FLUID MECHANICS D203
SAE SOLUTIONS TUTORIAL 4 – FLOW THROUGH POROUS PASSAGES

SELF ASSESSMENT EXERCISE No.1

Q.1
Outline briefly the derivation of the Carman-Kozeny equation.

$$\frac{dp}{dl} = -\frac{180\,\mu\, u(1-\varepsilon)^2}{d_s^2\varepsilon^3}$$

dp/dl is the pressure gradient, μ is the fluid viscosity, u is the superficial velocity, d_s is the particle diameter and ε is the void fraction.

A cartridge filter consists of an annular piece of material of length 150 mm and internal diameter and external diameters 10 mm and 20 mm. Water at 25°C flows radially inwards under the influence of a pressure difference of 0.1 bar. Determine the volumetric flow rate. (21.53 cm^3/s)

For the filter material take d = 0.05 mm and ε = 0.35.
μ=0.89 x 10^{-3} N s/m^2 and ρ= 997 kg/m^3.

The solution for part 1 is as given in the tutorial.

For radial flow we change dl to dr $\frac{dp}{dr} = -\frac{180\,\mu\; u(1-\varepsilon)^2}{d_s^2\varepsilon^3}$

The surface area of the annulus is 2πrL L = 0.15 m and d = 0.05 x 10^{-3} m
The velocity is u = Q/2πrL

$$\frac{dp}{dr} = -\frac{180\,\mu\;(1-\varepsilon)^2}{d_s^2\varepsilon^3}\,x\,\frac{Q}{2\,\pi\, r\, L}$$

$$\frac{dp}{dr} = -\frac{180 \text{ x } 0.89 \text{ x}10^{-3}\;(1-0.35)^2}{(0.05 \text{ x } 10^{-3})^2\, 0.35^3}\,x\,\frac{Q}{2\,\pi\;\; r \text{ x } 0.15}$$

$$dp = -670 \text{ x } 10^{-6}\, Q\,\frac{dr}{r}$$

Integrate $\Delta p = -670 \text{ x } 10^{-6}\, Q \ln\left(\frac{r_1}{r_2}\right) = -670 \text{ x } 10^{-6}\, Q \ln(2)$

Δp = -0.1 x 10^5 = -464.4 x 10^6 Q
Q = 21.53 x 10^{-6} m^3/s

Q.2

(a) Discuss the assumptions leading to the equation of horizontal viscous flow through a packed bed

$$\frac{dp}{dL} = -\frac{180\mu u(1-\varepsilon)^2}{d_s^2\varepsilon^3}$$

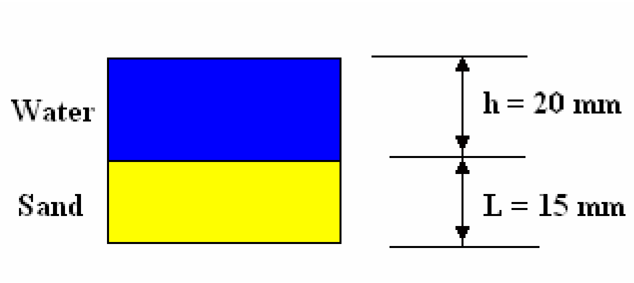
Δp is the pressure drop across a bed of depth L, void fraction ε and effective particle diameter d. u is the approach velocity and μ is the viscosity of the fluid.

(b) Water percolates downwards through a sand filter of thickness 15 mm, consisting of sand grains of effective diameter 0.3 mm and void fraction 0.45. The depth of the effectively stagnant clear water above the filter is 20 mm and the pressure at the base of the filter is atmospheric. Calculate the volumetric flow rate per m^2 of filter. (2.2 dm^3/s)

(Note the density and viscosity of water are given in the instructions on all exams papers)

μ=0.89 x 10^{-3} N s/m^2 and ρ= 997 kg/m^3

Part (a) is as stated in the tutorial.



Δp = ρ g h = 997 x 9.81 x 0.02 = 195.61 Pa

$$\frac{dp}{dL} = -\frac{195.61}{0.015} - \frac{180 \times 0.89 \times 10^{-3} u (1-0.45)^2}{(0.3 \times 10^{-3})^2 0.45^3}$$

u = 0.0022 m/s

Q = u A = 0.022 m^3/s per unit area

Q3.

Oil is extracted from a horizontal oil-bearing stratum of thickness 15 m into a vertical bore hole of radius 0.18 m. Find the rate of extraction of the oil if the pressure in the bore-hole is 250 bar and the pressure 300 m from the bore hole is 350 bar.

Take d= 0.05 mm, ε= 0.30 and μ = 5.0 x 10^{-3} N s/m^2.

$$\frac{dp}{dr} = -\frac{180\,\mu\; u(1-\varepsilon)^2}{d_s^2\varepsilon^3}$$

$$\frac{dp}{dr} = -\frac{180\,\mu\;(1-\varepsilon)^2}{d_s^2\varepsilon^3}\,x\,\frac{Q}{2\,\pi\,r\,L}$$

The surface area of the annulus is 2πrL L = 15 m and d = 0.05 x 10^{-3} m

The velocity is u = Q/2πrL

$$\frac{dp}{dr} = -\frac{180\,\mu\;(1-\varepsilon)^2}{d_s^2\varepsilon^3}\,x\,\frac{Q}{2\,\pi\,r\,L}$$

$$\frac{dp}{dr} = -\frac{180\text{ x }5\text{ x }10^{-3}\;(1-0.3)^2}{\left(0.05\text{ x }10^{-3}\right)^2 0.3^3}\,x\,\frac{Q}{2\,\pi\;\;r\text{ x }15}$$

$$dp = -69.32\text{ x }10^6\,Q\frac{dr}{r}$$

Integrate $\Delta p = -69.32\text{ x }10^6\,Q\ln\left(\frac{r_1}{r_2}\right) = -69.32\text{ x }10^6\,Q\ln\left(\frac{300}{0.18}\right) = 514.3\text{ x }10^6\,Q$

Δp = 250 – 350 = -100bar

Q = 100 x 10^5/514.3 x 10^6 = 0.01944 m^3/s