FLUID MECHANICS

TUTORIAL No.8A

WATER TURBINES

When you have completed this tutorial you should be able to

- Explain the significance of specific speed to turbine selection.

- Explain the general principles of
  Pelton Wheels
  Kaplan Turbines
  Francis Turbine

- Construct blade vector diagrams for moving vanes for a Pelton Wheels and a Francis Turbine

- Deduce formulae for power and efficiency for turbines.

- Solve numerical problems for a Pelton Wheels and a Francis Turbine

1. INTRODUCTION

A water turbine is a device for converting water (fluid) power into shaft (mechanical) power. A pump is a device for converting shaft power into water power.

Two basic categories of machines are the rotary type and the reciprocating type. Reciprocating motors are quite common in power hydraulics but the rotary principle is universally used for large power devices such as on hydroelectric systems.

Large pumps are usually of the rotary type but reciprocating pumps are used for smaller applications.

1.1 THE SPECIFIC SPEED FOR VARIOUS TYPES OF TURBINES

The power 'P' of any rotary hydraulic machine (pump or motor) depends upon the density 'ρ' , the speed 'N', the characteristic diameter 'D', the head change 'ΔH', the volume flow rate 'Q' and the gravitational constant 'g'. The general equation is:

$$P = f(\rho, N, D, \Delta H, Q, g)$$

It is normal to consider gΔH as one quantity. $P = f\{\rho, N, D, (g\Delta H), Q\}$

There are 6 quantities and 3 dimensions so there are three dimensionless groups Π_1, $\Pi_2$and Π_3. First form a group with P and ρND.

$$P = \varphi(\rho ND) = \Pi_1 \rho^a N^b D^c$$

$$M^1L^2T^{-3} = (ML^{-3})^a (T^{-1})^b (D^1)^c$$

Mass $1 = a$ Time $-3 = -b$ $b = 3$ Length $2 = -3a + c = -3 + c$ $c = 5$

$$P = \Pi_1 \rho^1 N^3 D^5 \qquad \Pi_1 = \frac{P}{\rho\ N^3 D^5} = \text{Power Coefficient}$$

Next repeat the process between Q and ρND

$$Q = \varphi(\rho ND) = \Pi_2 \rho^a N^b D^c$$

$$M^3T^{-1} = (ML^{-3})^a (T^{-1})^b (D^1)^c$$

Time $-1 = -b$ $b = 1$ Mass $0 = a$ Length $3 = -3a + c$ $c = 3$

$$Q = \Pi_2 \rho^0 N^1 D^3 \qquad \Pi_2 = \frac{Q}{ND^3} = \text{Flow Coefficient}$$

Next repeat the process between gΔH and ρND

$$(g\,\Delta H) = \varphi(\rho ND) = \Pi_3 \rho^a N^b D^c$$

$$M^0L^2T^{-2} = (ML^{-3})^a (T^{-1})^b (D^1)^c$$

Mass $0 = a$ Time $-2 = -b$ $b = 2$ Length $2 = -3a + c$ $c = 2$

$$Q = \Pi_3 \rho^0 N^2 D^2 \qquad \Pi_3 = \frac{Q}{N^2 D^2} = \text{Head Coefficient}$$

Finally the complete equation is $\frac{P}{\rho N^3 D^5} = \varphi\left(\frac{Q}{ND^2}\right)\left(\frac{g\,\Delta H}{N^2 D^2}\right)$

SPECIFIC SPEED Ns

The specific speed is a parameter used for pumps and turbines to determine the best design to match a given pumped system. The formula may be derived from consideration of the pump geometry or by dimensional analysis. The latter will be used here.

$$\frac{P}{\rho N^3 D^5} = \varphi\left(\frac{Q}{ND^2}\right)\left(\frac{g\,\Delta H}{N^2 D^2}\right)$$

The three dimensionless numbers represent the Power coefficient, the flow coefficient and the Head coefficient respectively. Now consider a family of geometrically similar machines operating at dynamically similar conditions. For this to be the case the coefficients must have the same values for each size. Let the 3 coefficients be Π_1, Π_2 and Π_3 such that

$$\Pi_1 = \frac{P}{\rho N^3 D^5} \qquad \Pi_2 = \frac{Q}{ND^3} \quad D = \left(\frac{Q}{N\Pi_2}\right)^{\frac{1}{3}} \qquad \Pi_3 = \frac{g\,\Delta H}{N^2 D^2} \quad D = \left(\frac{g\,\Delta H}{N^2 \Pi_3}\right)^{\frac{1}{2}}$$

Equating $\left(\frac{Q}{N\Pi_2}\right)^{\frac{1}{3}} = \left(\frac{g\,\Delta H}{N^2 \Pi_3}\right)^{\frac{1}{2}}$ $\qquad \frac{1}{N}\left(\frac{g\,\Delta H}{\Pi_3}\right)^{\frac{1}{2}} = \frac{Q^{\frac{1}{3}}}{\Pi_2^{\frac{1}{3}} N^{\frac{1}{3}}}$

$$\frac{(\Delta H)^{\frac{1}{2}}}{Q^{\frac{1}{3}} N^{\frac{2}{3}}} = \frac{\Pi_3^{\frac{1}{2}}}{\Pi_2^{\frac{1}{3}} g^{\frac{1}{2}}} = \text{constant} \qquad \frac{(\Delta H)^{\frac{1}{2}}}{KQ^{\frac{1}{3}}} = N^{\frac{2}{3}}$$

$$\left[\frac{(\Delta H)^{\frac{1}{2}}}{KQ^{\frac{1}{3}}}\right]^{\frac{3}{2}} = N = \frac{(\Delta H)^{\frac{3}{4}}}{K^{\frac{1}{2}} Q^{\frac{1}{2}}} \qquad \frac{NQ^{\frac{1}{2}}}{(\Delta H)^{\frac{3}{4}}} = K^{-\frac{1}{2}} = \text{constant} \qquad N_s = \frac{NQ^{\frac{1}{2}}}{(\Delta H)^{\frac{3}{4}}}$$

Ns is a dimensionless parameter that and the units used are normally rev/min for speed, m^3/s for flow rate and metres for head. Other units are often used and care should be taken when quoting Ns values.

It follows that for a given speed, the specific speed is large for large flows and low heads and small for small flows and large heads. The important value is the one that corresponds to the conditions that produce the greatest efficiency. The diagram illustrates how the design affects the specific speed.

