FLUID MECHANICS D203
SAE SOLUTIONS TUTORIAL 8A –TURBINES

SELF ASSESSMENT EXERCISE No. 1

1. The buckets of a Pelton wheel revolve on a mean diameter of 1.5 m at 1500 rev/min. The jet velocity is 1.8 times the bucket velocity. Calculate the water flow rate required to produce a power output of 2MW. The mechanical efficiency is 80% and the blade friction coefficient is 0.97. The deflection angle is 165°.

$D = 1.5$ m $N = 1500$ rev/min $v = 1.8\,u$ $\eta = 80\%$ $k = 0.97$ $\theta = 165^o$

Diagram Power = 2MW/0.8 = 2.5 MW
$u = \pi ND/60 = 117.8$ m/s
$v = 1.8 \times 117.8 = 212$ m/s
$DP = m\,u\,(v-u)\,(1- k\cos\theta) = 2.5 \times 10^6$
$m \times 117.8\,(94.24)(1- 0.97\cos 165) = 2.5 \times 10^6$
$m = 2.5 \times 10^6/21503 = 116.26$ kg/s

2. Calculate the diagram power for a Pelton Wheel 2m mean diameter revolving at 3000 rev/min with a deflection angle of 170° under the action of two nozzles , each supplying 10 kg/s of water with a velocity twice the bucket velocity. The blade friction coefficient is 0.98.

If the coefficient of velocity is 0.97, calculate the pressure behind the nozzles.
(Ans 209.8 MPa)

$D = 2$m $N = 3000$ rev/min $\theta = 170^o$ $v = 2u$ $k = 0.98$ $c_v = 0.97$ $m = 2 \times 10 = 20$ kg/s
$u = \pi ND/60 = 314.16$ m/s
$DP = m\,u\,(v-u)\,(1- k\cos\theta) = 20 \times 314.16 \times 314.16(1- 0.98\cos 170^o) = 3.879$ MW
$v = c_v \sqrt{2\Delta p/\rho}$
$\Delta p = (314.16 \times 2/0.97)^2 \times 1000/2 = 209.8$ MPa

3. A Pelton Wheel is 1.7 m mean diameter and runs at maximum power. It is supplied from two nozzles. The gauge pressure head behind each nozzle is 180 metres of water. Other data for the wheel is :

Coefficient of Discharge C_d= 0.99
Coefficient of velocity C_v= 0.995
Deflection angle = 165°.
Blade friction coefficient = 0.98
Mechanical efficiency = 87%
Nozzle diameters = 30 mm

Calculate the following.

i. The jet velocity (59.13 m/s)
ii. The mass flow rate (41.586 kg/s)
iii The water power (73.432 kW)
iv. The diagram power (70.759 kW)
v. The diagram efficiency (96.36%)
vi. The overall efficiency (83.8%)
vii. The wheel speed in rev/min (332 rev/min)

$D = 1.7$ m $\Delta H = 180$ m $c_d = 0.99$ $c_v = 0.995$ $c_c = c_d/c_v = 0.995$ $\rho = 1000$ kg/m^3

Power is maximum so $v = 2u$ 2 nozzles

$v = c_v \sqrt{2g\,\Delta H} = 0.995 \sqrt{(2g \times 180)} = 59.13$ m/s
$m = c_c\, \rho\, A\, v = 0.995 \times 1000\, (\pi \times 0.03^2/4) \times 59.13 = 41.587$ kg/s per nozzle
Water Power = $mg\,\Delta H = 41.587 \times 9.81 \times 180 = 73.43$ kW per nozzle.
$u = v/2 = 29.565$ m/s
Diagram Power = $m\, u\, (v-u)\, (1 - k\cos\theta)$
$DP = 41.587 \times 29.565\, (29.565)(1 - 0.98\cos 165) = 70.76$ kW per nozzle
$\eta_d = 70.76/73.43 = 83.8\%$
Mechanical Power = $70.76 \times 87\% = 61.56$ kW per nozzle.
$\eta_{oa} = 61.56/73.43 = 83.8\%$
$N = 60u/\pi D = 29.565 \times 60/(\pi \times 1.7) = 332.1$ rev/min

