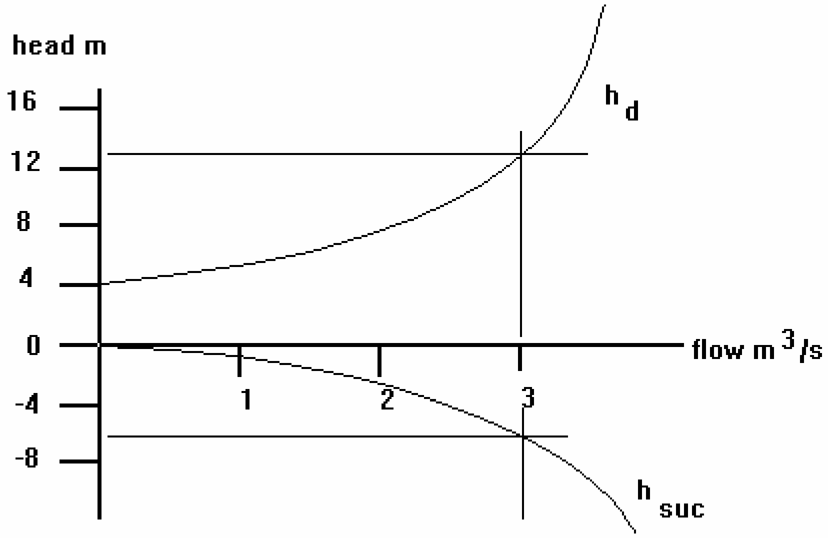
FLUID MECHANICS D203
SAE SOLUTIONS TUTORIAL 8C – PUMPED PIPED SYSTEMS

SELF ASSESSMENT EXERCISE No. 1

1. A pump has a suction pipe and a delivery pipe. The head required to pass water through them varies with flow rate as shown.



The pump must deliver 3 m^3/s at 2 000 rev/min. Determine the specific speed.

The vapour pressure is 0.025 bar and atmospheric pressure is 1.025 bar. Calculate the NPSH and the cavitation parameter.

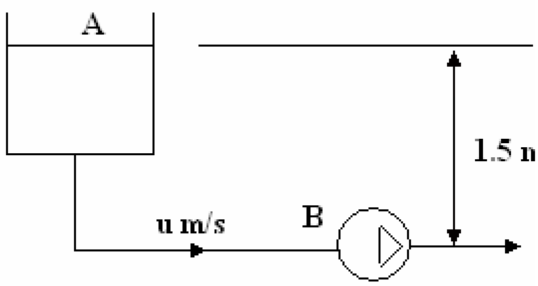
From the graph at 3 m/s h_d = 13 m $\qquad$ h_s = -6 m
NPSH = {1.025 x 10^5/(9.81 x 1000) – 6 } – 0.025 x 10^5/(9.81 x 1000)
NPSH = 4.448 – 0.2548 = 4.19 m
σ = NPSH/h_d = 4.19/13 = 0.323
Specific speed Ns = $NQ^{½}$ / $H^{3/4}$ = 2000 x $3^{½}$/$19^{3/4}$ = 380.6

2. Define the term "Net Positive Suction Head" and explain its significance in pump operation.

1.2 kg/s of acetone is to be pumped from a tank at 1 bar pressure. The acetone is at 40°C and the pump is 1.5 m below the surface. The suction pipe is 25 mm bore diameter. Calculate the NPSH at the pump inlet.

Losses in the suction pipe are equal to three velocity heads.
The vapour pressure of acetone is 55 kPa. The density is 780 kg/m^3.

The Net Positive Suction Head is the amount by which the absolute pressure on the suction side is larger than the vapour pressure (saturation pressure) of the liquid.



u = m/ρA = 1.2/(780 x π x 0.025^2) = 3.134 m/s

$h_A + z_A + u_A^2/2g = h_B + z_B + u_B^2/2g + h_L$

$h_L = 3\ u_B^2/2g$
$0 + 1.5 + 0 = h_B + 0 + 3.134^2/2g + 3\ u_B^2/2g$
$h_B = 1.5 - 4\ u_B^2/2g = -0.5$ m gauge

Atmospheric pressure = 1.0 bar $\rho = 780$ kg/m^3
Convert to pressure head $h = p/\rho g = 13.06$ m
Absolute head at pump = 13.06 – 0.5 = 12.56 m
Vapour pressure head = $55 \times 10^3 /\rho g = 7.19$ m
NPSH = 12.56 – 7.19 = 5.37 m

3. A centrifugal pump delivers fluid from one vessel to another distant vessel. The flow is controlled with a valve. Sketch and justify appropriate positions for the pump and valve when the fluid is a) a liquid and b) a gas.

