FLUID MECHANICS D203
SAE SOLUTIONS TUTORIAL 7 – FLUID FORCES

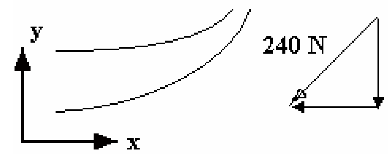
SELF ASSESSMENT EXERCISE No. 1

1. A pipe bends through an angle of 90^o in the vertical plane. At the inlet it has a cross sectional area of 0.003 m^2 and a gauge pressure of 500 kPa. At exit it has an area of 0.001 m^2 and a gauge pressure of 200 kPa.
Calculate the vertical and horizontal forces due to the pressure only.

$F_h = 500\,000 \times 0.003 = 1500$ N$\rightarrow$ $F_v = 200\,000 \times 0.001 = 200$ N$\downarrow$

2. A pipe bends through an angle of 45^o in the vertical plane. At the inlet it has a cross sectional area of 0.002 m^2 and a gauge pressure of 800 kPa. At exit it has an area of 0.0008 m^2 and a gauge pressure of 300 kPa.
Calculate the vertical and horizontal forces due to the pressure only.

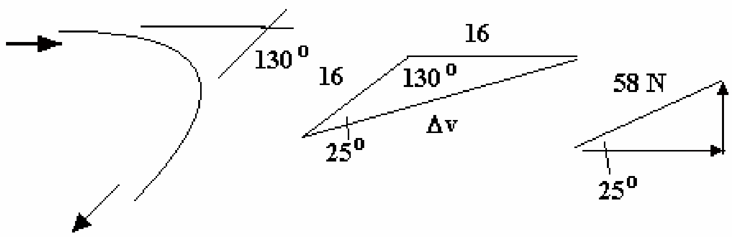
$F_{p1} = 800\,000 \times 0.002 = 1600$ N $F_{px1} = 1600$ N $F_{py1} = 0$
$F_{p2} = 300\,000 \times 0.0008 = 240$ N



$F_{py2} = 240 \sin 45^o = 169.7$ N $F_{px2} = 240 \cos 45^o = 169.7$ N

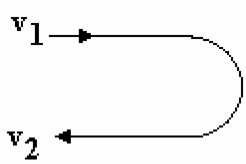
Totals $F_h = 1600 - 169.7 = 1430$ N $F_v = 0 - 169.7 = -169.7$ N

3. Calculate the momentum force acting on a bend of 130^o that carries 2 kg/s of water at 16m/s velocity.
Determine the vertical and horizontal components.



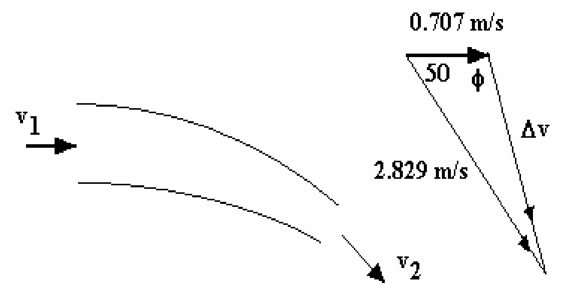
$\Delta v = 16 \sin 130/\sin 25 = 29$ m/s $F = m\,\Delta v = 2 \times 29 = 58$ N
$F_v = 58 \sin 25 = 24.5$ N $F_h = 58 \cos 25 = 52.57$ N

4. Calculate the momentum force on a 180^o bend that carries 5 kg/s of water. The pipe is 50 mm bore diameter throughout. The density is 1000 kg/m^3.



$v_1 = Q/A = m/\rho A = 5/(1000 \times \pi \times 0.025^2) = 2.546$ m/s
$v_2 = -2.546$ m/s
$\Delta v = 2.546 - (-2.546) = 5.093$ m/s $F = m\,\Delta v = 5 \times 5.093 = 25.25$ N

5. A horizontal pipe bend reduces from 300 mm bore diameter at inlet to 150 mm diameter at outlet. The bend is swept through 50^o from its initial direction.
The flow rate is 0.05 m^3/s and the density is 1000 kg/m^3. Calculate the momentum force on the bend and resolve it into two perpendicular directions relative to the initial direction.