4. Explain the significance and use of 'specific speed $Ns = \dfrac{NP^{1/2}}{\rho^{1/2}(gH)^{5/4}}$

Calculate the specific speed of a Pelton Wheel given the following.
d = nozzle diameter. D = Wheel diameter.
u = optimum blade speed = $0.46\, v_1$ v_1 = jet speed.
$\eta = 88\%$ C_V = coefficient of velocity = 0.98

$v_j = c_v\sqrt{2gH} = 0.98\sqrt{2gH} = 4.34H^{1/2}$ $u = 0.46\, v_j = \pi ND/60$

$$N = \frac{0.46\, v_j \times 60}{\pi D} = \frac{0.46 \times 4.34H^{1/2} \times 60}{\pi D} = 38.128\frac{H^{1/2}}{D}$$

$Q = A_j v_j = (\pi d^2/4) \times 4.34\, H^{½} = 3.41\, H^{½}\, d^2$

$P = \eta\, m\, g\, H = \eta \times \rho g\, Q\, H = 0.88 \times 1000 \times 9.81 \times 3.41 \times H^{½}\, d^2 = 29438\, H^{½}\, d^2\, H$

$$Ns = 38.128\frac{H^{1/2}}{D} \times \frac{\left(28438\, H^{3/2} d^2\right)^{1/2}}{\rho^{1/2}(gH)^{5/4}} = \frac{38.128 \times 28438^{1/2}}{1000^{1/2}\, 9.81^{5/4}} \times \frac{H^{1/2} H^{3/4} d}{D\, H^{5/4}} = 11.9\frac{d}{D}$$

5. A turbine is to run at 150 rev/min under a head difference of 22 m and an expected flow rate of 85 m^3/s.
A scale model is made and tested with a flow rate of 0.1 m^3/s and a head difference of 5 m. Determine the scale and speed of the model in order to obtain valid results.

When tested at the speed calculated, the power was 4.5 kW. Predict the power and efficiency of the full size turbine.

$N_1 = 150$ rev/min $Q_1 = 85\ m^3/s$ $\Delta H_1 = 22$ m
$Q_2 = 0.1\ m^3/s$ $\Delta H_2 = 55$ m

For similarity of Head Coefficient we have

$$\frac{\Delta H_1}{N_1^2 D_1^2} = \frac{\Delta H_2}{N_2^2 D_2^2} \qquad \frac{D_2^2}{D_1^2} = \frac{5 \text{x} 150^2}{22 N_2^2} = \frac{5114}{N_2^2} \qquad \frac{D_2}{D_1} = \sqrt{\frac{5114}{N_2^2}} = \frac{71.51}{N_2}$$

For similarity of Flow Coefficient we have

$$\frac{Q_1}{N_1 D_1^3} = \frac{Q_2}{N_2 D_2^3} \qquad \frac{D_2^3}{D_1^3} = \frac{0.1 \text{x} 150}{85 N_2} = \frac{0.176}{N_2} \qquad \frac{D_2}{D_1} = \sqrt[3]{\frac{71.51}{N_2}} = \frac{0.560}{N_2^{1/2}}$$

Equate $\dfrac{D_2}{D_1} = \dfrac{71.51}{N_2} = \dfrac{0.560}{N_2^{1/2}}$ $N_2^{2/3} = \dfrac{71.51}{0.56}$ $N_2 = 1443$ rev/min

$$\frac{D_2}{D_1} = 0.0496$$

Note if we use $\frac{N_1 Q_1^{1/2}}{H_1^{3/4}} = \frac{N_2 Q_2^{1/2}}{H_2^{3/4}}$ we get the same result.

