FLUID MECHANICS D203
SAE SOLUTIONS TUTORIAL 8B – CENTRIFUGAL PUMPS

SELF ASSESSMENT EXERCISE No. 1

1. A centrifugal pump must produce a head of 15 m with a flow rate of 40 dm^3/s and shaft speed of 725 rev/min. The pump must be geometrically similar to either pump A or pump B whose characteristics are shown in the table below.

Which of the two designs will give the highest efficiency and what impeller diameter should be used ?

Pump A D = 0.25 m N = 1 000 rev/min

Q (dm^3/s)	8	11	15	19	
H (m)		8.1	7.9	7.3	6.1
η%	48	55	62	56	

Pump B D = 0.55 m N = 900 rev/min

Q (dm^3/s)	6	8	9	11	
H (m)		42	36	33	27
η%	55	65	66	58	

$$Ns = \frac{NQ^{1/2}}{H^{3/4}} = \frac{725 \text{ x } 0.04^{1/2}}{15^{3/4}} = 19$$

PUMP A

Q (m^3/s)	0.008	0.011
H (m)	8.1	7.9
η%	48	55
Ns	18.6	22.26

Q (m^3/s)	0.06	0.008	0.009
H (m)	42	36	33
η%	55	65	66
Ns	13.36	17.32	19.6

Pump B gives the greater efficiency when Ns = 19

Drawing a graph or interpolating we find Q = 0.085 m^3/s H = 34.5 m , η = 65.5% when Ns=19

$$\frac{Q_1}{N_1 D_1^3} = \frac{Q_2}{N_2 D_2^3} \qquad \frac{0.04}{725 D_1^3} = \frac{0.085}{900 \text{ x } 0.55^3} \qquad D = 0.46 \text{ m}$$

or using the head coefficient

$$\frac{\Delta H_1}{N_1^2 D_1^2} = \frac{\Delta H_2}{N_2^2 D_2^2} \qquad \frac{0.15}{725^2 D_1^2} = \frac{34.5}{19002^2 \text{x} 0.55^2} \qquad D = 0.45 \text{ m}$$

Take the mean D = 0.455 m for the new pump

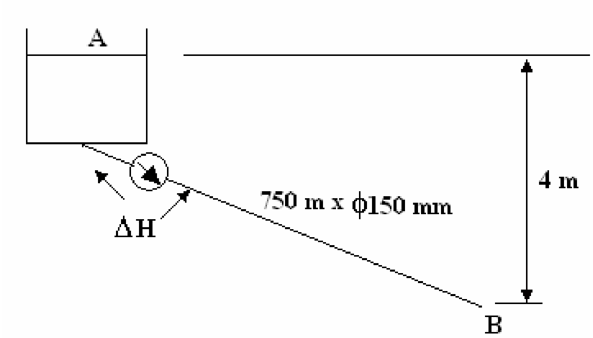
To commence pumping $\sqrt{2gH} = \pi ND/60$ $\quad D = \dfrac{60\sqrt{2g \text{ x } 15}}{\pi \text{ x } 725}$

Hence D = 0.452 m This seems to give the right answer more simply.

2. Define the Head and flow Coefficients for a pump.

Oil is pumped through a pipe 750 m long and 0.15 bore diameter. The outlet is 4 m below the oil level in the supply tank. The pump has an impeller diameter of 508 mm which runs at 600 rev/min. Calculate the flow rate of oil and the power consumed by the pump. It may be assumed $C_f=0.079(Re)^{-0.25}$. The density of the oil is 950 kg/m^3 and the dynamic viscosity is 5 x 10^{-3} N s/m^2. The data for a geometrically similar pump is shown below. D = 0.552 m N = 900 rev/min

Q (m^3/min)	0	1.14	2.27	3.41	4.55	5.68	6.86
H (m)	34.1	37.2	39.9	40.5	38.1	32.9	25.9
η%	0	22	41	56	67	72	65



