FLUID MECHANICS D203
SAE SOLUTIONS TUTORIAL 3 – BOUNDARY LAYERS

SELF ASSESSMENT EXERCISE 4

1. The BL over a plate is described by $u/u_1=\sin(\pi y/2\delta)$. Show that the momentum thickness is 0.137δ.

$$\theta=\int_0^\delta\left[\frac{u}{u_1}\right]\left[1-\frac{u}{u_1}\right]dy=\int_0^\delta\left[\frac{u}{u_1}-\left(\frac{u}{u_1}\right)^2\right]dy=\int_0^\delta\left[\sin\left\{\frac{\pi y}{2\delta}\right\}-\left(\sin\left\{\frac{\pi y}{2\delta}\right\}\right)^2\right]dy$$

We need the trig identity $\sin^2 A = ½ - ½ \cos 2A$

$$\theta=\int_0^\delta\left[\sin\left\{\frac{\pi y}{2\delta}\right\}-\frac{1}{2}+\frac{1}{2}\left(\cos\left\{\frac{\pi y}{2\delta}\right\}\right)\right]dy$$

$$\theta=\left[-\frac{2\delta}{\pi}\cos\left\{\frac{\pi y}{2\delta}\right\}-\frac{y}{2}+\frac{\delta}{2\pi}\sin\left\{\frac{\pi y}{2\delta}\right\}\right]_0^\delta$$

$$\theta=\left[-0-\frac{\delta}{2}+0\right]-\left[-\frac{2\delta}{\pi}-0+0\right]=0.137\delta$$

2. The velocity profile in a laminar boundary layer on a flat plate is to be modelled by the cubic expression $u/u_1=a_0+a_1y + a_2y^2+a^3y^3$
where u is the velocity a distance y from the wall and u_1 is the main stream velocity.

Explain why a_0 and a_2 are zero and evaluate the constants a_1 and a_3 in terms of the boundary layer thickness δ.

Define the momentum thickness θ and show that it equals $39\delta/280$
Hence evaluate the constant A in the expression $\delta/x = A\,(R_{ex})^{-0.5}$
where x is the distance from the leading edge of the plate. It may be assumed without proof that the friction factor $C_f = 2\, d\theta/dx$

At y = 0, u = 0 so it follows that $a_o = 0$

$d^2u/dy^2 = 0$ @ y = 0 so $a_2 = 0$. Show for yourself that this is so.

The law is reduced to $u/u_l = a_l y + a_3y^3$
At $y = \delta$, $u = u_1$ so $1 = a_1\delta + 3a_3\,\delta^2$
hence $a_l = (1-a_3\delta^3)/\delta$

Now differentiate and $du/dy = u_l\,(a_1 + 3a_3y^2)$
at $y = \delta$, du/dy is zero so $0 = a_1 + 3a_3\,\delta^2$ so $a_1 = -3a_3\delta^2$

Hence by equating $a_1 = 3/2\delta$ and $a_3 = -1/2\delta^3$

Now we can write the velocity distribution as $u/u_1=3y/2\delta - (y/\delta)^3/2$

and $du/dy = u_1\,\{3/2\delta + 3y^2/2\delta^3\}$

If we let $y/\delta = \eta$ $u/u_1=\{3\eta/2 + (\eta)^3/2\}$

The momentum thickness is

$$\theta = \int_0^{\delta} \left[\frac{u}{u_1}\right]\left[1-\frac{u}{u_1}\right] dy \text{ but } dy = \delta d\eta$$

$$\theta = \int_0^{1} \left[\frac{3\eta}{2}-\frac{\eta^3}{2}\right]\left[1-\frac{3\eta}{2}-\frac{\eta^3}{2}\right] d\eta$$

Integrating gives: $\theta = \delta\left[\frac{3\eta^2}{4}-\frac{\eta^4}{8}-\frac{9\eta^3}{12}-\frac{\eta^7}{28}+\frac{3\eta^5}{10}\right]$

Between the limits $\eta = 0$ and $\eta = 1$ this evaluates to $\theta = 39\delta/280$