(a) Minimum suction is required to avoid cavitation so put the valve on the pump outlet and this will also keep the pump primed when closed. The pump should be as close to the tank as possible.

(b) With gas cavitation is not a problem but for minimal friction the velocity must be kept low. If the gas is kept under pressure by putting the valve at the end of the pipe, it will be more dense and so the velocity will be lower for any given mass flow rate. The pump should be close to the supply tank.

<u>SELF ASSESSMENT EXERCISE No. 2</u>

The density of water is 1000 kg/m^3 and the bulk modulus is 4 GPa throughout.

1. A pipe 50 m long carries water at 1.5 m/s. Calculate the pressure rise produced when
   a) the valve is closed uniformly in 3 seconds.
   b) when it is shut suddenly.

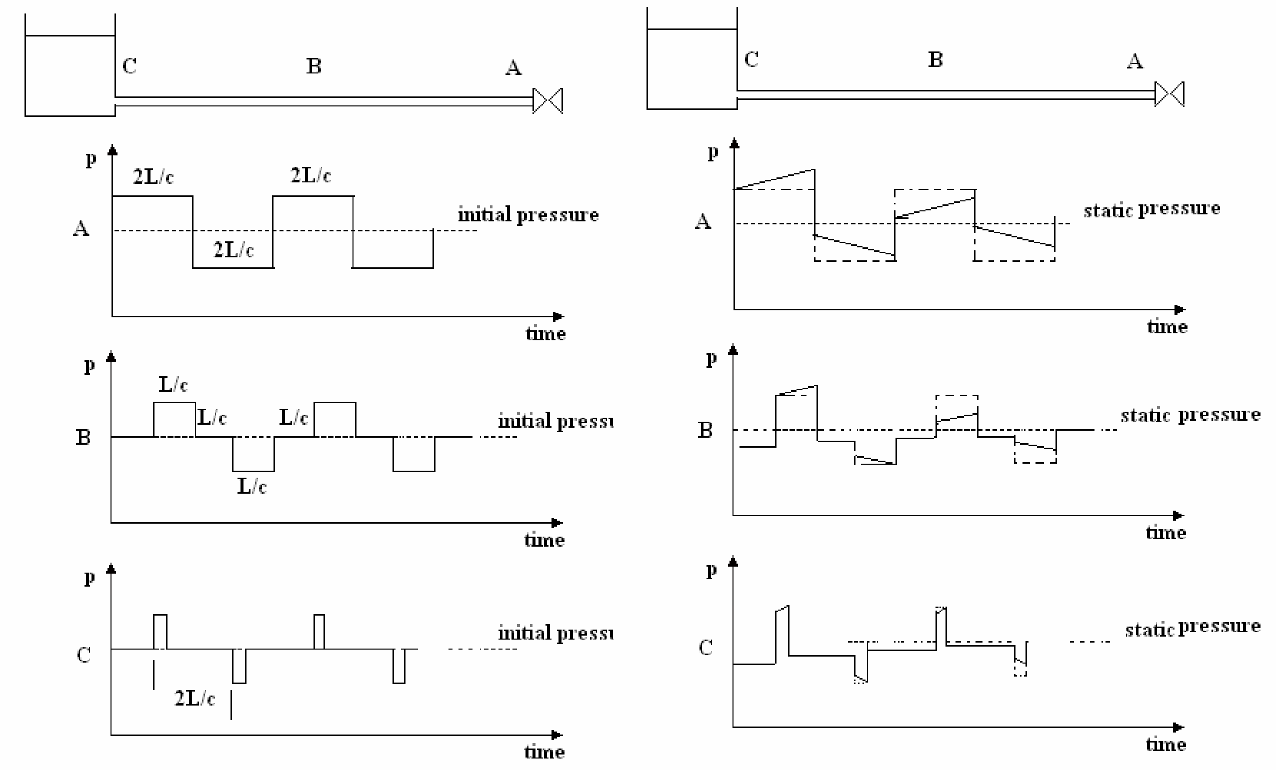
(a) $\Delta p = \rho Lu/t = 1000 \times 50 \times 1.5/3 = 25$ kPa
(b) $\Delta p = u(K\rho)^{0.5} = 1.5 \times (4 \times 10^9 \times 1000)^{0.5} = 3$ MPa

2. A pipe 2000 m long carries water at 0.8 m/s. A valve is closed. Calculate the pressure rise when
   a) it is closed uniformly in 10 seconds. a)
   b) it is suddenly closed.