ΔH = Head Added to the system by the pump.
This may be put into Bernoulli's equation.
$h_A + z_A + u_A^2/2g + \Delta H = h_B + z_B + u_B^2/2g + h_L$
$0 + 4 + 0 + \Delta H = 0 + 0 + u_B^2/2g + h_L$
$4 + \Delta H = u_B^2/2g + h_L$
$u_B = Q/A = Q/(\pi \times 0.075^2) = 56.588\ Q$
$h_L = 4\ C_f\ Lu^2/2gD$ $R_e = \rho uD/\mu = 950 \times (56.588\ Q)0.15/0.005 = 1612758Q$
$C_f = 0.079\ R_e^{-0.25} = 0.079\ (1612758Q)^{-0.25} = 0.002217\ Q^{-0.25}$
$h_L = 4\ (0.002217\ Q^{-0.25}) \times 750\ (56.588\ Q)^2/(2g \times 0.15) = 7236\ Q^{1.75}$
$4 + \Delta H = (56.588\ Q)^2/2g + 7236\ Q^{1.75}$
$\Delta H = 163.2\ Q^2 + 7236\ Q^{1.75} - 4$
If Q is given in m^3/min this becomes
$\Delta H = 0.0453\ Q^2 + 5.594\ Q^{1.75} - 4$ ow create a table for the system.

Q (m^3/min)	0	1	2	3	4	5
ΔH (m)	-4	1.64	15	34.7	60	90.6

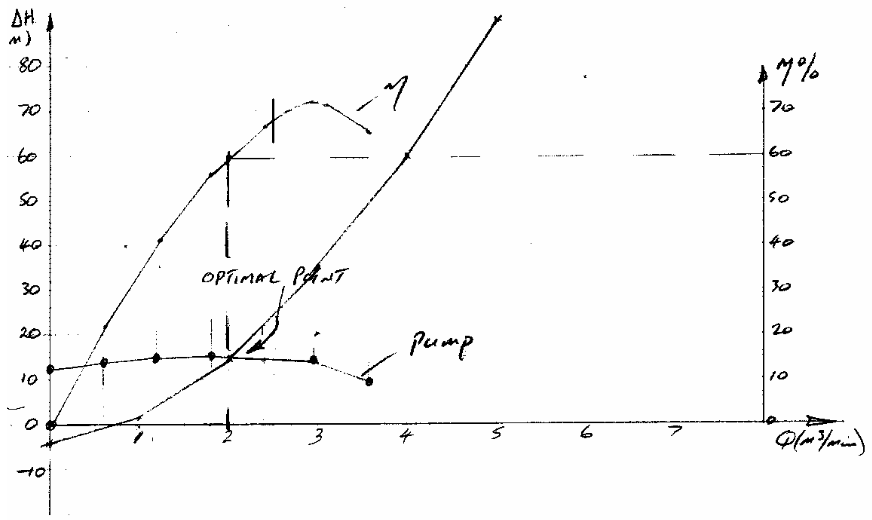
Now compile a table for a similar pump using $Q_2 = \dfrac{N_2 D_2^3 Q_1}{N_1 D_1^3}$ $\Delta H_2 = \dfrac{N_2^2 D_2^2 \Delta H_1}{N_1^2 D_1^2}$

$N_2 = 600$ $D_2 = 508$ $N_1 = 900$ $D_1 = 552$ produces $Q_2 = 0.52Q_1$ $\Delta H_2 = 0.38\Delta.\ _1$

Q_1 (m^3/min)	0	1.14	2.27	3.41	4.55	5.68	6.86
ΔH_1 (m)	34.1	37.2	39.9	40.5	38.1	32.9	25.9
η%	0	22	41	56	67	72	65
Q_2 (m^3/min)	0	0.59	1.18	1.8	2.4	2.9	3,6
ΔH_1 (m)	12.83	14	15	15.2	14.3	12.4	9.8

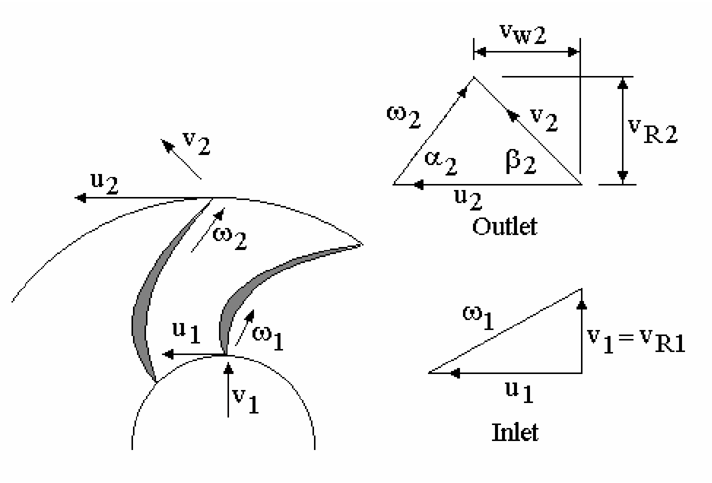
Plotting ΔH for the system against Q and η produces a matching point ΔH = 15 m Q = 2 (m^3/min) and
η = 59%