Now must first go back to the basic relationship. $du/dy = u_1 \{3/2\delta + 3y^2/2\delta^3\}$

At the wall where y = 0 the shear stress is

$$\tau_o = \mu\, du/dy = \mu\, u_l \{3/2\delta + 3y^2/2\delta^3\} = (\mu\, u_l/\delta)\, \delta\, [(3/2\delta) + 3y^2/2\delta^3]$$

Putting $y/\delta = \eta$ we get

$$\tau_o = (\mu\, u_l/\delta)\, \delta\, [(3/2\delta) + 3\delta^2/2\delta]$$

$$\tau_o = (\mu\, u_l/\delta)\, [(3/2) + 3\delta^2/2]$$

at the wall $\eta = 0$ $\qquad \tau_o = (\mu\, u_l/\delta)\,(3/2)$...... ....(2.1)

The friction coefficient C_f is always defined as

$$C_f = 2\tau_o/(\rho\, u_2^2) \text{............. } (2.2)$$

It has been shown elsewhere that $C_f = 2d\theta/dx$. The student should search out this information from test books.

Putting $\theta = 39\delta/280$ (from the last example) then

$$C_f = 2d\theta/dx = (2 \times 39/280)\, d\delta/dx \qquad \text{..............(2.3)}$$

Equating (2.2) and (2.3) gives

$$\tau_o = (\rho u1^2)(39/280) d\delta/dx \text{ ....(2.4)}$$

Equating (2.1) and (2.4) gives

$$(\rho u_1^2)(39/280) d\delta/dx = (\rho u/\delta)(3/2)$$

Hence $\qquad (3 \times 280)/(2 \times 39)(\mu\, dx)/\rho u) = \delta d\delta$

Integrating $\qquad 10.77(\mu x/\rho u_l) = \delta^2/2 + C$

Since $\delta = 0$ at $x = 0$ (the leading edge of the plate) then C=0

Hence $\qquad \delta = \{21.54\, \mu x/\rho u_l\}^{½}$

Dividing both sides by x gives $\delta/x = 4.64(\mu/\rho u_l x)^{-½} = 4.64 Re^{-½}$

NB $R_{ex} = \rho u_l x/\mu$. and is based on length from the leading edge.

3.(a) The velocity profile in a laminar boundary layer is sometimes expressed in the formula

$$\frac{u}{u_1} = a_0 + a_1\frac{y}{\delta} + a_2\left(\frac{y}{\delta}\right)^2 + a_3\left(\frac{y}{\delta}\right)^3 + a_4\left(\frac{y}{\delta}\right)^4$$

where u_1 is the velocity outside the boundary layer and δ is the boundary layer thickness. Evaluate the coefficients a_0 to a_4 for the case when the pressure gradient along the surface is zero.

(b) Assuming a velocity profile $u/u_1=2(y/\delta) - (y/\delta)^2$ obtain an expression for the mass and momentum fluxes within the boundary layer and hence determine the displacement and momentum thickness.

Part A

$$\frac{u}{u_1} = a_0 + a_1\frac{y}{\delta} + a_2\left(\frac{y}{\delta}\right)^2 + a_3\left(\frac{y}{\delta}\right)^3 + a_4\left(\frac{y}{\delta}\right)^4$$

Boundary conditions

Where $y = 0$, $u = 0$ hence $a_0 = 0$

Where $y = \delta$, $u = u_1$ $\quad 1 = a_1 + a_2 + a_3 + a_4$ ....(A)

Differentiate with respect to y

$$\frac{1}{u_1}\frac{du}{dy} = \frac{a_1}{\delta} + 2a_2\left(\frac{y}{\delta^2}\right) + 3a_3\left(\frac{y^2}{\delta^3}\right) + 4a_4\left(\frac{y^3}{\delta^4}\right)$$