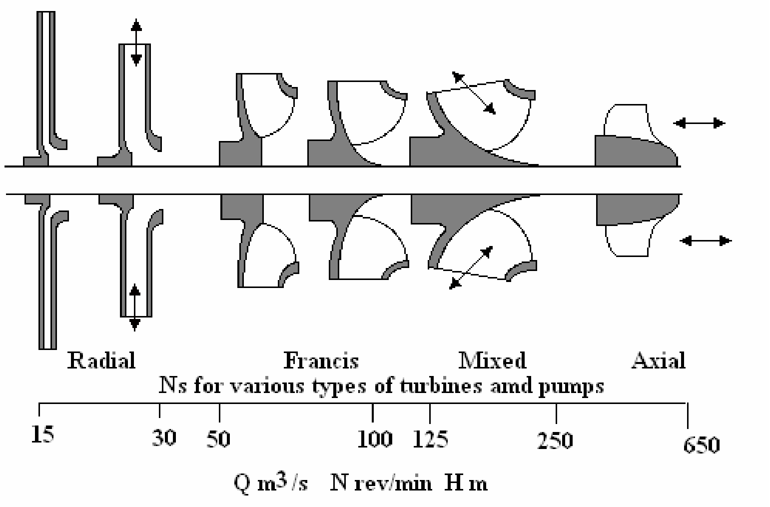


Figure 1

2. GENERAL PRINCIPLES OF TURBINES.

WATER POWER

This is the fluid power supplied to the machine in the form of pressure and volume.

Expressed in terms of pressure head the formula is W.P. = $mg\Delta H$

M is the mass flow rate in kg/s and ΔH is the pressure head difference over the turbine in metres. Remember that $\Delta p = \rho g \Delta H$

Expressed in terms of pressure the formula is W.P. = $Q\Delta p$

Q is the volume flow rate in m^3/s. Δp is the pressure drop over the turbine in N/m^2 or Pascals.

SHAFT POWER

This is the mechanical, power output of the turbine shaft. The well known formula is

$$S.P. = 2\pi NT$$

Where T is the torque in Nm and N is the speed of rotation in rev/s

DIAGRAM POWER

This is the power produced by the force of the water acting on the rotor. It is reduced by losses before appearing as shaft power. The formula for D.P. depends upon the design of the turbine and involves analysis of the velocity vector diagrams.

HYDRAULIC EFFICIENCY

This is the efficiency with which water power is converted into diagram power and is given by

$$\eta_{hyd} = D.P./W.P.$$

MECHANICAL EFFICIENCY

This is the efficiency with which the diagram power is converted into shaft power. The difference is the mechanical power loss.

$$\eta_{mech} = S.P./D.P.$$

OVERALL EFFICIENCY

This is the efficiency relating fluid power input to shaft power output.

$$\eta_{o/a} = S.P./W.P.$$

It is worth noting at this point that when we come to examine pumps, all the above expressions are inverted because the energy flow is reversed in direction.

The water power is converted into shaft power by the force produced when the vanes deflect the direction of the water. There are two basic principles in the process, ***IMPULSE and REACTION.***

IMPULSE occurs when the direction of the fluid is changed with no pressure change. It follows that the magnitude of the velocity remains unchanged.

REACTION occurs when the water is accelerated or decelerated over the vanes. A force is needed to do this and the reaction to this force acts on the vanes.

Impulsive and reaction forces are determined by examining the changes in velocity (magnitude and direction) when the water flows over the vane. The following is a typical analysis.

The vane is part of a rotor and rotates about some centre point. Depending on the geometrical layout, the inlet and outlet may or may not be moving at the same velocity and on the same circle. In order to do a general study, consider the case where the inlet and outlet rotate on two different diameters and hence have different velocities.

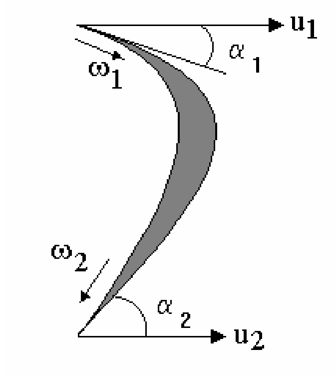


Fig.2

u_1 is the velocity of the blade at inlet and u_2 is the velocity of the blade at outlet. Both have tangential directions.ω_1 is the relative velocity at inlet and ω_2 is the relative velocity at outlet.

The water on the blade has two velocity components. It is moving tangentially at velocity u and over the surface at velocity ω. The absolute velocity of the water is the vector sum of these two and is denoted v. At any point on the vane $v = \omega + u$

At inlet, this rule does not apply unless the direction of v_1 is made such that the vector addition is true. At any other angle, the velocities will not add up and the result is chaos with energy being lost as the water finds its way onto the vane surface. The perfect entry is called ***"SHOCKLESS ENTRY"*** and the entry angle β_1 must be correct. This angle is only correct for a given value of v_1.

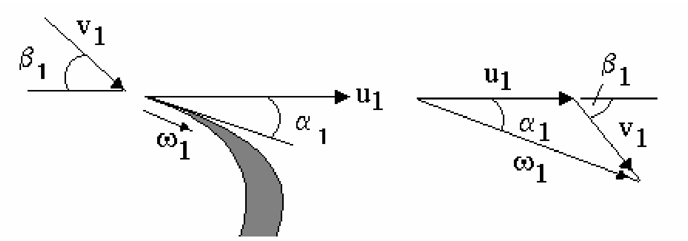


Fig.3

INLET DIAGRAM

For a given or fixed value of u_1 and v_1, shockless entry will occur only if the vane angle α_1 is correct or the delivery angle β_1 is correct. In order to solve momentum forces on the vane and deduce the flow rates, we are interested in two components of v_1. These are the components in the direction of the vane movement denoted v_w (meaning velocity of whirl) and the direction at right angles to it v_R (meaning radial velocity but it is not always radial in direction depending on the wheel design). The suffix (1) indicates the entry point. A typical vector triangle is shown.

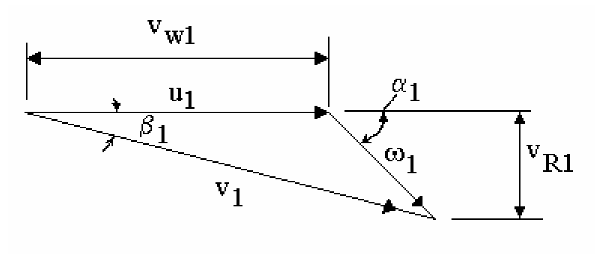


Fig.4

OUTLET DIAGRAM

At outlet, the absolute velocity of the water has to be the vector resultant of u and ω and the direction is unconstrained so it must come off the wheel at the angle resulting. Suffix (2) refers to the outlet point. A typical vector triangle is shown.