(a) $\Delta p = \rho Lu/t = 1000 \times 2000 \times 0.8/10 = 160$ kPa
(b) $\Delta p = u(K\rho)^{0.5} = 0.8 \times (4 \times 10^9 \times 1000)^{0.5} = 1.6$ MPa

SELF ASSESSMENT EXERCISE No. 3

1. Derive the water hammer equation for a long elastic pipe carrying water from a large upstream reservoir with a constant water level to a lower downstream reservoir. Flow is controlled by a valve at the downstream end.
Sketch the variation in pressure with time for both ends and the middle of the pipe following sudden closure of the valve. Sketch these variations for when friction is negligible and for when both friction and cavitation occur.
Assuming the effective bulk modulus is given by $K' = \{(D/tE) + 1/K\}^{-1}$ and that the maximum stress in the pipe is σ, derive a formula for the maximum allowable discharge.
Part (a) is given in the tutorial. Part (b) below – no friction on left.



When cavitation occurs the minimum pressure is the vapour pressure so the bottom part of the cycle will be at this pressure.
Part (c)

For a thin walled cylinder $\sigma = \dfrac{pD}{2t}$ $\quad p = \dfrac{2t\sigma}{D}$ $\quad u = Q/A = 4Q/\pi D^2$

$$\Delta p = u\sqrt{\frac{\rho}{\left\{\dfrac{D}{2tE} + \dfrac{1}{K}\right\}}} \qquad \frac{2t\sigma}{D} = \frac{4Q}{\pi D^2}\sqrt{\frac{\rho}{\left\{\dfrac{D}{2tE} + \dfrac{1}{K}\right\}}} \qquad Q = \frac{\sigma \pi D t}{2}\sqrt{\frac{\left\{\dfrac{D}{2tE} + \dfrac{1}{K}\right\}}{\rho}}$$

2a. Explain the purpose and features of a surge tank used to protect hydroelectric installations.

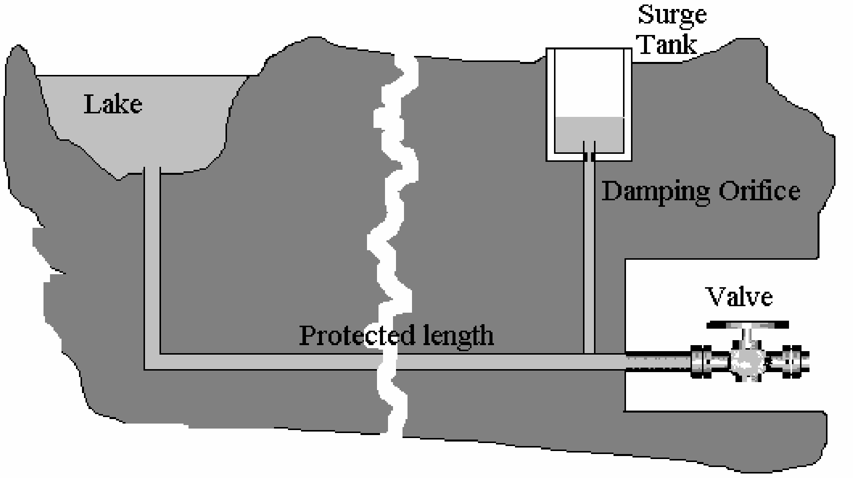
b. Derive an expression for the amplitude of oscillation of the water surface in a surge tank of cross sectional area A_T connected to a pipe of cross sectional area A_p and length L following a complete stoppage of the flow. The normal mean velocity in the pipe is u_o and friction may be ignored.

The general solution to the standard second order differential equation

$$\frac{d^2z}{dt^2} + m^2 z = c^2 \text{ is } z = E\sin(mt) + F\cos(mt) + \frac{c^2}{m^2}$$

Part (a)

On hydroelectric schemes or large pumped systems, a surge tank is used. This is an elevated reservoir attached as close to the equipment needing protection as possible. When the valve is closed, the large quantity of water in the main system is diverted upwards into the surge tank. The pressure surge is converted into a raised level and hence potential energy. The level drops again as the surge passes and an oscillatory trend sets in with the water level rising and falling. A damping orifice in the pipe to the surge tank will help to dissipate the energy as friction and the oscillation dies away quickly.