Where $y = \delta$, $du/dy = 0$

$$0 = \frac{a_1}{\delta} + 2a_2\left(\frac{1}{\delta}\right) + 3a_3\left(\frac{1}{\delta}\right) + 4a_4\left(\frac{1}{\delta}\right)$$

$$0 = a_1 + 2a_2 + 3a_3 + 4a_4 \text{ ....(B)}$$

Differentiate a second time.

$$\frac{1}{u_1}\frac{d^2u}{dy^2} = 2a_2\left(\frac{1}{\delta^2}\right) + 6a_3\left(\frac{y}{\delta^3}\right) + 12a_4\left(\frac{y^2}{\delta^4}\right)$$

Where $y = 0$, $d^2u/dy^2 = 0$ hence $0 = 2a_2\left(\frac{1}{\delta^2}\right)$ Hence $a_2 = 0$

(A) becomes $\quad 1 = a_1 + a_3 + a_4$

(B) becomes $\quad 0 = a_1 + 3a_3 + 4a_4$

Subtract $\quad 1 = 0 - 2a_3 - 3a_4$ ....(C)

The second differential becomes

$$\frac{1}{u_1}\frac{d^2u}{dy^2} = 6a_3\left(\frac{y}{\delta^3}\right) + 12a_4\left(\frac{y^2}{\delta^4}\right)$$

Where $y = \delta$, $d^2u/dy^2 = 0$

$$0 = 6a_3\left(\frac{y}{\delta^3}\right) + 12a_4\left(\frac{y^2}{\delta^4}\right) 6a_3\left(\frac{1}{\delta^2}\right) + 12a_4\left(\frac{1}{\delta^2}\right) = 6a_3 + 12a_4 \text{ ....(D)}$$

Divide through by 3 $\quad 0 = 2a_3 + 4a_4$ ....(E)

Add (C) and (E) $\quad a_4 = 1$

Substitute into (E) $\quad 0 = 2a_3 + 4 \quad a_3 = -2$

Substitute into (A) $\quad 1 = a_1 -2 + 1 \qquad a_1 = 2$

Hence

$$\frac{u}{u_1} = 2\frac{y}{\delta} - 2\left(\frac{y}{\delta}\right)^3 + 2\left(\frac{y}{\delta}\right)^4$$

PART B

$$\frac{u}{u_1} = 2\eta - \eta^2 \qquad \delta^* = \int_0^\delta \left[1 - \frac{u}{u_1}\right] dy \qquad \delta^* = \delta \int_0^1 \left[1 - 2\eta + \eta^2\right] d\eta$$

$$\delta^* = \delta\left[\eta - \eta^2 + \frac{\eta^3}{3}\right]_0^1 \qquad \delta^* = \delta\left[1 - 1 + \frac{1}{3}\right]_0^1 = \frac{\delta}{3}$$

$$\theta = \int_0^\delta \left[\frac{u}{u_1}\right]\left[1 - \frac{u}{u_1}\right] dy = \int_0^\delta \left[2\eta - \eta^2\right]\left[1 - 2\eta + \eta^2\right] dy$$

$$\theta = \delta \int_0^1 \left(2\eta - 5\eta^2 + 4\eta^3 - d\eta^4\right) d\eta \qquad \theta = \delta\left[\eta^2 - 5\eta^2/3 + \eta^4 - \eta^5/5\right]_0^1$$

$$\theta = \delta\left[1 - 5/3 + 1 - 1/5\right] \qquad \theta = 2\delta/15$$

4. When a fluid flows over a flat surface and the flow is laminar, the boundary layer profile may be represented by the equation

$$u/u_1 = 2(\eta) - (\eta)^2 \quad \text{where } \eta = y/\delta$$

y is the height within the layer and δ is the thickness of the layer. u is the velocity within the layer and u_1 is the velocity of the main stream.

Show that this distribution satisfies the boundary conditions for the layer.
Show that the thickness of the layer varies with distance (x) from the leading edge by the equation

$$\delta = 5.48x(R_{ex})^{-0.5}$$

It may be assumed that $\tau_o = \rho u_1^2\, d\theta/dx$

Where y = 0, $u = 0$ $\quad \eta = y/\delta = 0$ so the condition is satisfied.

