FLUID MECHANICS D203
SAE SOLUTIONS TUTORIAL9 – COMPRESSIBLE FLOW

SELF ASSESSMENT EXERCISE No. 1

1. Calculate the specific entropy change when a perfect gas undergoes a reversible isothermal expansion from 500 kPa to 100 kPa. R = 287 J/kg K.

T is constant so $\Delta s = mR\ln(p_1/p_2)$ = 1 x 287 x ln(5/1) = 462 J/kg K

2. Calculate the total entropy change when 2 kg of perfect gas is compressed reversibly and isothermally from 9 dm^3 to 1 dm^3. R=300 J/kg K.

$\Delta s = mR\ln(V_2/V_1)$ = 1 x 300 x ln(1/9) = 470 J/kg K

3. Calculate the change in entropy when 2.5 kg of perfect gas is heated from 20°C to 100°C at constant volume. Take c_v= 780 J/kg K (Answer 470 J/K)

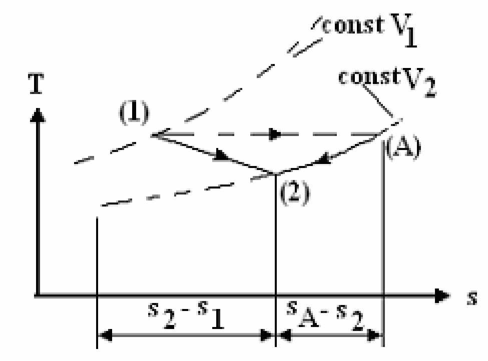
$\Delta s = m\, c_v \ln(T_2/T_1)$ = 2.5 x 780 x ln(373/293) = -1318 J/kg K

4. Calculate the total entropy change when 5 kg of gas is expanded at constant pressure from 30°C to 200°C. R = 300 J/kg K c_v= 800 J/kg K (Answer 2.45 kJ/K)

$\Delta s = m\, c_p \ln(T_2/T_1)$ $\quad c_p = R + c_v$ = 1100 J/kg K

Δs = = 5 x 1100 x ln(473/303) = 2450 J/kg K

5. Derive the formula for the specific change in entropy during a polytropic process using a constant volume process from (A) to (2).



$s_2-s_1 = (s_A-s_1) - (s_A-s_2)$

$s_2-s_1 = (s_A-s_1) + (s_2-s_A)$

For the constant temperature process

$(s_A-s_1) = R\ln(p_1/p_A)$

For the constant volume process

$(s_2-s_A) = (c_v/R)\ln(T_2/T_A)$

Hence

$$\Delta s = R\ln\frac{p_1}{p_A} + C_p\ln\frac{T_2}{T_A} + s_2 - s_1 \quad T_A = T_1$$

Then $\quad \Delta s = s_2 - s_1 = \Delta s = R\ln\left(\frac{p_1}{p_A}\right) + c_v\ln\left(\frac{T_2}{T_A}\right)$

Divide through by R $\quad \Delta s/R = \ln\left(\frac{p_1}{p_A}\right) + \frac{c_v}{R}\ln\left(\frac{T_2}{T_A}\right)$

From the relationship between c_p, c_v, R and γ we have $c_p/R = \gamma/(\gamma-1)$

From the gas laws we have $p_A/T_A = p_2/T_2$ $\quad p_A = p_2 T_A/T_2 = p_2 T_1/T_2$

Hence

$$\frac{\Delta s}{R} = \ln\left(\frac{p_1}{p_2}\right) + \frac{1}{\gamma-1}\ln\left(\frac{T_2}{T_1}\right) = \ln\left(\frac{p_1}{p_2}\right)\left(\frac{T_2}{T_1}\right)^{1+\frac{1}{\gamma-1}} = \ln\left(\frac{p_1}{p_2}\right)\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

6. A perfect gas is expanded from 5 bar to 1 bar by the law $pV^{1.6} = C$. The initial temperature is 200°C. Calculate the change in specific entropy.

R = 287 J/kg K $\gamma = 1.4$.

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{1-1/n} = 473\left(\frac{1}{5}\right)^{1-1/1.6} = 258.7\ \mathrm{K}$$

$$\Delta s = R\ln\left(\frac{p_1}{p_2}\right)\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = 287\ \ln(5)\left(\frac{258.7}{473}\right)^{\frac{1.4}{0.4}} = -144\ \mathrm{J/K}$$

7. A perfect gas is expanded reversibly and adiabatically from 5 bar to 1 bar by the law $pV^{\gamma} = C$. The initial temperature is 200°C. Calculate the change in specific entropy using the formula for a polytropic process. R = 287 J/kg K $\gamma = 1.4$.

$$T_2 = 473\left(\frac{1}{5}\right)^{1-1/1.4} = 298.6\ \mathrm{K}$$

$$\Delta s = R\ln\left(\frac{p_1}{p_2}\right)\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = 287\ \ln(5)\left(\frac{298.6}{473}\right)^{\frac{1.4}{0.4}} = 0$$

SELF ASSESSMENT EXERCISE No. 2

Take γ = 1.4 and R = 283 J/kg K in all the following questions.