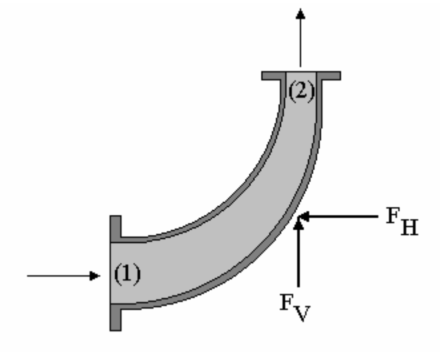
$$\Delta v = \sqrt{(2.82\sin 50)^2 + (2.825\sin 50 - 0.707)^2} = 2.4359\text{m/s}$$

$$\varphi = \tan^{-1}\left(\frac{2.825\sin 50}{2.82\cos 50 - .707}\right) = 62.8^o$$

F = m Δv = 50 x 2.43 = 121.5 N
Fv = 121.5 sin 6.84 = 108.1 N   Fh = 121.5 cos 62.84 = 55.46 N

SELF ASSESSMENT EXERCISE No. 2

Assume the density of water is 1000 kg/m^3 throughout.
1. A pipe bends through 90^o from its initial direction as shown in fig.13. The pipe reduces in diameter such that the velocity at point (2) is 1.5 times the velocity at point (1). The pipe is 200 mm diameter at point (1) and the static pressure is 100 kPa. The volume flow rate is 0.2 m^3/s. Assume there is no friction. Calculate the following.
a) The static pressure at (2).
b) The velocity at (2).
c) The horizontal and vertical forces on the bend F_H and F_V.
d) The total resultant force on the bend.



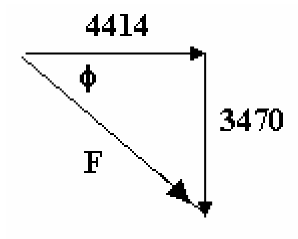
$u_2 = 1.5\ u_1$ $D_1 = 200$ mm $p_1 = 100$ kPa $Q = 0.02\ m^3/s$
$m = 200$ kg/s $A_1 = \pi D_1^2/4 = 0.0314\ m^2$
$u_1 = Q/A_1 = 6.37$ m/s $u_2 = 1.5\ u_1 = 9.55$ m/s

Bernoulli $p_1 + \rho u_1^2/2 = p_2 + \rho u_2^2/2$
Gauge pressures assumed.
$100\ 000 + 1000 \times 6.37^2/2 = p_2 + 1000 \times 9.55^2/2$
$p_2 = 74.59$ kPa $A_2 = Q/u_2 = 0.0209\ m^2$

$F_{p1} = p_1 A_1 = 3140\ N \rightarrow$ $F_{p2} = p_2 A_2 = 1560\ N \downarrow$

$F_{m1} = m\Delta v$ (hor) = 200 (0 – 6.37) = -1274 N on water and 1274 N on bend →
$F_{m2} = m\Delta v$ (vert) = 200 (9.55 - 0) = 1910 N on water and -1910 N on bend ↓
Total horizontal force on bend = 3140 + 1274 = 4414→
Total vertical force on bend = 1560↓ + 1910 = 3470 N ↓

$F = \sqrt{(4414^2 + 3470^2)} = 561$ N $\phi = \tan^{-1}(3470/4414) = 38.1^o$



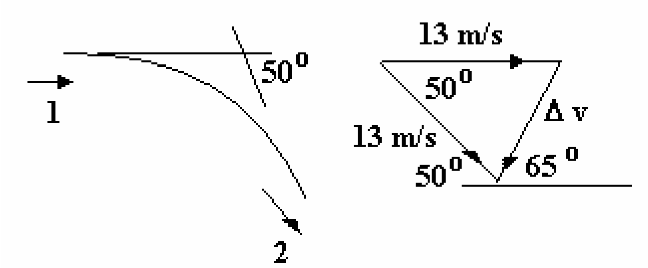
2. A nozzle produces a jet of water. The gauge pressure behind the nozzle is 2 MPa. The exit diameter is 100 mm. The coefficient of velocity is 0.97 and there is no contraction of the jet. The approach velocity is negligible. The jet of water is deflected 165º from its initial direction by a stationary vane. Calculate the resultant force on the nozzle and on the vane due to momentum changes only.