Part (b)

Mean velocity in surge tank $u_T = \frac{dz}{dt} = \frac{Q}{A_T}$ $\quad Q = A_T \frac{dz}{dt}$

Mean velocity in the pipe $u_p = \frac{Q}{A_p}$ $\quad$ Substitute for Q $\quad u_p = \frac{dz}{dt}\frac{A_T}{A_p}$ ....(1)

The diversion of the flow into the surge tank raises the level by z. This produces an increased pressure at the junction point of $\Delta p = \rho gz$

The pressure force produced $F = A_p\Delta p = A_p\Delta gz$

The inertia force required to decelerate the water in the pipe is

F = mass x deceleration = - mass x acceleration = - $\rho A_p L$ du/dt

Equating forces we have the following.

$$A_p\rho gz = -\rho A_p L\frac{du}{dt} \qquad gz = -L\frac{du}{dt} \qquad z = -\frac{L}{g}\frac{du}{dt} \text{..................(2)}$$

Putting (1) into (2) we get

$$z = -\frac{L}{g}\frac{A_T}{A_p}\frac{d^2z}{dt^2} \qquad \frac{d^2z}{dt^2} = -\frac{gA_p}{LA_T}z \text{...........................(3)}$$

By definition this is simple harmonic motion since the displacement z is directly proportional to the acceleration and opposite in sense. It follows that the frequency of the resulting oscillation is

$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}\frac{A_p}{A_T}}$ $\quad$ The periodic time will be $\quad T=1/f$

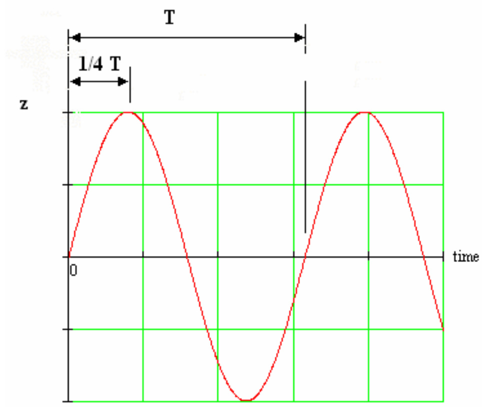
The amplitude and periodic time are referred to as the APO (amplitude and period of oscillation).

Equation (3) maybe re-written as follows.

$$\frac{d^2z}{dt^2} = -\frac{gA_p}{LA_T}z = -\omega^2 z$$

$$\frac{1}{\omega^2}\frac{d^2z}{dt^2} + z = 0$$

$$\frac{d^2z}{dt^2} + \omega^2 z = 0$$



The standard solution to this equation is $z = z_o \sin(\omega t)$

z_o is the amplitude, that is, the amount by which the height in the tank will move up and down from the mean level. The following is a direct way of finding the amplitude.

$$\text{The mean change in height} = \frac{z_o}{2}$$

$$\text{The weight of water entering the surge tank} = \rho g A_T z_o$$

$$\text{The potential energy stored in the tank} = \rho g A_T z_o \frac{z_o}{2} = \rho g A_T \frac{z_o^2}{2}$$

$$\text{The kinetic energy lost} = \text{Mass x } \frac{u^2}{2} = \rho L A_p \frac{u^2}{2}$$

$$\text{Equate the energies.} \quad \rho L A_p \frac{u^2}{2} = \rho g A_T \frac{z_o^2}{2} \qquad\qquad z_o = u_o \sqrt{\frac{LA_p}{gA_T}}$$

The equation for the motion in full is $z = u_o \sqrt{\frac{LA_p}{gA_T}} \sin(\omega t)$

The peak of the surge occurs at T/4 seconds from the disturbance.

3.a. A hydroelectric turbine is supplied with 0.76 m^3/s of water from a dam with the level 51 m above the inlet valve. The pipe is 0.5 m bore diameter and 650 m long.
Calculate the pressure at inlet to the turbine given that the head loss in the pipe is 8.1 m.
Calculate the maximum pressure on the inlet valve if it is closed suddenly. The speed of sound in the pipe is 1200 m/s.

b. The pipe is protected by a surge tank positioned close to the inlet valve.
Calculate the maximum change in level in the surge tank when the valve is closed suddenly (ignore friction).
Calculate the periodic time of the resulting oscillation.