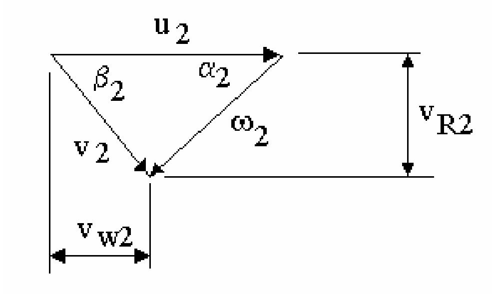


Fig. 5

DIAGRAM POWER

Diagram power is the theoretical power of the wheel based on momentum changes in the fluid. The force on the vane due to the change in velocity of the fluid is $F = m\Delta v$ and these forces are vector quantities. m is the mass flow rate. The force that propels the wheel is the force developed in the direction of movement (whirl direction). In order to deduce this force, we should only consider the velocity changes in the whirl direction (direction of rotation) Δv_w. The power of the force is always the product of force and velocity. The velocity of the force is the velocity of the vane (u). If this velocity is different at inlet and outlet it can be shown that the resulting power is given by

$$D.P. = m\,\Delta v_w = m\,(u_1 v_{w1} - u_2\, v_{w2})$$

3. PELTON WHEEL

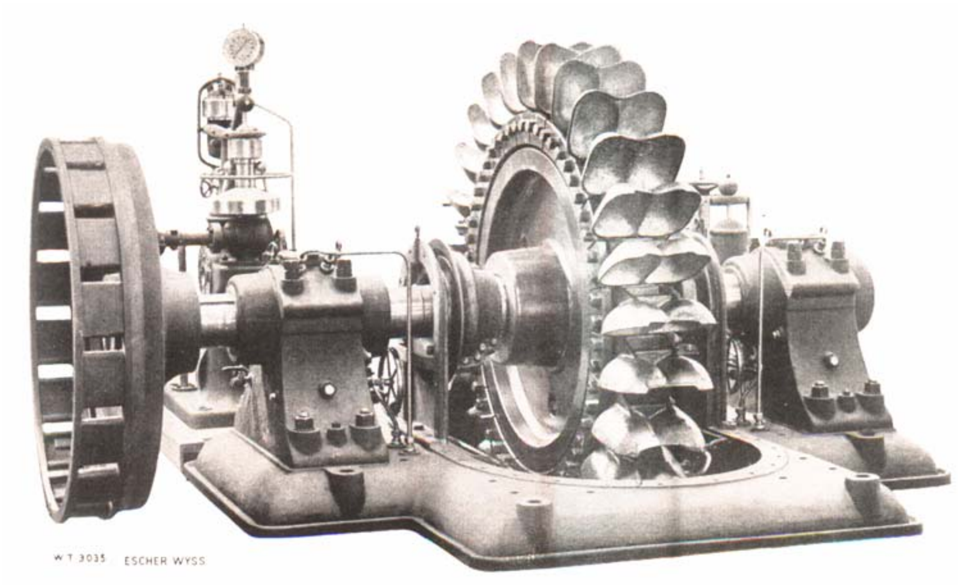


Fig. 6 Pelton Wheel With Case Removed

Pelton wheels are mainly used with high pressure heads such as in mountain hydroelectric schemes. The diagram shows a layout for a Pelton wheel with two nozzles.

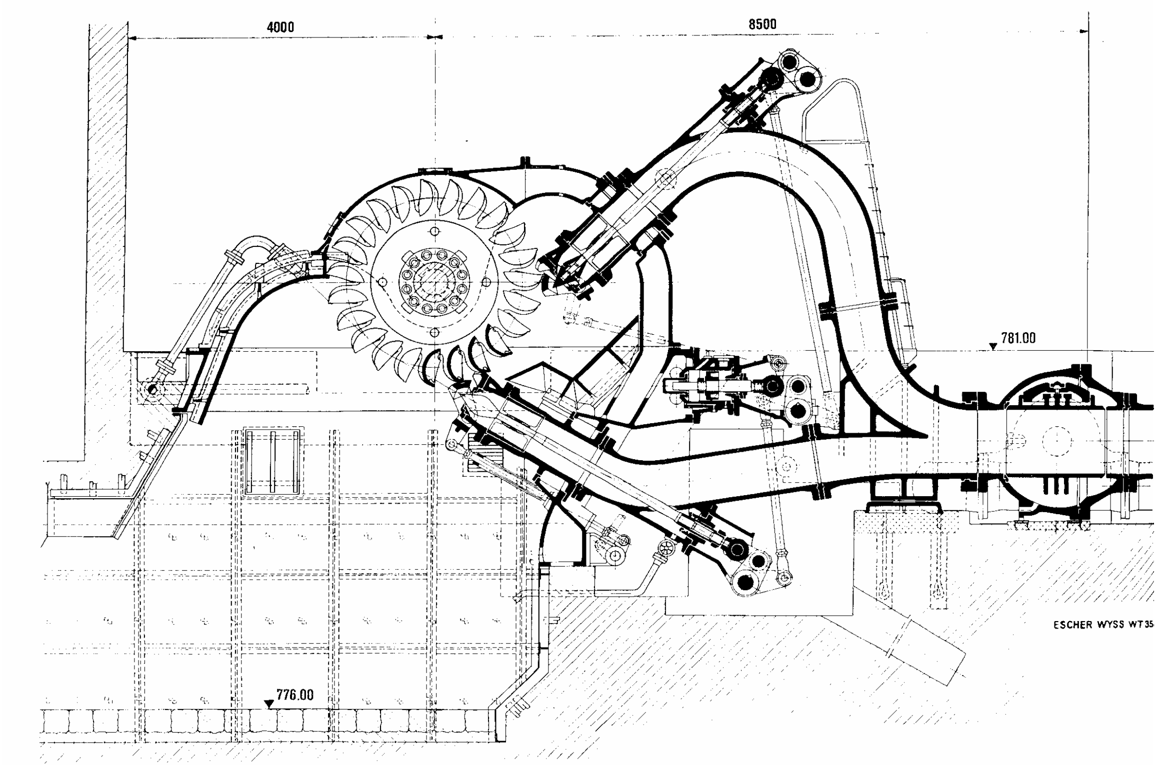


Fig.7 Schematic Diagram Of Pelton Wheel With Two Nozzles

3.1 GENERAL THEORY

The Pelton Wheel is an impulse turbine. The fluid power is converted into kinetic energy in the nozzles. The total pressure drop occurs in the nozzle. The resulting jet of water is directed tangentially at buckets on the wheel producing impulsive force on them. The buckets are small compared to the wheel and so they have a single velocity $u = \pi ND$
D is the mean diameter of rotation for the buckets.