Where y = δ, $u = u_1$ $\quad \eta = 1$ $\quad u/u_1 = 2(\eta) - (\eta)^2 = 1$ so the condition is satisfied.

Where y = δ, $du/dy = 0$ $\quad \dfrac{1}{u_1}\dfrac{du}{dy} = \dfrac{2}{\delta} + 2\left(\dfrac{y}{\delta^2}\right) = \dfrac{4}{\delta}$

Where y = 0, $d_2u/dy^2 = 0$ $\quad \dfrac{1}{u_1}\dfrac{d^2u}{dy^2} = \left(\dfrac{2}{\delta^2}\right)$

The last two are apparently not satisfactory conditions.

Starting with $\quad \dfrac{du}{dy} = u_1\left\{\dfrac{2}{\delta} + \dfrac{2y}{\delta^2}\right\}$

At the wall where y =0 the shear stress is

$$\tau_o = \mu\, du/dy = \mu u_1 \{2/\delta + 2y/\delta^2\} = (\mu u_1/\delta)\ [2 + 2y/\delta]$$

Putting $y/\delta = \eta$ we get $\quad \tau_o = (\mu\, u_1/\delta)\, \delta\, [2 + 2\eta]$

at the wall $\eta = 0$ $\quad \tau_o = (2\mu u_1/\delta)$……….......(1)

Putting $\theta = 2\delta/15$ (last example) then $\tau_o = (\rho u_1^2)\, d\theta/dx = (\rho u_1^2)(2/15)d\delta/dx$ ....(2)

Equating (1) and (2) $\quad (\rho u_1^2)(2/15)d\delta/dx = (2\mu u_1/\delta)$

Hence $\quad 15\,(\mu/\rho u_1)dx = \delta\, d\delta$

Integrating $\quad 15(\mu x/\rho u_1) = \delta^2/2 + C$

Since $\delta = 0$ at $x = 0$ (the leading edge of the plate) then C=0

Hence $\quad \delta^2 = 30(\mu x/\rho u_1) \qquad \delta = 5.478(\mu x/\rho u_1) = 5.478 R_{ex}^{-1/2}$

5. Define the terms *displacement thickness* δ^* and *momentum thickness* θ.

Find the ratio of these quantities to the boundary layer thickness δ if the velocity profile within the boundary layer is given by $u/u_1=\sin(\pi y/2\delta)$

Show, by means of a momentum balance, that the variation of the boundary layer thickness δ with distance (x) from the leading edge is given by $\delta = 4.8(R_{ex})^{-0.5}$

It may be assumed that $\tau_o = \rho u_1^2\, d\theta/dx$

Estimate the boundary layer thickness at the trailing edge of a plane surface of length 0.1 m when air at STP is flowing parallel to it with a free stream velocity u_1 of 0.8 m/s. It may be assumed without proof that the friction factor C_f is given by $C_f = 2\, d\theta/dx$

N.B. standard data $\mu = 1.71 \times 10^{-5}$ N s/m^2. $\rho = 1.29$ kg/m^3.

DISPLACEMENT THICKNESS δ*

The flow rate within a boundary layer is less than that for a uniform flow of velocity u_1. If we had a uniform flow equal to that in the boundary layer, the surface would have to be displaced a distance δ* in order to produce the reduction. This distance is called the displacement thickness.

MOMENTUM THICKNESS θ

The momentum in a flow with a BL present is less than that in a uniform flow of the same thickness. The momentum in a uniform layer at velocity u_1 and height h is $\rho h u_1^2$. When a BL exists this is reduced by $\rho u_1^2\theta$. Where θ is the thickness of the uniform layer that contains the equivalent to the reduction.

$$\theta = \int_0^\delta \left[\frac{u}{u_1}\right]\left[1-\frac{u}{u_1}\right]dy = \int_0^\delta \left[\frac{u}{u_1}-\left(\frac{u}{u_1}\right)^2\right]dy = \int_0^\delta \left[\sin\left\{\frac{\pi y}{2\delta}\right\}-\left(\sin\left\{\frac{\pi y}{2\delta}\right\}\right)^2\right]dy$$