1. An aeroplane flies at Mach 0.8 in air at 15° C and 100 kPa pressure. Calculate the stagnation pressure and temperature. (Answers 324.9 K and 152.4 kPa)

$$\frac{\Delta T}{T} = M^2 \frac{k-1}{2} = 0.8^2 \frac{1.4}{2} = 0.128 \quad \Delta T = 0.128 \text{ x } 288 = 36.86 \text{ K}$$

$$\frac{p_2}{p_1} = \left(M^2 \frac{k-1}{2} + 1\right)^{\frac{k}{k-1}} = 1.128^{3.5} = 1.5243 \qquad p_2 = 100 \text{ x } 1.5243 = 152.43 \text{ kPa}$$

2. Repeat problem 1 if the aeroplane flies at Mach 2.

$$\frac{\Delta T}{T} = M^2 \frac{k-1}{2} = 2^2 \frac{1.4}{2} = 0.8 \qquad \Delta T = 0.8 \text{ x } 288 = 230.4 \text{ K}$$

T_2 = 288 + 230.4 = 518.4 K

$$\frac{p_2}{p_1} = \left(M^2 \frac{k-1}{2} + 1\right)^{\frac{k}{k-1}} = 1.8^{3.5} = 7.824 \qquad p_2 = 100 \text{ x } 7.824 = 782.4 \text{ kPa}$$

3. The pressure on the leading edges of an aircraft is 4.52 kPa more than the surrounding atmosphere. The aeroplane flies at an altitude of 5 000 metres. Calculate the speed of the aeroplane.(Answer 109.186 m/s)

From fluids tables, find that a = 320.5 m/s p_1 = 54.05 kPa γ = 1.4

$$\frac{p_2}{p_1} = \frac{58.57}{54.05} = 1.0836 = \left(M^2 \frac{k-1}{2} + 1\right)^{\frac{k}{k-1}}$$

$$1.0836 = \left(M^2 \frac{1.4-1}{2} + 1\right)^{\frac{1.4}{1.4-1}} = \left(0.2\, M^2 + 1\right)^{3.5}$$

1.0232 = 0.2 M^2 + 1 M = 0.3407 = v/a v = 109.2 m/s

4. An air compressor delivers air with a stagnation temperature 5 K above the ambient temperature. Determine the velocity of the air. (Answer 100.2 m/s)

$$\frac{\Delta T}{T_1} = \frac{v_1^2 (k-1)}{2\gamma R T_1} \qquad \Delta T = \frac{v_1^2 (1.4-1)}{2 \text{ x } 1.4 \text{ x } 287} = 5\,\text{K} \qquad v_1 = 100.2 \text{ m/s}$$

SELF ASSESSMENT EXERCISE No. 3

1. A Venturi Meter must pass 300g/s of air. The inlet pressure is 2 bar and the inlet temperature is 120°C. Ignoring the inlet velocity, determine the throat area. Take C_d as 0.97.
Take γ =1.4 and R = 287 J/kg K (assume choked flow)

$$m = C_d A_2 \sqrt{\left[\frac{2\gamma}{\gamma - 1}\right] p_1 \rho_1 \left\{ (r_c)^{\frac{2}{\gamma}} - (r_c)^{1+\frac{1}{\gamma}} \right\}} \qquad r_c = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{2}{2.4}\right)^{3.5} = 0.528$$

$\rho_1 = p_1/RT_1 = 2 \times 10^5 /(287 \times 393) = 1.773$ kg/m^3

$$0.3 = 0.97 A_2 \sqrt{7 \text{ x } 2 \text{ x} 10^5 \text{ x} 1.773 \left\{(0.528)^{1.428} - (0.528)^{1.714}\right\}} = 0.97 A_2 \sqrt{166307}$$

$A_2 = 758 \times 10^{-6}$ m^2 and the diameter = 31.07 mm

2. Repeat problem 1 given that the inlet is 60 mm diameter and the inlet velocity must not be neglected.

$$m = C_d A_2 \sqrt{\frac{\left[\frac{2\gamma}{\gamma - 1}\right] p_1 \rho_1 \left\{ (r_c)^{\frac{2}{\gamma}} - (r_c)^{1+\frac{1}{\gamma}} \right\}}{1 - \left(\frac{A_2}{A_1}\right)^2 \left(\frac{p_2}{p_1}\right)^{2/\gamma}}} \qquad 0.3 = C_d A_2 \sqrt{\frac{166307}{1 - \left(\frac{A_2}{A_1}\right)^2 (0.4)}}$$