The theoretical velocity issuing from the nozzle is given by

$$v_1 = (2gH)^{1/2} \text{ or } v_1 = (2p/\rho)^{1/2}$$

Allowing for friction in the nozzle this becomes

$$v_1 = C_v(2gH)^{1/2} \text{ or } v_1 = C_v(2p/\rho)^{1/2}$$

H is the gauge pressure head behind the nozzle, p the gauge pressure and c_v the coefficient of velocity and this is usually close to unity.

The mass flow rate from the nozzle is

$$m = C_c\,\rho A v_1 = C_c\rho A C_v(2gH)^{1/2} = C_d\,\rho A(2gH)^{1/2}$$

C_c is the coefficient of contraction (normally unity because the nozzles are designed not to have a contraction).

C_d is the coefficient of discharge and $C_d = C_cC_v$

In order to produce no axial force on the wheel, the flow is divided equally by the shape of the bucket. This produces a zero net change in momentum in the axial direction.

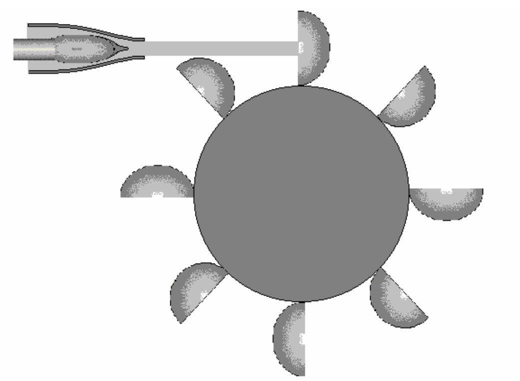


Fig.8
Layout of Pelton Wheel with One Nozzle

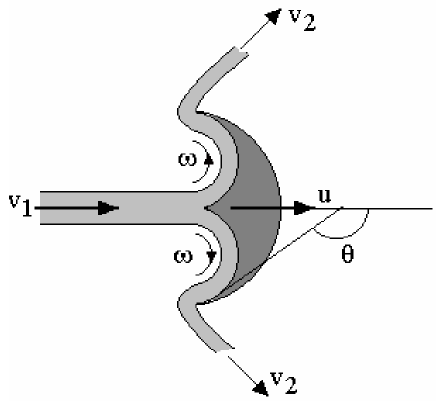


Fig. 9
Cross Section Through Bucket

The water is deflected over each half of the bucket by an angle of θ degrees. Since the change in momentum is the same for both halves of the flow, we need only consider the vector diagram for one half. The initial velocity is v_1 and the bucket velocity u_1 is in the same direction. The relative velocity of the water at inlet (in the middle) is ω_1 and is also in the same direction so the vector diagram is a straight line.

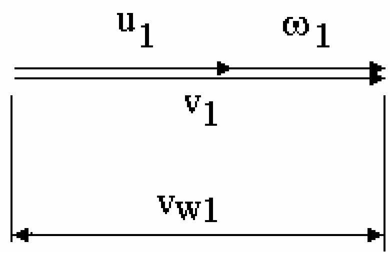


Fig. 10

If the water is not slowed down as it passes over the bucket surface, the relative velocity ω_2 will be the same as ω_1. In reality friction slows it down slightly and we define a blade friction coefficient as $k = \omega_2/\omega_1$

The exact angle at which the water leaves the sides of the bucket depends upon the other velocities but as always the vectors must add up so that

Note that $u_2 = u_1 = u$ since the bucket has a uniform velocity everywhere.

The vector diagram at exit is as shown.

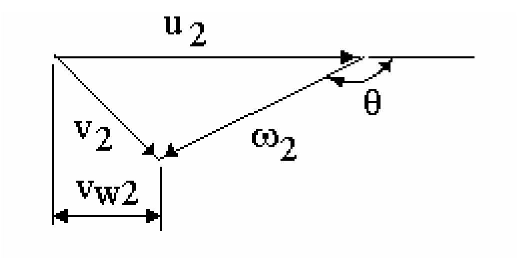


Fig. 11

It is normal to use ω_1 and u as common to both diagrams and combine them as shown.

Since $u_2 = u_1 = u$ the diagram power becomes I

Examining the combined vector diagram shows that $\Delta v_W = \omega_1 - \omega_2 \cos\theta$

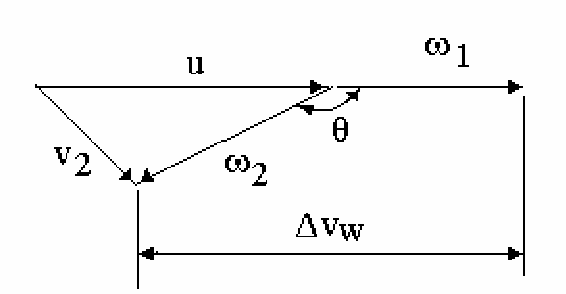


Fig. 12

Hence D.P.= $mu(\omega_1 - \omega_2\cos\theta)$ but $\omega_2 = k\omega_1$

D.P.= $mu\omega_1(1 - k\cos\theta)$ but $\omega_1 = v_1 - u$

D.P.= $mu(v_1 - u)(1 - k\cos\theta)$

WORKED EXAMPLE No. 1

A Pelton wheel is supplied with 1.2 kg/s of water at 20 m/s. The buckets rotate on a mean diameter of 250 mm at 800 rev/min. The deflection angle is 165^o and friction is negligible. Determine the diagram power. Draw the vector diagram to scale and determine Δv_W.

SOLUTION

$u = \pi ND/60 = \pi \times 800 \times 0.25/60 = 10.47$ m/s

D.P = $mu(v_1 - u)(1 - k\cos\theta)$

D.P = 1.2 x 10.47 x (20 - 10.47)(1 - cos 165) = 235 Watts

You should now draw the vector diagram to scale and show that Δv_W= 18.5 m/s

3.2 CONDITION FOR MAXIMUM POWER

If the equation for diagram power is used to plot D.P against u, the graph is as shown below.

Clearly the power is zero when the buckets are stationary and zero when the buckets move so fast that the water cannot catch up with them and strike them. In between is a velocity which gives maximum power. This may be found from max and min theory.