We need the trig identity $\sin^2 A = ½ - ½ \cos 2A$

$$\theta = \int_0^\delta \left[\sin\left\{\frac{\pi y}{2\delta}\right\}-\frac{1}{2}+\frac{1}{2}\left(\cos\left\{\frac{\pi y}{2\delta}\right\}\right)\right]dy \qquad \theta = \left[-\frac{2\delta}{\pi}\cos\left\{\frac{\pi y}{2\delta}\right\}-\frac{y}{2}+\frac{\delta}{2\pi}\sin\left\{\frac{\pi y}{2\delta}\right\}\right]_0^\delta$$

$$\theta = \left[-0-\frac{\delta}{2}+0\right]-\left[-\frac{2\delta}{\pi}-0+0\right] = 0.137\delta$$

$\tau_o = \mu\, du/dy = \mu\, u_1 \sin(\pi y/2\delta) = \mu\, u_1(\pi/2\delta)\cos(\pi y/2\delta)$

At the wall y = $\tau_o = \mu\, u_1\, (\pi/2\delta)$ ....(1)

$C_f = 2\tau_o/\rho u_1^2$ ....(2)

$C_f = 2\, d\theta/dx$ and $\theta = 0.137\delta$ $C_f=2\, d(0.137\delta)/dx = 0.274\delta\, d\delta/dx$ ....(3)

Equate (2) and (3) $2\tau_o/\rho u_1^2 = 0.274\delta\, d\delta/dx$

$\tau_o = \rho\, u_1^2(0.137\delta)\, d\delta/dx$............... (4)

Equate (1) and (4) $\mu\, u_1\, (\pi/2\delta) = \rho\, u_1^2(0.137\delta) d\delta/dx$

$\mu\pi x/(0.274\rho\, u_1^2) = \delta^2/2\, C$ but where x =0, δ = 0 so C = 0

$\delta/x = \{(2\pi)/0.274)\}^{½}\ R_{ex}^{-½} = 4.8\, R_{ex}^{-½}$

x = 0.1 m u = 0.8 m/s $\rho = 1.29$ kg/m^3 $\mu = 1.7I \times 10^{-5}$ (from fluids tables)

$R_{ex} = (1.29)(0.8)(0.1)/ 1.71 \times 10^{-5} = 6035$

$\delta/0.1 = 4.8(6035)^{-½}$ δ = 0.006 m

Extra ...

$\tau_o = \mu\, u_1\pi/2\delta$ $C_f = 2\tau_o\, \rho\, u_1^2\, 2(\mu\, u_l\, \pi/2\delta) / \rho\, u_1^2 = \mu\pi x/(\rho\, u_1\, \delta x) = \pi x/ R_{ex}\delta$

$\delta/x = 4.8\, R_{ex}^{-½}$ $C_f = (\pi / R_{ex})\, (R_{ex}^{½} /4.8) = 0.65\, R_{ex}^{-½}$

6. In a laminar flow of a fluid over a flat plate with zero pressure gradient an approximation to the velocity profile is $u/u_1=(3/2)(\eta) - (1/2)(\eta)^3$

$\eta = y/\delta$ and u is the velocity at a distance y from the plate and u_1 is the mainstream velocity. δ is the boundary layer thickness.

Discuss whether this profile satisfies appropriate boundary conditions.

Show that the local skin-friction coefficient C_f is related to the Reynolds' number (Re_x) based on distance x from the leading edge by the expression $C_f = A (Re_x)^{-0.5}$

and evaluate the constant A.