$c_v = 0.97$ $\Delta p = 2$ MPa $\rho = 1000$ kg/m^3

$v_1 = c_v \sqrt{(2\Delta p/\rho)} = 0.97\sqrt{(2 \times 2 \times 10^6/1000)} = 61.35$ m/s
$m = \rho A_1 v_1 = 1000 \times \pi \times 0.1^2/4 \times 61.35 = 481.8$ kg/s
Force on Nozzle = $m \Delta v = 481.8 \times (61.35 - 0) = 29.56$ kN
Force on vane = $m \Delta v$ $\Delta v = 61.35\sqrt{\{2(1 - \cos 165^o)\}} = 121.6$ m/s
Force on vane = $m \Delta v = 481.8 \times 121.6 = 58.6$ kN

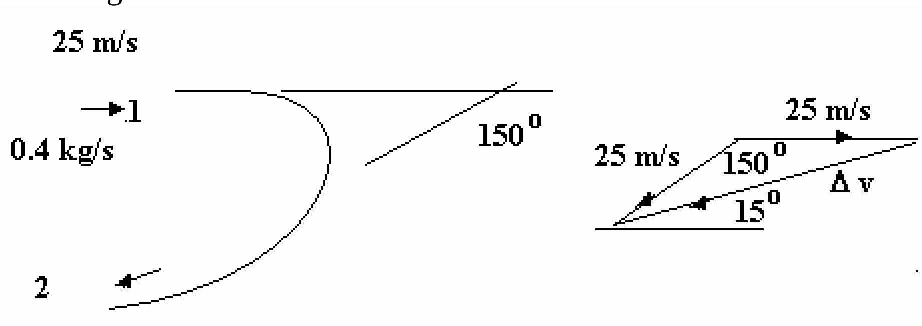
3. A stationary vane deflects 5 kg/s of water 50º from its initial direction. The jet velocity is 13 m/s. Draw the vector diagram to scale showing the velocity change. Deduce by either scaling or calculation the change in velocity and go on to calculate the force on the vane in the original direction of the jet.

$v_1 = 13$ m/s $m = 5$ kg/s
$\Delta v = 13 \sin 50^o/\sin 65^o = 10.99$ m/s
Force on vane = $m \Delta v = 5 \times 10.99 = 54.9$ N
Horizontal component is $F\cos 65^o = 23.2$ N



4. A jet of water travelling with a velocity of 25 m/s and flow rate 0.4 kg/s is deflected 150º from its initial direction by a stationary vane. Calculate the force on the vane acting parallel to and perpendicular to the initial direction.

$v_1 = 25$ m/s $m = 0.4$ kg/s

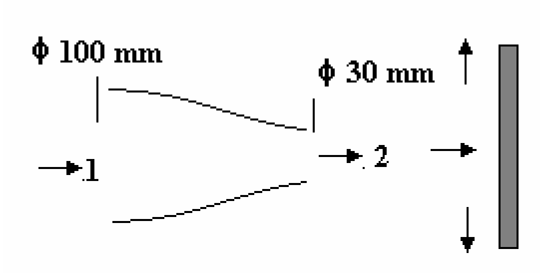


$\Delta v = 25 \sqrt{\{2(1 - \cos 150^o)\}} = 48.3$ m/s $F = m \Delta v = 0.4 \times 48.3 = 19.32$ N
$F_v = 19.32 \sin 15^o = 5$ N
$F_h = 19.32 \cos 15^o = 18.66$ N

5. A jet of water discharges from a nozzle 30 mm diameter with a flow rate of 15 dm^3/s into the atmosphere. The inlet to the nozzle is 100 mm diameter. There is no friction nor contraction of the jet. Calculate the following.

i. the jet velocity. ii. the gauge pressure at inlet. iii. the force on the nozzle.

The jet strikes a flat stationary plate normal to it. Determine the force on the plate.
$Q = 0.015\ m^3/s$ $\rho = 1000\ kg/m^3$ $m = 15\ kg/s$



$A_1 = \pi \times 0.1^2/4 = 0.00785\ m^2$
$v_1 = Q/A_1 = 0.015 \div 0.00785 = 1.901\ m/s$
$A_2 = \pi \times 0.03^2/4 = 0.0007068\ m^2$
$v_2 = Q/A_2 = 0.015 \div 0.0007068 = 21.22\ m/s$

Bernoulli $p_1 + \rho v_1^2/2 = p_2 + \rho v_2^2/2$
Gauge pressures assumed.
$p_1 + 1000 \times 1.901^2/2 = 0 + 1000 \times 21.22^2/2$
$p_1 = 223.2\ kPa$

Force on nozzle $= (p_1A_1 - p_2A_2) + m(v_2 - v_1)$ v_1 is approximately zero.
$= (223.2 \times 10^3 \times 0.00785 - 0) + 15(21.22 - 0) = 2039\ N \leftarrow$
Force on Plate = m Δv Δv in horizontal direction is 21.22
Force on Plate = 15 x 21.22 = 311.8 N $\rightarrow$
Some common sense is needed determining the directions.

