FLUID MECHANICS D203
SAE SOLUTIONS TUTORIAL 3 – BOUNDARY LAYERS

SELF ASSESSMENT EXERCISE 1

1. A smooth thin plate 5 m long and 1 m wide is placed in an air stream moving at 3 m/s with its length parallel with the flow. Calculate the drag force on each side of the plate. The density of the air is 1.2 kg/m^3 and the kinematic viscosity is 1.6 x 10^{-5} m^2/s.

$R_{ex} = u L/\nu = 3 \times 5/1.6 \times 10^{-5} = 937.5 \times 10^3$
$C_{DF} = 0.074\ R_{ex}^{-1/5} = 4.729 \times 10^{-3}$
Dynamic Pressure = $\rho u_o^2/2 = 1.2 \times 3^2/2 = 5.4$ Pa
$\tau_w = C_{DF}$ x dyn press = 0.0255 Pa
$R = \tau_w \times A = 0.0255 \times 5 = 0.128$ N

2. A pipe bore diameter D and length L has fully developed laminar flow throughout the entire length with a centre line velocity u_o. Given that the drag coefficient is given as $C_{Df} = 16/Re$ where $Re = \frac{\rho u_o D}{\mu}$, show that the drag force on the inside of the pipe is given as $R=8\pi\mu u_o L$ and hence the pressure loss in the pipe due to skin friction is
$p_L = 32\mu u_o L/D^2$

$C_{DF} = 16/R_e$
$R = \tau_w \times \rho u_o^2/2 = C_{DF} \times (\rho u_o^2/2) \times A$
$R = (16/R_e)(\rho u_o^2/2)\ A$
$R = (16\mu/\rho u_o D)(\rho u_o^2/2)\ \pi DL$
$R = (16\ \mu\ u_o\ \pi\ L/2) = 8\ \pi\ \mu\ u_o\ L$
$p_L = R/A = 8\ \pi\ \mu\ u_o\ L/(\pi D^2/4) = 32\ \mu\ u_o\ L/D^2$

SELF ASSESSMENT EXERCISE No. 2

1. Calculate the drag force for a cylindrical chimney 0.9 m diameter and 50 m tall in a wind blowing at 30 m/s given that the drag coefficient is 0.8.
The density of the air is 1.2 kg/m^3.

$C_D = 0.8 = 2R/(\rho u^2 A)$ $R = 0.8\ (\rho u^2/2)A = 0.8\ (1.2 \times 30^2/2)(50 \times 0.9) = 19440$ N

2 Using the graph (fig.1.12) to find the drag coefficient, determine the drag force per metre length acting on an overhead power line 30 mm diameter when the wind blows at 8 m/s. The density of air may be taken as 1.25 kg/m^3 and the kinematic viscosity as 1.5 x 10^{-5} m^2/s. (1.8 N).

$R_e = u\ d/\nu = 8 \times 0.03/1.5 \times 10^{-5} = 16 \times 10^3$
From the graph
$C_D = 1.5$

$R = C_D\ (\rho u_o^2/2)A = 1.5\ (1.25 \times 8^2/2)(0.03 \times 1) = 1.8$ N

SELF ASSESSMENT EXERCISE No. 3

1. a. Explain the term Stokes flow and terminal velocity.
   b. Show that the terminal velocity of a spherical particle with Stokes flow is given by the formulae = $d^2g(\rho_s - \rho_f)/18\mu$. Go on to show that $C_D=24/R_e$

Stokes flow –for ideal fluid - no separation - Re <0.2
R = Buoyant weight = $(\pi d^3/6)g\,(\rho_s - \rho_f) = 3\,\pi\,d\,\mu\,u_t$
$u_t = d^2\,g\,(\rho_s - \rho_f)/18\,\mu$ R = $C_D(\rho\,u_t^2/2)(\pi\,d^2/4)$ $C_D = 26\mu/(\rho\,u_t\,d) = 24/R_e$

2. Calculate the largest diameter sphere that can be lifted upwards by a vertical flow of water moving at 1 m/s. The sphere is made of glass with a density of 2630 kg/m^3. The water has a density of 998 kg/m^3 and a dynamic viscosity of 1 cP.