It may be assumed without proof that $C_f = 2\, d\theta/dx$

and that θ is the integral of $(u/u_1)(1 - u/u_1)dy$ between the limits 0 and δ

This is the same as Q2 whence $\theta = 39\delta/280$ $C_f = 2\, d\theta/dx = (78/280)d\delta/dx$

$\tau_o = \rho u^2\pi/2\delta$

$C_f = 2\, \tau_o/\rho u^2 = (\rho\, u_1\, \pi\, x)/\delta\, \rho\, u_1^2 x) = (\pi\, x /\delta)\, R_{ex}^{½}$

$\delta/x = 4.64\, R_{ex}^{½}$

$C_f = \pi/(4.64\, R_{ex}^{½})\, x\, (1/ R_{ex}) = 0.65\, R_{ex}^{½}$

SELF ASSESSMENT EXERCISE No. 5

1. Under what circumstances is the velocity profile in a pipe adequately represented by the 1/7 th power law $u/u_1=(y/R)^{1/7}$ where u is the velocity at distance y from the wall, R is the pipe radius and u_1 is the centre-line velocity ?

The table shows the measured velocity profile in a pipe radius 30 mm. Show that these data satisfy the 1/7 th power law and hence evaluate

(i) the centre-line velocity
(ii) the mean velocity u_m
(iii) the distance from the wall at which the velocity equals u_m.

1.0	2.0	5.0	10.0	15.0	20.0	y (mm)
1.54	1.70	1.94	2.14	2.26	2.36	u (m/s)

Limitations are that the flow must be turbulent, with $R_e > 10^7$ and the velocity gradient must be the same at the junction between laminar sub layer and the turbulent layer.
$u/u_1 = (y/R)^{1/7}$ a = radius = 30 mm

Evaluate at various values of y

y	1	2	5	10	15	20	mm
u	1540	1700	1940	2140	2260	2350	mm/s
u_l	2503	2503	2506	2504	2495	2500	mm/s

Since u_l is constant the law is true. Take u_l = 2502 mm/s

$$Q = 2\pi\int_0^R (R-y)\left(\frac{y}{R}\right)^{1/7} = 2\,\pi \text{ x } 2502\int_0^R \left(R^{6/7}y^{1/7} - y^{8/7}R^{-1/7}\right)dy$$

$$Q = 2\pi \text{ x } 2502\left[\frac{49R^2}{120}\right]$$

Mean velocity $u_m = Q/A = \dfrac{2\pi \text{ x } 2502}{\pi R^2}\left[\dfrac{49R^2}{120}\right] = 2403$

$2043/2502 = (y/30)^{1/7}$ y = 7.261 mm. Note this fits with $u_m = (49/60)\,u_l$ and if this was the starting point the question would be simple.

2.

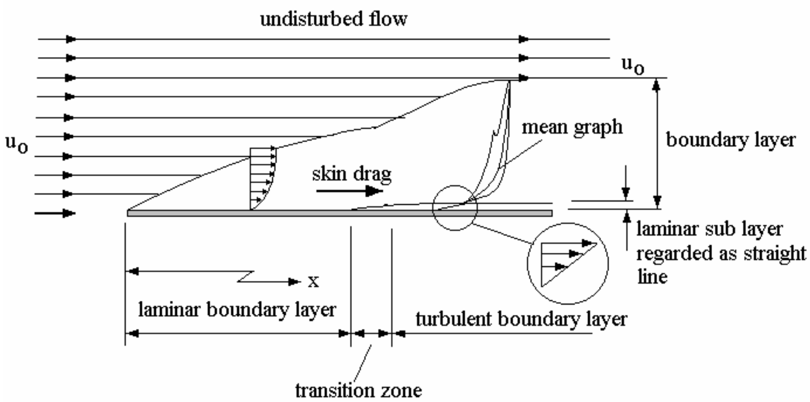
(a) Discuss the limitations of the 1/7th power law $u/u_1=(y/R)^{1/7}$ for the velocity profile in a circular pipe of radius R, indicating the range of Reynolds numbers for which this law is applicable.

(b) Show that the mean velocity is given by $49u_1/60$.

(c) Water flows at a volumetric flow rate of 1.1 x 10^{-3} m^3/s in a tube of diameter 25 mm. Calculate the centre-line velocity and the distance from the wall at which the velocity is equal to the mean velocity.

