameter. The outlet is 4 m below the oil level in the supply tank. The pump has an impeller diameter of 508 mm which runs at 600 rev/min. Calculate the flow rate of oil and the power consumed by the pump. It may be assumed $C_f=0.079(Re)^{-0.25}$. The density of the oil is 950 kg/m^3 and the dynamic viscosity is 5×10^{-3} N s/m^2.

   The data for a geometrically similar pump is shown below.

   D = 0.552 m N = 900 rev/min

Q (m^3/min)	0	1.14	2.27	3.41	4.55	5.68	6.86
H (m)	34.1	37.2	39.9	40.5	38.1	32.9	25.9
η%	0	22	41	56	67	72	65

   Answer 2 m^3/min and 7.89 kWatts

3. GENERAL THEORY

A Centrifugal pump is a Francis turbine running backwards. The water between the rotor vanes experiences centrifugal force and flows radially outwards from the middle to the outside. As it flows, it gains kinetic energy and when thrown off the outer edge of the rotor, the kinetic energy must be converted into flow energy. The use of vanes similar to those in the Francis wheel helps. The correct design of the casing is also vital to ensure efficient low friction conversion from velocity to pressure. The water enters the middle of the rotor without swirling so we know v_{w1} is always zero for a c.f. pump. Note that in all the following work, the inlet is suffix 1 and is at the inside of the rotor. The outlet is suffix 2 and is the outer edge of the rotor.

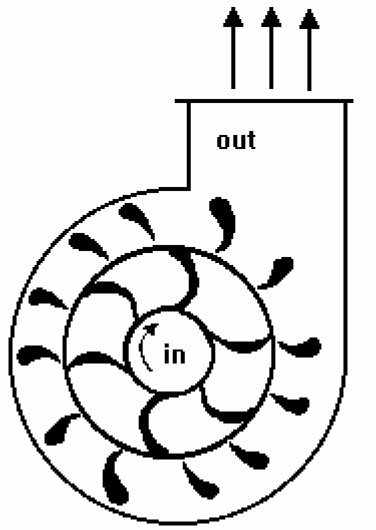


Fig. 1 Basic Design

The increase in momentum through the rotor is found as always by drawing the vector diagrams. At inlet v_1 is radial and equal to v_{r1} and so v_{w1} is zero. This is so regardless of the vane angle but there is only one angle which produces shockless entry and this must be used at the design speed.

At outlet, the shape of the vector diagram is greatly affected by the vane angle. The diagram below shows a typical vector diagram when the vane is swept backwards (referred to the vane velocity u).

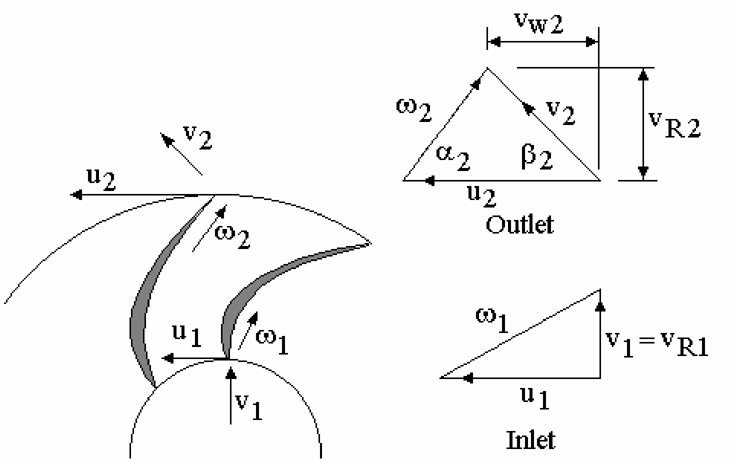


Fig. 2

v_{w2} may be found by scaling from the diagram. We can also apply trigonometry to the diagram as follows.

$$v_{w2} = u_2 - \frac{v_{r2}}{\tan\alpha_2} = u_2 - \frac{Q}{A_2\tan(\alpha_2)} = u_2 - \frac{Q}{\pi k D_2 t_2 \tan(\alpha_2)} \qquad u_2 = \pi N D_2$$

t is the height of the vane and k is the correction factor for the blade thickness.