$C_D =(2/\rho\,u^2A)R = \{(2 \times 4)/(\rho\,u^2\,\pi d^2)\}\,(\pi d^3/6)g\,(\rho_s - \rho_f) = 21.38\,d$
Try Newton Flow first
D = 0.44/21.38 = 0.206 m R_e =(998 x 1 x 0.0206)/0.001 = 20530 therefore this is valid.

3. Using the same data for the sphere and water as in Q2, calculate the diameter of the largest sphere that can be lifted upwards by a vertical flow of water moving at 0.5 m/s. (5.95 mm).

$C_D = 85.52\,d$ Try Newton Flow
D = 0.44/85.52 = 0.0051 R_e =(998 x 0.5 x 0.0051)/0.001 = 2567 therefore this is valid.

4. Using the graph (fig. 1.12) to obtain the drag coefficient of a sphere, determine the drag on a totally immersed sphere 0.2 m diameter moving at 0.3 m/s in sea water. The density of the water is 1025 kg/m^3 and the dynamic viscosity is 1.05 x 10^{-3} Ns/m^2.

$R_e = (\rho u d/\mu) = (1025 \times 0.3 \times 0.2/1.05 \times 10^{-3}) = 58.57 \times 10^3$ From the graph $C_D = 0.45$
$R = C_D\,(\rho u^2/2)A = 0.45(1025 \times 0.3^2/2)(\pi \times 0.2^2/4) = 0.65$ N

5. A glass sphere of diameter 1.5 mm and density 2 500 kg/m^3 is allowed to fall through water under the action of gravity. The density of the water is 1000 kg/m^3 and the dynamic viscosity is 1 cP.
   Calculate the terminal velocity assuming the drag coefficient is $C_D = 24\,R_e^{-1}\,(1+ 0.15R_e^{0.687})$

C_D = F/(Area x Dynamic Pressure)

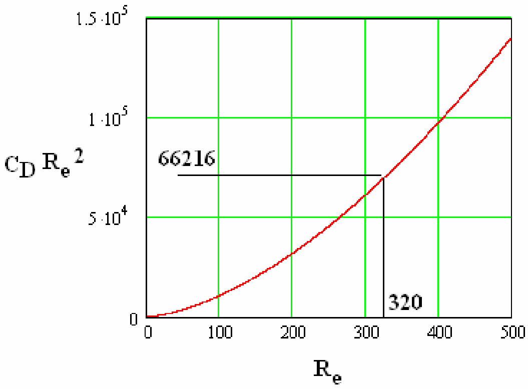
$$R = \frac{\pi d^3 g(\rho_s - \rho_f)}{6} \qquad C_D = \frac{\pi d^3 g(\rho_s - \rho_f)}{(\pi\ d^2/4)(\rho\,u^2/2)} \qquad C_D = \frac{4d^3 g(\rho_s - \rho_f)}{3\rho_f\ u^2 d^2}$$

Arrange the formula into the form $C_D\,R_e^2$ as follows.

$$C_D = \frac{4d^3 g(\rho_s - \rho_f)}{3\rho_f\ u^2 d^2} \times \frac{\rho_f \mu^2}{\rho_f \mu^2} = \frac{4d^3 g(\rho_s - \rho_f)\rho_f}{3\mu^2} \times \frac{\mu^2}{\rho_f^2 u^2 d^2} = \frac{4d^3 g(\rho_s - \rho_f)\rho_f}{3\mu^2} \times \frac{1}{R_e^2}$$

$C_D R_e^2 = \dfrac{4d^3 g(\rho_s - \rho_f)\rho_f}{3\mu^2}$ and evaluating this we get 66217

From $C_D = \dfrac{24}{R_e}\left[1 + 0.15R_e^{0.687}\right]$ we may solve by plotting $C_D R_e^2$ against Re



From the graph R_e = 320 hence
$u = R_e\,\mu/\rho d = 0.215$ m/s

6. A glass sphere of density 2 690 kg/m^3 falls freely through water. Find the terminal velocity for a 4 mm diameter sphere and a 0.4 mm diameter sphere. The drag coefficient is

$$C_D = 8F/\{\pi d^2 \rho u^2\}$$

This coefficient is related to the Reynolds number as shown for low values of R_e.