Power Coefficient

$$\frac{P_1}{\rho N_1^3 D_1^5} = \frac{P_2}{\rho N_2^3 D_2^5} \qquad P_1 = \frac{P_2\left(\rho N_1^3 D_1^5\right)}{\rho N_2^3 D_2^5} = \frac{P_2 N_1^3 D_1^5}{N_2^3 D_2^5} = \frac{4.5 \text{ x } 150^3 \text{ x}\left(\frac{1}{0.05}\right)^5}{1443^3} = 16.2\text{MW}$$

Water Power = mgΔH = (85 x 1000) x 9.81 x 22 = 18.3 MW

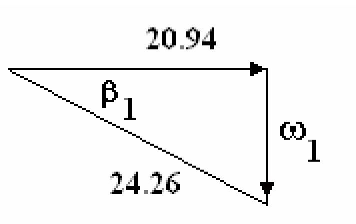
H = 16.2/18.3 = 88%

SELF ASSESSMENT EXERCISE No.2

1. The following data is for a Francis Wheel
   Radial velocity is constant
   No whirl at exit.
   Flow rate=0.4 m^3/s D_1=0.4 m D_2=0.15 m k =0.95 α_1=90^o N=1000 rev/min
   Head at inlet = 56 m head at entry to rotor = 26 m head at exit = 0 m
   Entry is shock less.

Calculate i. the inlet velocity v_1 (24.26 m/s)
ii. the guide vane angle (30.3^o)
iii. the vane height at inlet and outlet (27.3 mm, 72.9 mm)
iv. the diagram power (175.4 MW)
v. the hydraulic efficiency (80%)

$v_1 = (2gh)^{½}$ = {2 x 9.81 x (56 – 26)} ^{½} = 24.26 m/s
u_1 = πND/60 = π x 1000 x 0.4/60 = 20.94 m/s
$\beta_1 = \cos^{-1}$ (20.94/24.26) = 30.3^o
$\omega_1 = v_{r1}$ = 12.25 m/s
Q = 0.4 = π D t k v_r
t_1 = 0.4/(π x 0.4 x 0.95 x 12.25) =0.0273 m
t_2 = 0.4/(π x 0.15 x 0.95 x 12.25) =0.0729 m
v_{w1} = 20.94 v_{w2} = 0
P = mu_1 v_{w1} = 400 x 20.94 x 20.94 = 174.4 kW
Water Power = m g H = 400 x 9.81 x 56 = 219.7 kW
η = 174.4/219.7 = 80%

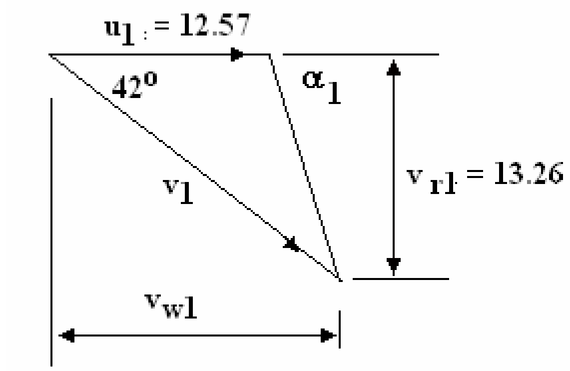


2. A radial flow turbine has a rotor 400 mm diameter and runs at 600 rev/min. The vanes are 30 mm high at the outer edge. The vanes are inclined at 42^o to the tangent to the inner edge. The flow rate is 0.5 m^3/s and leaves the rotor radially. Determine

   i. the inlet velocity as it leaves the guide vanes. (19.81 m/s)
   ii. the inlet vane angle. (80.8^o)
   iii. the power developed. (92.5 kW)

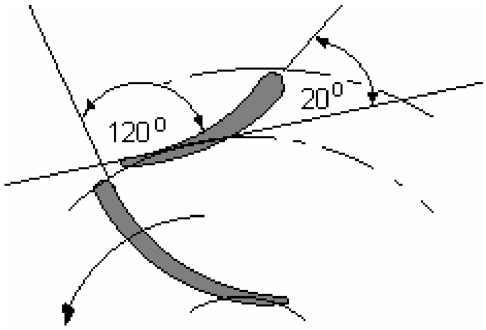
Radial Flow Turbine Inlet is the outer edge. u_1 = πND/60 = π x 600 x 0.4/60 = 12.57 m/s
v_{r1} = Q/πDt = 0.5/(π x 0.4 x 0.03) = 13.26 m/s