3.1. DIAGRAM POWER D.P.= $m\Delta u v_W$

since usually v_{W1} is zero this becomes D.P.= $m u_2 v_{W2}$

3.2. WATER POWER W.P. = $mg\Delta h$

Δh is the pressure head rise over the pump.

3.3. MANOMETRIC HEAD Δh_m

This is the head that would result if all the energy given to the water is converted into pressure head. It is found by equating the diagram power and water power.

$$m u_2 v_{w2} = mg\Delta g_m \qquad \Delta h_m = \frac{u_2 v_{w2}}{g} = \frac{u_2}{g}\left\{u_2 - \frac{Q}{A_2 \tan(\alpha_2)}\right\}$$

3.4. MANOMETRIC EFFICIENCY η_m

$$\eta_m = \frac{\text{Water Power}}{\text{Diagram Power}} = \frac{mg\,\Delta h}{m u_2 v_{w2}} = \frac{mg\,\Delta h}{mg\,\Delta h_m} = \frac{\Delta h}{\Delta h_m}$$

3.5. SHAFT POWER S.P. = $2\pi NT$

3.6. OVERALL EFFICIENCY $\eta_{o/a} = \dfrac{\text{Water Power}}{\text{Shaft Power}}$

3.7. KINETIC ENERGY AT ROTOR OUTLET $\text{K.E.} = \dfrac{m v_2^2}{2}$

Note the energy lost is mainly in the casing and is usually expressed as a fraction of the K.E. at exit.

3.8. NO FLOW CONDITION

There are two cases where you might want to calculate the head produced under no flow condition. One is when the outlet is blocked say by closing a valve, and the other is when the speed is just sufficient for flow to commence.

Under normal operating conditions the developed head is given by the following equation.

$$\Delta h = \frac{u_2 v_{w2}}{g} = \frac{u_2}{g}\left\{u_2 - \frac{Q}{A_2 \tan(\alpha_2)}\right\}$$

When the outlet valve is closed the flow is zero. The developed head is given by the following equation. $\Delta h = \dfrac{u_2}{g}\{u_2 - 0\} = \dfrac{u_2^2}{g}$

When the speed is reduced until the head is just sufficient to produce flow and overcome the static head, the radial velocity v_{r2} is zero and the fluid has a velocity u_2 as it is carried around with the rotor. The kinetic energy of the fluid is $\dfrac{m u_2^2}{2}$ and this is converted into head equal to the static head.

It follows that $h_s = \dfrac{u_2^2}{2g}$. Substituting $u_2 = \dfrac{\pi N D_2}{60}$ we find that $N = 83.5\dfrac{\sqrt{h_s}}{D}$.

WORKED EXAMPLE No.4

A centrifugal pump has the following data :

Rotor inlet diameter	D_1 = 40 mm
Rotor outlet diameter	D_2 = 100 mm
Inlet vane height	h_1 = 60 mm
Outlet vane height	h_2 = 20 mm
Speed	N =1420 rev/min
Flow rate	Q = 0.0022 m^3/s
Blade thickness coefficient	k = 0.95

The flow enters radially without shock.
The blades are swept forward at 30^o at exit.
The developed head is 5 m and the power input to the shaft is 170 Watts.

Determine the following.

i. The inlet vane angle
ii. The diagram power
iii. The manometric head
iv. The manometric efficiency
v. The overall efficiency.
vi. The head produced when the outlet valve is shut.
vii. The speed at which pumping commences for a static head of 5 m.