<u>SELF ASSESSMENT EXERCISE No.3</u>

1. A vane moving at 30 m/s has a deflection angle of 90°. The water jet moves at 50 m/s with a flow of 2.5 kg/s. Calculate the diagram power assuming that all the mass strikes the vane.

$\rho = 100\ kg/m^3$ $m = 2.5\ kg/s$ $u = 30\ m/s$ $v = 50\ m/s$

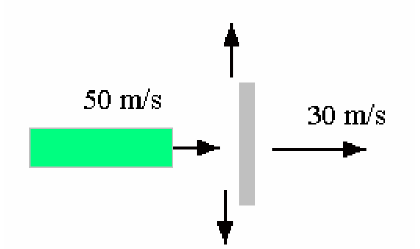
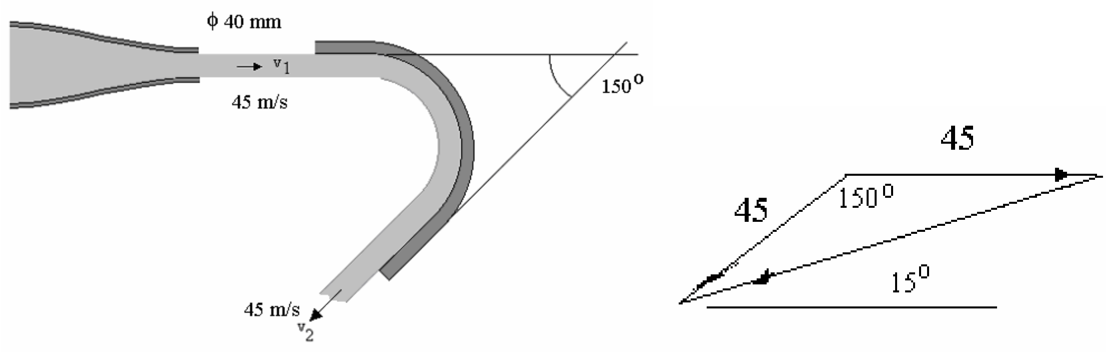


Diagram Power = mu (v – u) = 2.5x 30 (50 – 30) = 1500 Watts

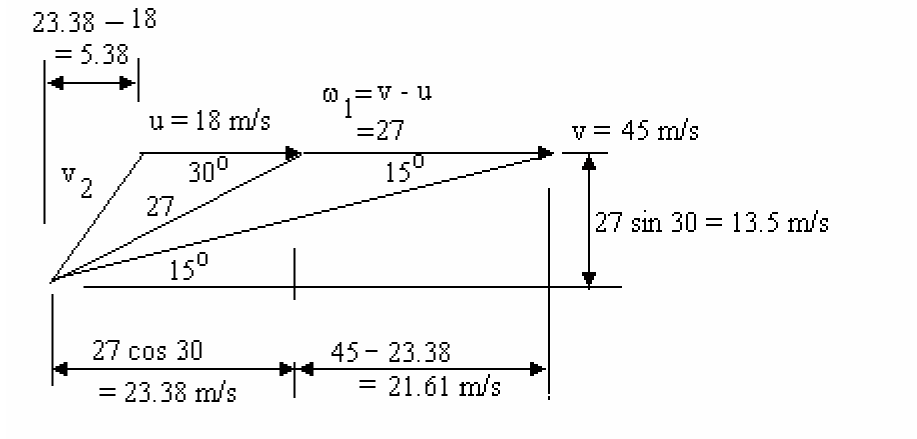
2. Figure 10 shows a jet of water 40 mm diameter flowing at 45 m/s onto a curved fixed vane. The deflection angle is 150°. There is no friction. Determine the magnitude and direction of the resultant force on the vane.

The vane is allowed to move away from the nozzle in the same direction as the jet at a velocity of 18 m/s. Draw the vector diagram for the velocity at exit from the vane and determine the magnitude and direction of the velocity at exit from the vane.



STATIONARY VANE
$\Delta v = 45\sqrt{\{2(1-\cos 150^o)\}} = 86.93$ m/s $m = \rho A v = 1000 \times \pi \times 0.04^2/4 \times 45 = 56.54$ kg/s
$F = m\,\Delta v = 4916$ N

MOVING VANE



The relative velocity at exit is $\omega_2 = 27$ m/s
The absolute velocity $v_2 = \sqrt{(13.5^2 + 5.38^2)} = 14.53$ m/s