13.26/v_{w1} = tan 42^o
v_{w1} = 14.72 m/s
$v_1 = (13.26^2 + 14.72^2)^{½}$ = 19.81 m/s



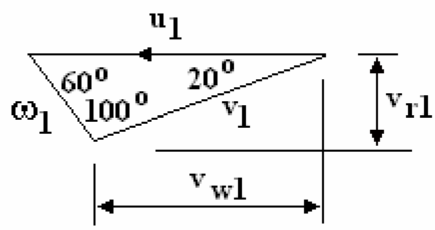
$13.26/(14.72 – 12.57) = \tan \alpha_1$
$\alpha_1 = 80.8^o$
$v_{w2} = 0$
$DP = mu \ = v_{w1}$
$DP = 500 \times 12.57 \times 14.72 = 92.5$ kW

3. The runner (rotor) of a Francis turbine has a blade configuration as shown. The outer diameter is 0.45 m and the inner diameter is 0.3 m. The vanes are 62.5 mm high at inlet and 100 mm at outlet. The supply head is 18 m and the losses in the guide vanes and runner are equivalent to 0.36 m. The water exhausts from the middle at atmospheric pressure. Entry is shock less and there is no whirl at exit. Neglecting the blade thickness, determine :



i. The speed of rotation.
ii. The flow rate.
iii. The output power given a mechanical efficiency of 90%.
iv. The overall efficiency.
v. The outlet vane angle.

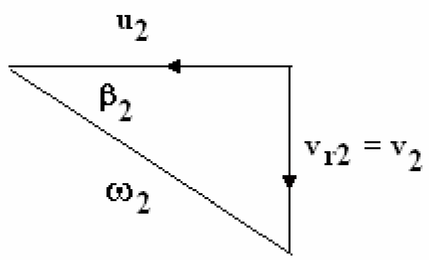
INLET
Useful head is 18 – 0.36 = 17.64 m



$m\, u_1\, v_{w1} = m\, u_2\, v_{w2}$
$u_1\, v_{w1} = u_2\, v_{w2}$

$(u_1\, v_{w1}/g) = \Delta H = 17.64$

sine rule $(v_1/\sin 60) = (u_1/\sin 100)$
$v_1 = 0.879\, u_1$
$(v_{r1}/ v_1) = \sin 20$ $\qquad v_1 = 2.923\, v_{r1}$
Equate $\qquad 0.879\, u_1 = 2.923\, v_{r1} \qquad v_{r1} = 0.3\, u_1$
$v_{w1} = v_{r1}/\tan 20 = 0.824\, u_1$
$17.64 = u_1 \times 0.824\, u_1 /g \qquad u_1^2 = 210 \quad u_1 = 14.5$ m/s
$v_{r1} = 0.3\, u_1 = 4.35$ m/s



EXIT
$u = \pi N D \quad N = u_1 / \pi D_1 = u_2 / \pi D_2$
$u_2 = u_1 D_1 / D_2 = 14.4 \times 300/450 = 9.67$ m/s
$N = u_1/ \pi D_1 = 14.5 \times 60/(\pi \times 0.45) = 615$ rev/min
$v_r = Q/\pi Dh$
$v_{r1} = 4.35 = Q/\pi D_1 h_1 = Q/(\pi \times 0.45 \times 0.0625)$
$Q = 0.384\ m^3/s$
$v_{r2} = Q/\pi D_2 h_2 = Q/(\pi \times 0.3 \times 0.1) = 10.61\, Q = 4.08$ m/s
$4.08/9.67 = \tan \beta_2$
$\beta_2 = 22.8^o$

$P = m\, g\, \Delta H = 384 \times 9.81 \times 17.64 = 66.45$ kW

Output Power = 66.45 x 90% = 59.8 kW

Overall efficiency = $59800/(m\, g\ \Delta H) = 58805/(384 \times 9.81 \times 18) = 88.2$ %