$A = \pi \times 0.5^2/4 = 0.1963\ m^2$ $\quad u = Q/A = 0.76/0.1963 = 3.87$ m/s

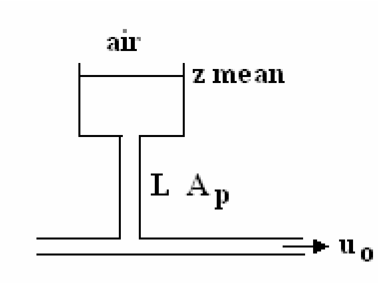
$h_A + z_A + u_A^2/2g = h_B + z_B + u_B^2/2g + h_L$
$0 + 189 + 0 = h_B + 138 + 3.87^2/2g + 8.4$
$h_B = 41.83$ m $\quad p_B = \rho g\, h_B = 0.41 \times 10^6$ Pa

Sudden closure $\quad \Delta p = \rho\, u\, a'$ $\quad a' = 1200$ m/s $\quad \Delta p = 998 \times 3.87 \times 1200 = 4.635 \times 10^6$ Pa

The maximum pressure is 0.41 + 4.635 = 5.045 MPa

This will occur at T/4 seconds

Part (b)



$u_o = Q/A_p$ $\quad dz/dt = Q/A_T$ $\quad u_o = (A_T/A_p)dz/dt$ ....(1)

$\Delta p = \rho\, g\, z$ $\quad \Delta F = A_p\, \rho\, g\, z$

This force decelerates the fluid and the mass decelerated is $m = \rho\, A_p\, L$ $\quad \Delta F = m\, a$
Acceleration is $-du_o/dt$
$A_p\, \rho\, g\, z = -\rho\, A_p\, L\, du_o/dt$
$g\, z = -\, L\, du_o/dt$
$z = -\,(L/g)\, du_o/dt$ ....(2)
Put (1) into (2) $z = -\frac{L}{g}\frac{A_T}{A_p}\frac{d^2z}{dt^2}$ ....(3)

Simple Harmonic Motion so $\omega^2 = \frac{L}{g}\frac{A_T}{A_p}$ and the amplitude is $u_o\left\{\frac{A_p}{A_T}\frac{L}{g}\right\}^{1/2}$

$A_T = \pi\, 4^2/4 = 12.566\ m^2$
$\Delta p = 4.635 \times 10^6$ Pa $\quad \Delta h = \Delta p/\rho g = 473.4$ m

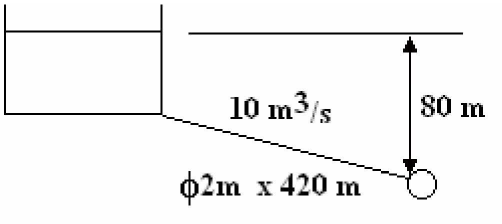
Amplitude = $3.87\left\{\frac{0.1963}{12.566} \times \frac{650}{9.81}\right\}^{1/2} = 3.987$ m

4. A pipe 2 m bore diameter and 420 m long supplies water from a dam to a turbine. The turbine is located 80 m below the dam level. The pipe friction coefficient f is 0.01 ($f = 4C_f$).

Calculate the pressure at inlet to the turbine when 10 m^3/s of water is supplied.

Calculate the pressure that would result on the inlet valve if it was closed suddenly. The speed of sound in the pipe is 1432 m/s.

Calculate the fastest time the valve could be closed normally if the pressure rise must not exceed 0.772 MPa).



$A = \pi\ 2^2/4 = 3.142\ m^2$ $\quad u = Q/A = 10/3.142 = 3.18$ m/s

Sudden closure $\quad \Delta p = \rho\ u\ a' = 998 \times 3.18 \times 1432 = 4.55$ MPa
Gradual closure $\quad \Delta p = \rho Lu/t = 998 \times 420 \times 3.18/t = 1.333/t$ MPa
$h_B = 80 - h_L$
Loss in pipe $= 4C_f\ Lu^2/2gD = f\ Lu^2/2gD = 0.01 \times 420 \times 3.18^2/(2 \times 9.81 \times 2) = 1.082$ m
$h_B = 78.92$ m
$p = \rho g\ h_B = 0.772$ MPa
To avoid cavitation Δp is about 0.772 MPa

$T = 1.333/0.772 = 1.72$ seconds

5.
a) Sketch the main features of a high-head hydro-electric scheme.

b) Deduce from Newton's laws the amplitude and period of oscillation (APO) in a cylindrical surge tank after a sudden stoppage of flow to the turbine. Assume there is no friction.

c) State the approximate effect of friction on the oscillation.

d) An orifice of one half the tunnel diameter is added in the surge pipe near to the junction with the tunnel. What effect does this have on the APO ?

All the answers to this question are contained in the tutorial.