SOLUTION

$u_1 = \pi ND_1$ = 2.97 m/s
$u_2 = \pi ND_2$ = 7.435 m/s
$v_{r1} = Q/k\pi D_1 h_1$ = 0.307 m/s
$v_{r2} = Q/k\pi D_2 h_2$ = 0.368 m/s
Since the flow enters radially $v_1 = v_{r1}$ = 0.307 m/s and v_{w1} = 0
From the inlet vector diagram the angle of the vane that produces no shock is found as follows:
$\tan\alpha_1$ = 0.307/2.97 hence $\alpha_1 = 5.9^o$.

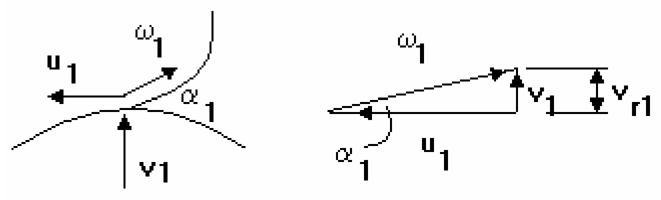


Fig. 41
Inlet vector diagram

From the outlet vector diagram we find :

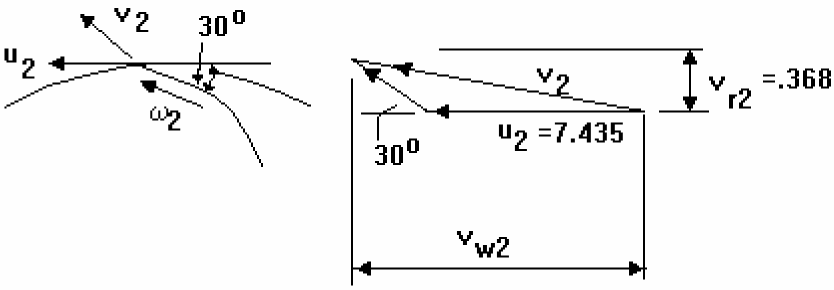


Fig. 42
Outlet vector diagram

v_{w_2} = 7.435 + 0.368/tan 30o = 8.07 m/s
D.P.= $mu_2v_{w_2}$
D.P.= 2.2 x 7.435 x 8.07 = 132 Watts
W.P. = $mg\Delta h$ = 2.2 x 9.81 x 5 = 107.9 Watts
Δh_m = W.P./D.P.= 107.9/132 = 81.7%
$\Delta h_m = u_2v_{w_2}/g$ = 7.435 x 8.07/9.81 =6.12 m
$\Delta_m = \Delta h/\Delta h_m$ = 5/6.12 = 81.7%
$\eta_{o/a}$ = W.P./S.P. = 107.9/170 = 63.5%

When the outlet valve is closed the static head is $\Delta h = \frac{u_2^2}{g} = \frac{7.435^2}{9.81} = 5.63\ \text{m}$

The speed at which flow commences is $N = 83.5\frac{\sqrt{h_s}}{D} = 83.5\frac{\sqrt{5}}{0.1} = 1867$ rev/min

SELF ASSESSMENT EXERCISE No. 2

1. The rotor of a centrifugal pump is 100 mm diameter and runs at 1 450 rev/min. It is 10 mm deep at the outer edge and swept back at 30^{o}. The inlet flow is radial. the vanes take up 10% of the outlet area. 25% of the outlet velocity head is lost in the volute chamber. Estimate the shut off head and developed head when 8 dm^3/s is pumped. (5.87 m and 1.89 m)

2. The rotor of a centrifugal pump is 170 mm diameter and runs at 1 450 rev/min. It is 15 mm deep at the outer edge and swept back at 30^{o}. The inlet flow is radial. the vanes take up 10% of the outlet area. 65% of the outlet velocity head is lost in the volute chamber. The pump delivers 15 dm^3/s of water.

   Calculate
   i. The head produced. (9.23 m)
   ii. The efficiency. (75.4%)
   iii. The power consumed. (1.8 kW)