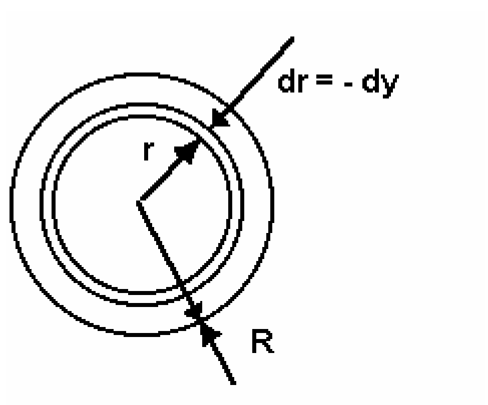
(d) Assuming that $C_f=0.079(Re)^{-0.25}$ evaluate the wall shear stress and hence estimate the laminar sub-layer thickness.

$\mu = 0.89 \times 10^{-3}$ N s/m^2. $\rho = 998$ kg/m^3.



Limitations are that the flow must be turbulent with $R_e>10^7$ and the velocity gradient must be the same at the junction between the laminar sub layer and the turbulent level.

Flow through an elementary cylinder. For a pipe, the B.L. extends to the centre so δ = radius = R. Consider an elementary ring of flow.



The velocity through the ring is u.
The volume flow rate through the ring is $2\pi rudr$
The volume flow rate in the pipe is $Q = 2\pi\int rudr$

Since δ = R then $u = u_1(y/R)^{1/7}$

also $r = R-y$

$$Q = 2\pi\int (R-y)udr = 2\pi\int u_1R^{-1/7}(R-y)y^{1/7}dy$$

$$Q = 2\pi u_1R^{-1/7}[Ry^{1/7}-y^{8/7}]$$

$$Q = 2\pi u_1R^{-1/7}[(7/8)Ry^{8/7}-(7/15)y^{15/7}]$$

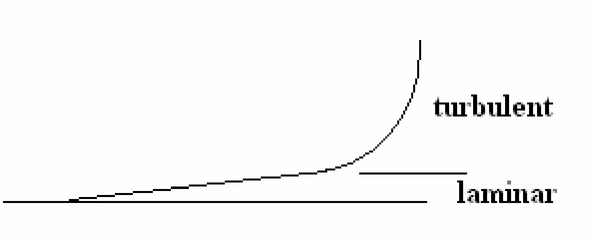
$$Q = (49/60)\pi R^2u_1.$$

The mean velocity is defined by $u_m=Q/\pi R^2$ hence $u_m=(49/60)u_1$

$Q = 1.1 \times 10^{-3}$ m^3/s R = 0.025/2 = 0.0125 m u_{mean} = 2.241 m/s u_1 = 2.744 m/s

$u = u_1 (y/R)^{1/7}$ y = 3.0285 m when $u = u_{mean}$

At the junction, the gradients are the same.



Laminar sub layer $\tau_o = \mu\, du/dy$

$R_e = \rho u D/\mu = 2.241 \times 0.025 \times 998/0.89 \times 10^{-3} = 62820$
$C_f = 0.005 = 2\tau_o/\rho u^2$ $\tau_o = 0.005 \times 998 \times 2.241^2/(2 \times 0.005) = 12.5\ \mathrm{N/m^2}$

For the turbulent layer $\tau_o = \mu\, du/dy$

$$12.5 = 0.89\mathrm{x}10^{-3}\frac{d\left\{u_1\left(\frac{y}{R}\right)^{1/7}\right\}}{dy}$$

$$y^{-6/7} = \frac{7 \times 14045 \times 0.0125^{1/7}}{2.744} = 52.196 \times 10^{-6}$$

$y = 10.09 \times 10^{-6}$ m

Some text uses the following method.

$y = 5\mu/\rho u^*$ where $u^* = \sqrt{(\tau_o/\rho)} = \sqrt{(12.7/998)} = 0/112$
In this case $y = 39.81 \times 10^{-6}$ m (the thickness of the laminar sub-layer)