$1 - (A_2/A_1)^2 \times 0.4 = 1738882 A_2^2$ $\qquad A_1^2 = (\pi \times 0.06^2/4)^2 = 7.99 \times 10^{-6}$ m^2

$1 - 50062\, A_2^2 = 1738882 A_2^2$ $\qquad A_2^2 = 559 \times 10^{-9}$ $\qquad A_2 = 747.6 \times 10^{-6}$ m^2

The diameter is 30.8 mm. Neglecting the inlet velocity made very little difference.

3. A nozzle must pass 0.5 kg/s of steam with inlet conditions of 10 bar and 400°C. Calculate the throat diameter that causes choking at this condition. The density of the steam at inlet is 3.263 kg/m^3. Take γ for steam as 1.3 and C_d as 0.98.

$$m = C_d A_2 \sqrt{\left[\frac{2\gamma}{\gamma - 1}\right] p_1 \rho_1 \left\{ (r_c)^{\frac{2}{\gamma}} - (r_c)^{1+\frac{1}{\gamma}} \right\}} \qquad r_c = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{2}{2.3}\right)^{4.33} = 0.5457$$

$$0.5 = 0.98 A_2 \sqrt{8.667 \text{ x } 3.2626 \text{ x } 10 \text{ x } 10^5 \left\{(0.5457)^{1.538} - (0.5457)^{1.538}\right\}} = 0.98 A_2 \sqrt{1.4526 \text{ x } 10^6}$$

$A_2 = 423 \times 10^{-6}$ m^2 and the diameter =23.2 mm

4. A Venturi Meter has a throat area of 500 mm^2. Steam flows through it, and the inlet pressure is 7 bar and the throat pressure is 5 bar. The inlet temperature is 400°C. Calculate the flow rate. The density of the steam at inlet is 2.274 kg/m^3. Take γ= 1.3. R = 462 J/kg K. C_d = 0.97.
From the steam tables v_1 = 0.4397 m^3/ kg so ρ_1 = 1/0.4397 = 2.274 kg/m^3

$$m = C_d A_2 \sqrt{\left[\frac{2\gamma}{\gamma - 1}\right] p_1 \rho_1 \left\{ (r_c)^{\frac{2}{\gamma}} - (r_c)^{1+\frac{1}{\gamma}} \right\}}$$

$$m = 0.97 \text{ x } 500 \text{ x } 10^{-6} \sqrt{\left[\frac{2 \text{ x } 1.3}{1.3 - 1}\right] 7 \text{ x} 10^5 \text{ x } 2.274 \left\{(5/7)^{1.538} - (5/7)^{1.764}\right\}}$$

$m = 485 \times 10^{-6} \times 783$ $\qquad m = 0.379$ kg/s

5. A pitot tube is pointed into an air stream which has an ambient pressure of 100 kPa and temperature of 20ºC. The pressure rise measured is 23 kPa. Calculate the air velocity. Take γ = 1.4 and R = 287 J/kg K.

$$\frac{p_2}{p_1} = \frac{123}{100} = 1.23 = \left(M^2 \frac{\gamma - 1}{2} + 1 \right)^{\frac{\gamma}{\gamma - 1}} \qquad 1.23 = \left(0.2M^2 + 1\right)^{3.5}$$

$1.0609 = 0.2M^2 + 1$ $\quad$ $0.0609 = 0.2M^2$ $\quad$ M 0.5519

$a = \gamma RT^{1/2} = (1.4 \times 287 \times 293)^{1/2} = 343.1$ m/s

v = 0.5519 x 343.1 = 189.4 m/s

6. A fast moving stream of gas has a temperature of 25ºC. A thermometer is placed into it in front of a small barrier to record the stagnation temperature. The stagnation temperature is 28ºC. Calculate the velocity of the gas. Take γ= 1.5 and R = 300 J/kg K. (Answer 73.5 m/s)

$\Delta T = 3$ K $\quad$ $\Delta T/T_1 = v^2/\gamma RT$ $\quad$ $c_p = \gamma R/(\gamma-1)$

$\Delta T = 3 = v^2/2\, c_p = v^2(\gamma - 1)/(2\, \gamma R) = v^2(1.5 - 1)/(2 \times 1.5 \times 300)$

$v^2 = 5400$ $\quad$ v = 73.48 m/s