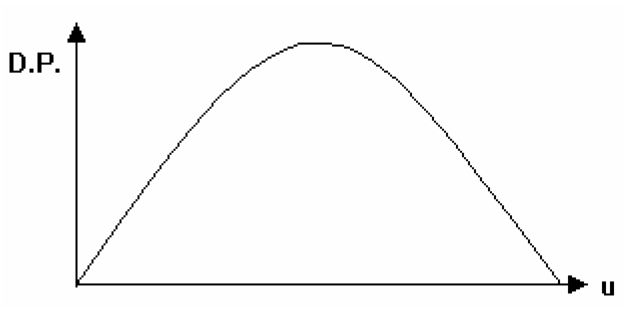


Fig. 13

$$\frac{d(D.P.)}{du} = \frac{d\{mu(v_1 - u)(1 - k\cos\theta)\}}{du} \qquad \frac{d(D.P.)}{du} = \frac{d\{m(uv_1 - u^2)(1 - k\cos\theta)\}}{du}$$

$$\frac{d(D.P.)}{du} = m(v_1 - 2u)(1 - k\cos\theta)$$

For a maximum value $m(v_1 - 2u)(1 - k\cos\theta) = 0$ Hence for maximum power $v_1 = 2u$

3.3 SPECIFIC SPEED N_S FOR PELTON WHEELS

You may have already covered the theory for specific speed in dimensional analysis but for those who have not, here is a brief review.

Specific speed is a parameter which enables a designer to select the best pump or turbine for a given system. It enables the most efficient matching of the machine to the head and flow rate available. One definition of specific speed for a turbine is : $N_S = NQ^{1/2}(H)^{-3/4}$

N is the speed in rev/min, Q is the volume flow rate in m^3/s and H is the available head in metres. The equation may be developed for a Pelton Wheel as follows.

$u = \pi ND/60 = K_1ND$ D = mean wheel diameter $N = u/(K_1D)$

u = bucket velocity $v_j = K_2H^{1/2}$ H = head behind the nozzle

v_j = nozzle velocity Now for a fixed speed wheel, $u = K_3v_j$ Hence

$$N = \frac{K_3 v_j}{K_1 D} = \frac{K_3 K_2 H^{\frac{1}{2}}}{K_1 D} = \frac{K_4 H^{\frac{1}{2}}}{D}$$

$$Q = A_j v_j = \frac{\pi d^2}{4} v_j \quad d = \text{nozzle diameter} \qquad Q = \frac{\pi d^2}{4} K_2 H^{\frac{1}{2}} = K_5 d^2 H^{\frac{1}{2}}$$

Substituting all in the formula for N_S we get $N_s = k\frac{d}{D}$

The value of k has to be deduced from the data of the wheel and nozzle. Note that Ns is sometimes defined in terms of water power as $N_s = \frac{NP^{\frac{1}{2}}}{\rho^{\frac{1}{2}}(gH)^{\frac{5}{4}}}$

This is just an alternative formula and the same result can be easily obtained other ways. You will need the substitution $P = \rho QgH$

SELF ASSESSMENT EXERCISE No. 1

1. The buckets of a Pelton wheel revolve on a mean diameter of 1.5 m at 1500 rev/min. The jet velocity is 1.8 times the bucket velocity. Calculate the water flow rate required to produce a power output of 2MW. The mechanical efficiency is 80% and the blade friction coefficient is 0.97. The deflection angle is 165^o.
(Ans. 116.3 kg/s)

2. Calculate the diagram power for a Pelton Wheel 2m mean diameter revolving at 3000 rev/min with a deflection angle of 170^o under the action of two nozzles , each supplying 10 kg/s of water with a velocity twice the bucket velocity. The blade friction coefficient is 0.98.
(Ans. 3.88 MW)

If the coefficient of velocity is 0.97, calculate the pressure behind the nozzles.
(Ans 209.8 MPa)

3. A Pelton Wheel is 1.7 m mean diameter and runs at maximum power. It is supplied from two nozzles. The gauge pressure head behind each nozzle is 180 metres of water. Other data for the wheel is :

Coefficient of Discharge C_d = 0.99
Coefficient of velocity C_v = 0.995
Deflection angle = 165^o.
Blade friction coefficient = 0.98
Mechanical efficiency = 87% Nozzle diameters = 30 mm

Calculate the following.

i. The jet velocity (59.13 m/s)
ii. The mass flow rate (41.586 kg/s)
iii The water power (73.432 kW)
iv. The diagram power (70.759 kW)
v. The diagram efficiency (96.36%)
vi. The overall efficiency (83.8%)
vii. The wheel speed in rev/min (332 rev/min)

4. Explain the significance and use of 'specific speed' $Ns = NP^{1/2}/\{\rho^{1/2}(gH)^{5/4}\}$
Explain why in the case of a Pelton wheel with several nozzles, P is the power per nozzle.
Explain why a Francis Wheel is likely to be preferred to a Pelton wheel when site conditions suggest that either could be used.
Calculate the specific speed of a Pelton Wheel given the following.

d = nozzle diameter.
D = Wheel diameter.
u = optimum blade speed = 0.46 v_1
v_1= jet speed.
η= 88% C_v = coefficient of velocity = 0.98

Answer Ns = 11.9 d/D

5. Explain the usefulness of specific speed in the selection of pumps and turbines.

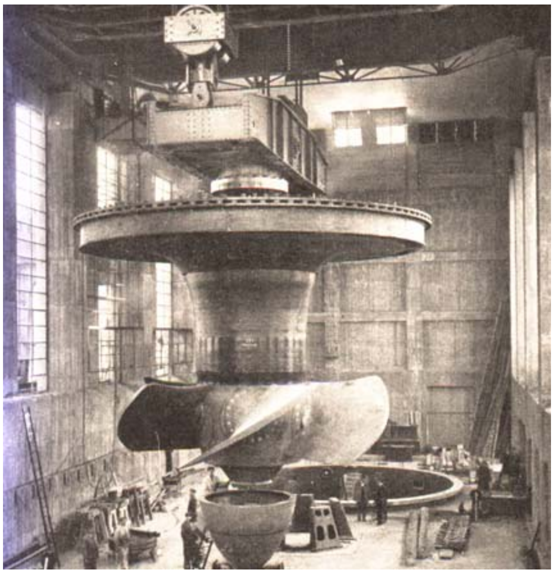
A turbine is to run at 150 rev/min under a head difference of 22 m and an expected flow rate of 85 m^3/s.

A scale model is made and tested with a flow rate of 0.1 m^3/s and a head difference of 5 m. Determine the scale and speed of the model in order to obtain valid results.

When tested at the speed calculated, the power was 4.5 kW. Predict the power and efficiency of the full size turbine.

Answers 0.05 scale 16.17 MW and 88%.