P = mgΔH/η = 2 x (950/60) x 9.81 x 15/0.59 = 7.89 kW



SELF ASSESSMENT EXERCISE No. 2

1. The rotor of a centrifugal pump is 100 mm diameter and runs at 1 450 rev/min. It is 10 mm deep at the outer edge and swept back at 30°. The inlet flow is radial. the vanes take up 10% of the outlet area. 25% of the outlet velocity head is lost in the volute chamber. Estimate the shut off head and developed head when 8 dm^3/s is pumped. (5.87 m and 1.89 m)



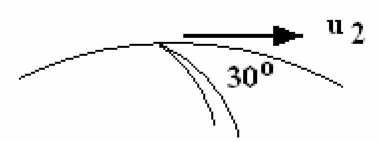
$v_{R2} = Q/A_2$
$= 0.008/(\pi \times 0.1 \times 0.01 \times 0.9) = 2.829$ m/s

OUTLET
$u_2 = \pi N D_2$
$= \pi \times (450/60) \times 0.1 = 7.592$ m/s
$v_{w2} = 7.592 – 2.829/\tan 30^o = 2.692$ m/s
$v_2 = (2.692^2 + 2.829^2)^{½} = 3.905$ m/s
Kinetic Head = $v_2^2/2g$
$= 3.905^2/2g = 0.777$ m
Loss in chamber = 25% x 0.777 = 0.194 m
Manometric Head = $u_2\, v_{w2}/g$
= 7.592 x 2.692/9.81 = 2.08 m
Developed Head = 2.08 – 0.194 = 1.89 m

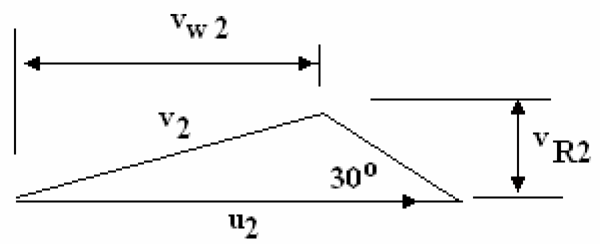
$\Delta h = u_2\, v_{w2}/g = (u_2 – Q/A_2 \tan\alpha_2)$
When there is no flow Q = 0 so $\Delta h = u_2\, v_{w2}/g$ - u_2 = (7.592/9.81) x 7.592 = 5.875 m

2. The rotor of a centrifugal pump is 170 mm diameter and runs at 1 450 rev/min. It is 15 mm deep at the outer edge and swept back at 30°. The inlet flow is radial. the vanes take up 10% of the outlet area. 65% of the outlet velocity head is lost in the volute chamber. The pump delivers 15 dm^3/s of water.

Calculate
   i. The head produced. (9.23 m)
   ii. The efficiency. (75.4%)
   iii. The power consumed. (1.8 kW)



$u_2 = \pi N D_2 = \pi \times (1450/60) \times 0.17 = 12.906$ m/s
$v_{R2} = Q/A_2$
$v_{R2} = 0.015/(\pi \times 0.17 \times 0.015 \times 0.9) = 2.08$ m/s
$v_{w1} = 0$



OUTLET
$v_{w2} = 12.906 – 2.08/\tan 30^o = 9.3$ m/s
$v_2 = (9.3^2 + 2.08^2)^{½} = 9.53$ m/s
Kinetic Head = $v_2^2/2g = 9.53^2/2g = 4.628$ m
Head Recovered = 35% x 4.628 = 1.62 m
Head Loss = 3 m
Manometric Head = $u_2\, v_{w2}/g$
= 12.906 x 9.3/9.81 = 12.23 m
Developed Head 12.23 – 3= 9.23 m

$\eta_{man} = 9.23/12.23 = 75.3\%$

DP = m $u_2\, v_{w2}$ = 15 x 12.906 x 9.3 = 1.8 kW
WP = m g Δh = 15 x 9.81 x 9.23 = 1.358 kW
$\eta = 1.358/1.8 = 75.4\%$