Re	15	20	25	30	35
C_D	3.14	2.61	2.33	2.04	1.87

The density and viscosity of the water is 997 kg/m^3 and 0.89 x 10^{-3} N s/m^2.
For Re >1000 C_D = 0.44

For a 4 mm sphere we might guess from the question that R_e is greater than 1000 and hence C_D = 0.44

$$R = \frac{\pi d^3 g(\rho_s - \rho_f)}{6} \qquad C_D = \frac{\pi d^3 g(\rho_s - \rho_f)}{(\pi\ d^2/4)(\rho u^2/2)} \qquad C_D = \frac{4d^3 g(\rho_s - \rho_f)}{3\rho_f\ u^2 d^2}$$

$$u = \sqrt{\frac{4d^3 g(\rho_s - \rho_f)}{3\rho_f\ C_D d^2}}$$ Putting in values ρ = 997 μ = 0.0089 d = 0.004 C_D = 0.44

u = 0.45 m/s Check $R_e = \rho u d/\mu$ = 2013 so this is valid

For the 0.4 mm sphere we might guess from the question that $C_D = \frac{8F}{\pi d^2 \rho u^2}$

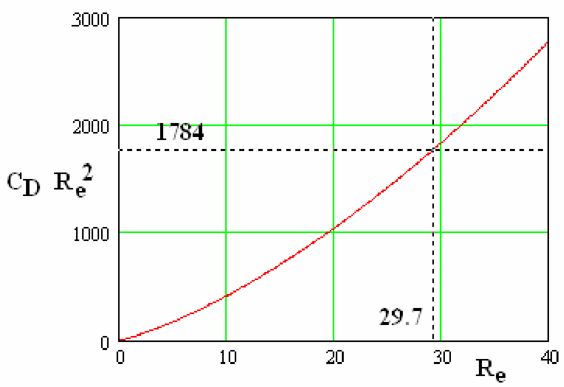
$$C_D = \frac{8F}{\pi d^2 \rho u^2} \times \frac{\rho\mu^2}{\rho\mu^2} = \frac{8F\rho}{\pi\mu^2} \times \frac{\mu^2}{\rho^2 u^2 d^2} = \frac{8F\rho}{\pi\mu^2} \times \frac{1}{R_e^2}$$

$$C_D R_e^2 = \frac{8F\rho}{\pi\mu^2} = 3.205 \times 10^9 F$$

$$R = \frac{\pi d^3 g(\rho_s - \rho_f)}{6}$$ (The buoyant weight)

For a 0.4 mm sphere F = 556.55 x 10^{-9} N

$$C_D R_e^2 = 3.205 \times 10^9 F = 1784$$



Plot graph for the 0.4 mm sphere
The 0.4 mm sphere fits the table
u = R_e μ/ρd = 0.066 m/s

7. A glass sphere of diameter 1.5 mm and density 2 500 kg/m^3 is allowed to fall through water under the action of gravity. Find the terminal velocity assuming the drag coefficient is
$C_D = 24\ R_e^{-1}(1 + 0.15 R_e^{0.687})$

$$R = \frac{\pi d^3 g(\rho_s - \rho_f)}{6}$$ (The buoyant weight)

$$R = \frac{\pi \times 0.0015^3 \times 9.81(2500 - 997)}{6} = 26.056 \times 10^{-6}\ N$$

$$C_D = \frac{8F}{\pi d^2 \rho u^2} = \frac{8 \times 26 \times 10^{-6}}{\pi\,(0.0015)^2 \times 997 \times u^2} = \frac{29.578 \times 10^3}{u^2}$$

$$C_D = \frac{29.578 \times 10^3}{u^2} = \frac{24}{R_e}\left[1 + 0.15 R_e^{0.687}\right]$$

$$\frac{29.578 \times 10^3}{u^2} = \frac{24 \times 0.00089}{997\,u\,(0.0015)}\left[1 + 0.15\left(\frac{997\,u\,(0.0015)}{0.00089}\right)^{0.687}\right]$$

$$2.0709 = u\left[1 + 24.657u^{0.687}\right] = u + 24.657u^{1.687}$$

Solve for u and u = 0.215 m/s (plotting might be the best way)