4. KAPLAN TURBINE



The Kaplan turbine is a pure reaction turbine. The main point concerning this is that all the flow energy and pressure is expended over the rotor and not in the supply nozzles. The picture shows the rotor of a large Kaplan turbine. They are most suited to low pressure heads and large flow rates such as on dams and tidal barrage schemes.

The diagram below shows the layout of a large hydroelectric generator in a dam.

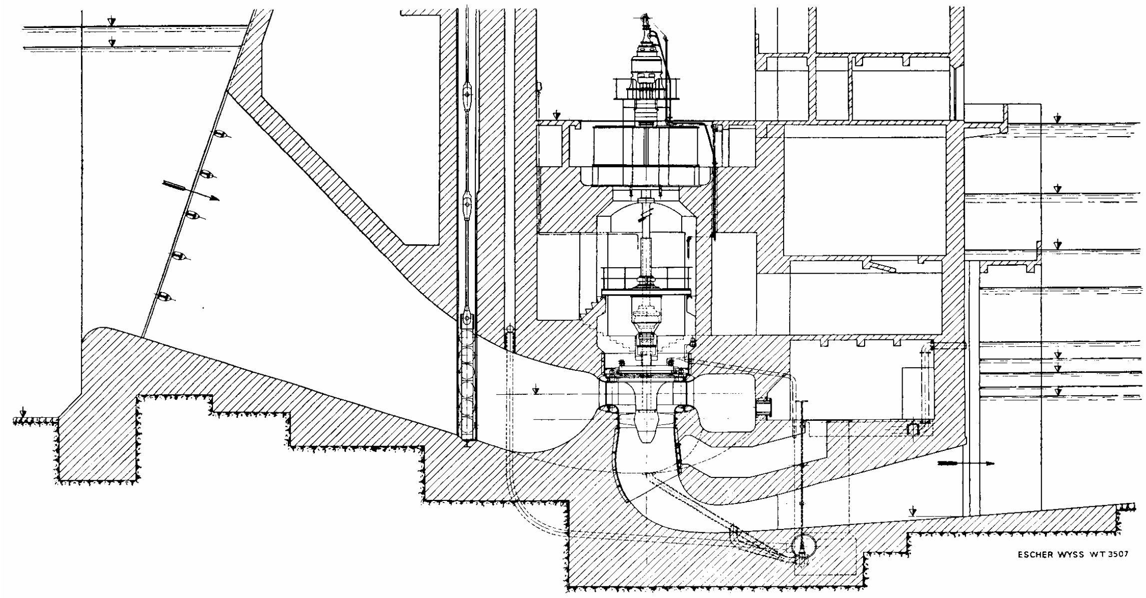


Fig.14 Picture and schematic of a Kaplan Turbine

5. FRANCIS WHEEL

The Francis wheel is an example of a mixed impulse and reaction turbine. They are adaptable to varying heads and flows and may be run in reverse as a pump such as on a pumped storage scheme. The diagram shows the layout of a vertical axis Francis wheel.

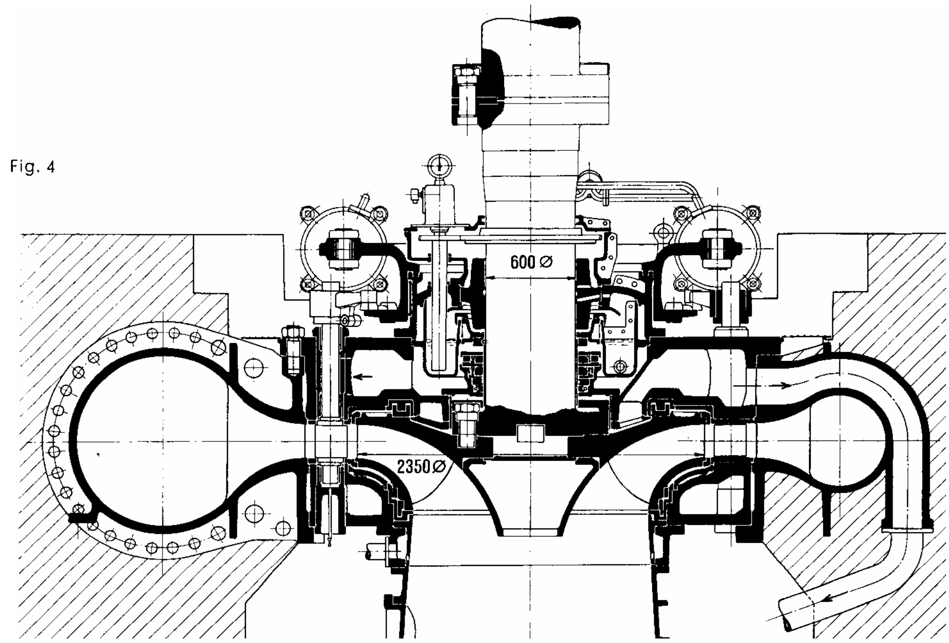


Fig.15

The Francis Wheel is an inward flow device with the water entering around the periphery and moving to the centre before exhausting. The rotor is contained in a casing that spreads the flow and pressure evenly around the periphery.

The impulse part comes about because guide vanes are used to produce an initial velocity v_1 that is directed at the rotor.

Pressure drop occurs in the guide vanes and the velocity is $v_1 = k (\Delta H)^{1/2}$ where ΔH is the head drop in the guide vanes.

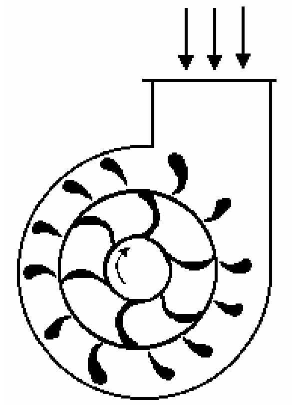


Fig.16

The angle of the guide vanes is adjustable so that the inlet angle β_1 is correct for shockless entry.

The shape of the rotor is such that the vanes are taller at the centre than at the ends. This gives control over the radial velocity component and usually this is constant from inlet to outlet. The volume flow rate is usually expressed in terms of radial velocity and circumferential area.

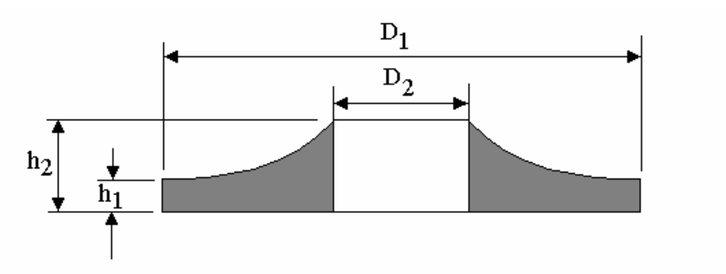


Fig.17

v_R = radial velocity   A = circumferential area = $\pi D h k$

$Q = v_R \pi D h k$ h = height of the vane.

k is a factor which allows for the area taken up by the thickness of the vanes on the circumference. If v_R is constant then since Q is the same at all circumferences,

$$D_1 h_1 = D_2 h_2.$$

VECTOR DIAGRAMS

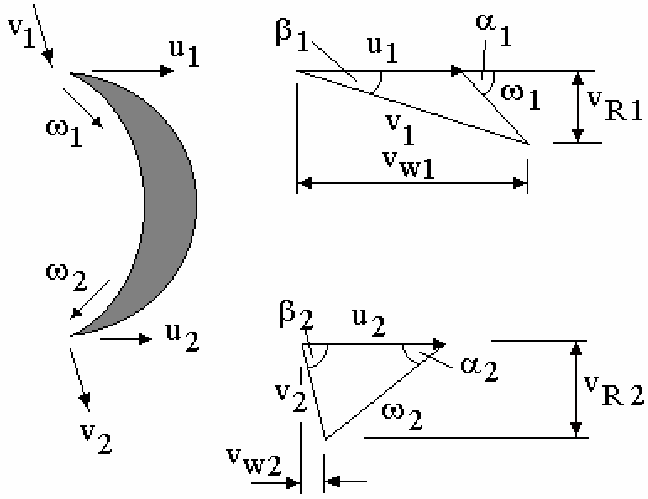


Fig. 18

The diagram shows how the vector diagrams are constructed for the inlet and outlet. Remember the rule is that the vectors add up so that u + v = ω

If u is drawn horizontal as shown, then V_W is the horizontal component of v and v_R is the radial component (vertical).

MORE DETAILED EXAMINATION OF VECTOR DIAGRAM

Applying the sine rule to the inlet triangle we find

$$\frac{v_1}{\sin(180-\alpha_1)} = \frac{u_1}{\sin\{180-\beta_1-(180-\alpha_1)\}}$$

$$\frac{v_1}{\sin(\alpha_1)} = \frac{u_1}{\sin(\alpha_1-\beta_1)} \qquad v_1 = \frac{u_1\sin(\alpha_1)}{\sin(\alpha_1-\beta_1)}\text{.................(1)}$$

Also $v_1 = \frac{v_{r1}}{\sin(\beta_1)}$............(2) $\qquad v_{r1} = v_{w1}\tan\beta_1$................(3)

equate (1) and (2) $\qquad \frac{u_1\sin(\alpha_1)}{\sin(\alpha_1-\beta_1)} = \frac{v_{r1}}{\sin(\beta_1)}$

$v_{r1} = \frac{u_1\sin(\alpha_1)\sin(\beta_1)}{\sin(\alpha_1-\beta_1)}$...........(4) $\qquad$ equate (3) and (4) $\qquad v_{w1} = \frac{u_1\sin(\alpha_1)\sin(\beta_1)}{\sin(\alpha_1-\beta_1)\tan\beta_1}$.............(5)

If all the angles are known, then v_{w1} may be found as a fraction of u_1.

DIAGRAM POWER

Because u is different at inlet and outlet we express the diagram power as :

$$\text{D.P.} = m\,\Delta(uv_w) = m\,(u_1v_{w1} - u_2v_{w2})$$

The kinetic energy represented by v_2 is energy lost in the exhausted water. For maximum efficiency, this should be reduced to a minimum and this occurs when the water leaves radially with no whirl so that $v_{w2} = 0$. This is produced by designing the exit angle to suit the speed of the wheel. The water would leave down the centre hole with some swirl in it. The direction of the swirl depends upon the direction of v_2 but if the flow leaves radially, there is no swirl and less kinetic energy. Ideally then,

$$\text{D.P.} = m\,u_1\,v_{w1}$$

WATER POWER

The water power supplied to the wheel is mgΔH where ΔH is the head difference between inlet and outlet.

HYDRAULIC EFFICIENCY

The maximum value with no swirl at exit is η_{hyd} = D.P./W.P. = $u_1 v_{w1}/g\rho H$

OVERALL EFFICIENCY

$\eta_{o/a}$= Shaft Power/Water Power

$\eta_{o/a} = 2\pi NT/mg\Delta H$

LOSSES

The hydraulic losses are the difference between the water power and diagram power.

Loss = $mg\Delta H - mu_1 v_{w1} = mgh_L$ $h_L = \Delta H - u_1 v_{w1}/g$ $\Delta H - h_L = u_1 v_{w1}/g$

WORKED EXAMPLE No. 2

The following data is for a Francis Wheel.

Radial velocity is constant No whirl at exit.

Flow rate 0.189 m^3/s

D_1= 0.6 m D_2=0.4 m k =0.85 h_1=50 mm

$\alpha_1 = 110^o$ N=562 rev/min

Head difference from inlet to outlet is 32 m. Entry is shockless. Calculate

i. the guide vane angle
ii. the diagram power
iii. the hydraulic efficiency
iv. the outlet vane angle
v. the blade height at outlet.

SOLUTION

$u_1 = \pi ND_1$ = 17.655 m/s v_{r1}= $Q/(\pi D_1 h_1 k)$= 0.189/(π x 0.6 x 0.05 x 0.85) = 2.35 m/s

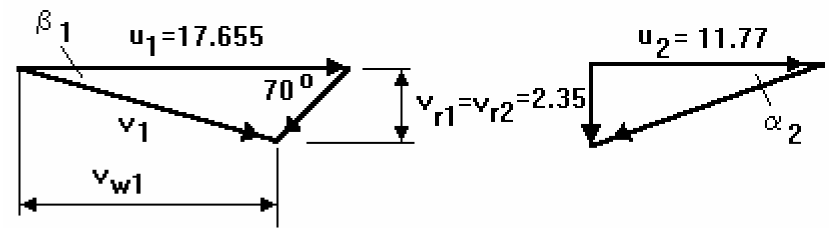


Fig. 19

v_{w1} and β_1 may be found by scaling or by trigonometry.

v_{w1}=16.47m/s $\beta_1 = 8.12^o$ $u_2 = \pi ND_2$ = 11.77 m/s

$\alpha_2 = \tan^{-1}$ (2.35/11.77) = 11.29^o

D.P. = m $u_1 v_{w1}$=189 (17.655 x 16.47) = 54 957 Watts

W.P.= mgΔH = 189 x 9.81 x 32 = 59 331 Watts

η_{hyd} = 54 957/59 331 = 92.6%

since $v_{r1} = v_{r2}$ then $D_1 h_1 = D_2 h_2$

h_2 = 0.6 x 0.05/0.4 = 0.075 m

WORKED EXAMPLE No. 3

The runner (rotor) of a Francis turbine has a blade configuration as shown. The outer diameter is 0.4 m and the inner diameter is 0.25 m. The vanes are 65 mm high at inlet and 100 mm at outlet. The supply head is 20 m and the losses in the guide vanes and runner are equivalent to 0.4 m. The water exhausts from the middle at atmospheric pressure. Entry is shockless and there is no whirl at exit. Neglecting the blade thickness, determine :

i. the speed of rotation.
ii. the flow rate.
iii. the output power given a mecanical efficiency of 88%.
iv. the overall efficiency.
v. The outlet vane angle.

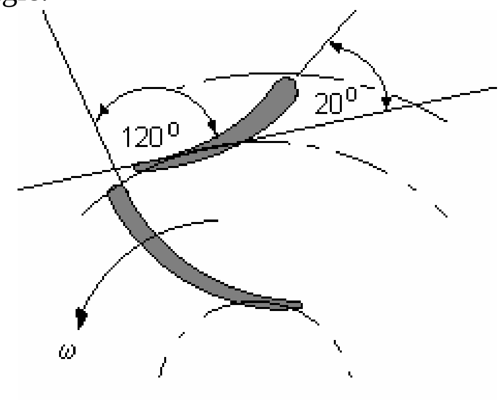


Fig.20

SOLUTION

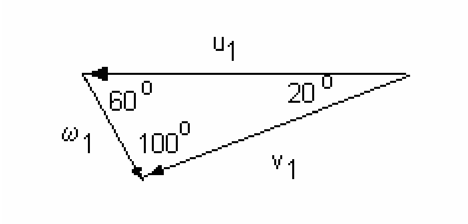


Fig.21

$$v_{w1} = \frac{u_1 \sin(\alpha_1) \sin(\beta_1)}{\sin(\alpha_1 - \beta_1) \tan\beta_1}$$

$$v_{w1} = \frac{u_1 \sin(120)\sin(20)}{\sin(100)\tan(20)} = 0.826 u_1$$

The inlet vector diagram is as shown. Values can be found by drawing to scale.
Since all angles are known but no flow rate, find v_{w1} in terms of u_1

$\Delta H - h_L = u_1 v_{w1}/g$

$20 - 0.4 = 19.6 = u_1 v_{w1}/g$

$19.6 = 0.826\ u_1^2/g$

$u_1 = 15.26$ m/s

$u_1 = \pi N D_1/60$

$N = 15.26 \times 60/(\pi \times 0.4) = 728.5$ rev/min

$$v_{r1} = \frac{u_1 \sin(\alpha_1)\sin(\beta_1)}{\sin(\alpha_1 - \beta_1)} = \frac{15.26 \sin(120)\sin(20)}{\sin(100_1)} = 4.589 \text{m/s}$$

$Q = v_{r1} \times \pi D_1 h_1 = 12.6 \times \pi \times 0.4 \times 0.065 = 0.375\ m^3/s$
m = 375 kg/s

$v_{w1} = 0.826\ u_1 = 12.6$ m/s

Diagram Power = $m\ u_1\ v_{w1} = 375 \times 15.26 \times 12.6 = 72.1$ kW

Output power = 0.88 x 72.1 = 63.45 kW

OUTLET TRIANGLE
$u_2 = \pi N D_2/60 = \pi \times 728.5 \times 0.25/60 = 9.54$ m/s
$Q = v_{r2} \times \pi D_2 h_2$
$0.375 = v_{r2} \times \pi \times 0.25 \times 0.1$
$v_{r2} = 4.775$ m/s = v_2 if no whirl.
$\tan \alpha_2 = 4.775/9.54 = 0.5$
$\alpha_2 = 26.6^o$.

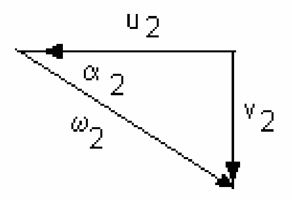


Fig. 22

SELF ASSESSMENT EXERCISE No. 2

1. The following data is for a Francis Wheel
   Radial velocity is constant
   No whirl at exit.
   Flow rate=0.4 m^3/s
   D_1=0.4 m
   D_2=0.15 m
   k =0.95
   $\alpha_1 = 90^o$
   N=1000 rev/min
   Head at inlet = 56 m
   head at entry to rotor = 26 m
   head at exit = 0 m
   Entry is shockless.

Calculate i. the inlet velocity v_1 (24.26 m/s)
ii. the guide vane angle (30.3^o)
iii. the vane height at inlet and outlet (27.3 mm, 72.9 mm)
iv. the diagram power (175.4 MW)
v. the hydraulic efficiency (80%)

2. A radial flow turbine has a rotor 400 mm diameter and runs at 600 rev/min. The vanes are 30 mm high at the outer edge. The vanes are inclined at 42^o to the tangent to the inner edge. The flow rate is 0.5 m^3/s and leaves the rotor radially. Determine

   i. the inlet velocity as it leaves the guide vanes. (19.81 m/s)
   ii. the inlet vane angle. (80.8^o)
   iii. the power developed. (92.5 kW)

3. The runner (rotor) of a Francis turbine has a blade configuration as shown. The outer diameter is 0.45 m and the inner diameter is 0.3 m. The vanes are 62.5 mm high at inlet and 100 mm at outlet. The supply head is 18 m and the losses in the guide vanes and runner are equivalent to 0.36 m. The water exhausts from the middle at atmospheric pressure. Entry is shockless and there is no whirl at exit. Neglecting the blade thickness, determine :

i. The speed of rotation. (1691 rev/min)
ii. The flow rate. (1.056 m^3/s)
iii. The output power given a mechanical efficiency of 90%. (182.2 MW)
iv. The overall efficiency. (88.2%)
v. The outlet vane angle. (22.97^o)

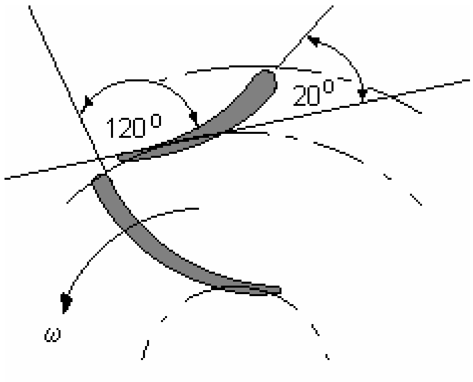


Fig. 23