FLUID MECHANICS D203
SAE SOLUTIONS TUTORIAL 2 – APPLICATIONS OF BERNOULLI

SELF ASSESSMENT EXERCISE 3

Take the density of water to be 997 kg/m^3 throughout unless otherwise stated.

1. A Venturi meter is 50 mm bore diameter at inlet and 10 mm bore diameter at the throat. Oil of density 900 kg/m^3 flows through it and a differential pressure head of 80 mm is produced. Given C_d = 0.92, determine the flow rate in kg/s.

$$Q = C_d A_1 \sqrt{\frac{2\Delta p}{\rho(r^2 - 1)}}$$

$r = A_1/A_2 = 25 \quad \Delta p = \rho g \Delta h = 900 \times 9.81 \times 0.08 = 706.3 \times 10^3$ Pa

$$Q = \frac{0.92 \text{ x } \pi \text{ x } 0.05^2}{4}\sqrt{\frac{2 \text{ x } 706300}{900(25^2 - 1)}} = 909.59 \text{ x } 10^{-6} \text{ m}^3/\text{s}$$

$m = \rho Q = 0.0815$ kg/s

2. A Venturi meter is 60 mm bore diameter at inlet and 20 mm bore diameter at the throat. Water of density 1000 kg/m^3 flows through it and a differential pressure head of 150 mm is produced. Given C_d = 0.95, determine the flow rate in dm^3/s.

$$Q = C_d A_1 \sqrt{\frac{2 \rho g \Delta h}{\rho(r^2 - 1)}}$$

r = 9

$$Q = \frac{0.95 \text{ x } \pi \text{ x } 0.06^2}{4}\sqrt{\frac{2 \text{ x } 1000 \text{ x } 9.81 \text{ x } 0.15}{1000(9^2 - 1)}} = 515 \text{ x } 10^{-6} \text{ m}^3/\text{s or } 0.515 \text{ dm3/s}$$

3. Calculate the differential pressure expected from a Venturi meter when the flow rate is 2 dm^3/s of water. The area ratio is 4 and C_d is 0.94. The inlet c.s.a. is 900 mm^2.

$$Q = 0.002 = C_d A_1 \sqrt{\frac{2\Delta p}{\rho(r^2 - 1)}}$$

r = 4

$$0.002 = 0.94 \text{ x } 900 \text{x} 10^{-6}\sqrt{\frac{2\Delta p}{1000(4^2 - 1)}} \qquad 2.3641 = \sqrt{\frac{\Delta p}{7500}} \qquad 5.589 = \frac{\Delta p}{7500}$$

Δp = 41916 Pa

4. Calculate the mass flow rate of water through a Venturi meter when the differential pressure is 980 Pa given C_d = 0.93, the area ratio is 5 and the inlet c.s.a. is 1000 mm^2.

r = 5

$$m = \rho C_d A_1 \sqrt{\frac{2\Delta p}{\rho(r^2 - 1)}} = 1000 \text{ x } 0.93 \text{ x } 1000 \text{ x } 10^{-6}\sqrt{\frac{2 \text{ x } 980}{1000(5^2 - 1)}} = 0.2658 \text{ kg/s}$$

5. Calculate the flow rate of water through an orifice meter with an area ratio of 4 given C_d is 0.62, the pipe area is 900 mm^2 and the d.p. is 586 Pa. (ans. 0.156 dm^3/3).

r = 4

$$Q = C_d A_1 \sqrt{\frac{2\Delta p}{\rho(r^2 - 1)}} = 900 \text{ x } 10^{-6} \text{ x } 0.62\sqrt{\frac{2 \text{ x } 586}{1000(4^2 - 1)}} = 155.9 \text{ x } 10^{-6} \text{ m}^3/\text{s}$$

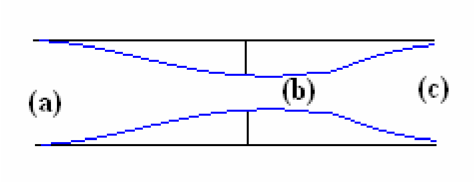
6. Water flows at a mass flow rate 0f 0.8 kg/s through a pipe of diameter 30 mm fitted with a 15 mm diameter sharp edged orifice.

   There are pressure tappings (a) 60 mm upstream of the orifice, (b) 15 mm downstream of the orifice and (c) 150 mm downstream of the orifice, recording pressure p_a, p_b and p_c respectively. Assuming a contraction coefficient 0f 0.68, evaluate

   (i) the pressure difference (p_a - p_b) and hence the discharge coefficient.
   (ii)the pressure difference (p_b - p_c) and hence the diffuser efficiency.
   (iii) the net force on the orifice plate.

d_o = 15 mm d_j = jet diameter $Cc = 0.68 = (A_b/A_o) = (d_b/15)^2$ d_b = 12.37 mm

No Friction between (a) and (b)
so Cv = 1.0 Cd = Cc Cv = Cc



$$m = \rho A_o C_d \sqrt{\frac{2\Delta p}{\rho\left(1 - Cc^2\beta^4\right)}} \qquad \beta = 15/30 = 0.5$$

$$0.8 = 997\,\frac{\pi \text{ x } 0.015^2}{4} \text{ x } 0.68\sqrt{\frac{2\Delta p}{997\left(1 - 0.68^2 \text{ x } 0.5^4\right)}}$$

$$6.677 = \sqrt{\frac{\Delta p}{484}} \qquad \Delta p = p_a - p_b = 21581\,\text{Pa}$$

Note the same answer may be obtained by applying Bernoulli's equation between (a) and (b)
Now apply Bernoulli's equation between (b) and (c)

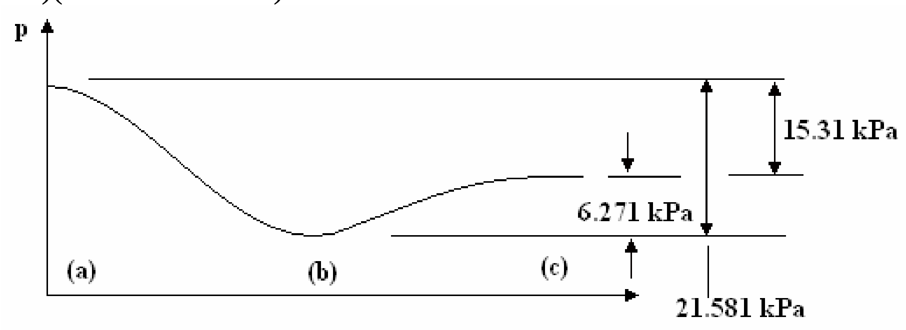
$p_b + \rho u_b^2/2 = p_c + \rho u_c^2/2 + \text{loss}$ $\text{loss} = \rho (u_b - u_c)^2/2$

$$u_b = \frac{m}{\rho A_b} = \frac{0.8}{997 \text{ x } \pi \text{ x } 0.01237^2/4} = 6.677 \text{ m/s}$$

$$u_c = \frac{m}{\rho A_c} = \frac{0.8}{997 \text{ x } \pi \text{ x } 0.03^2/4} = 1.135\,\text{m/s}$$

loss = 997 (6.677 – 1.135)^2/2 = 15311 Pa

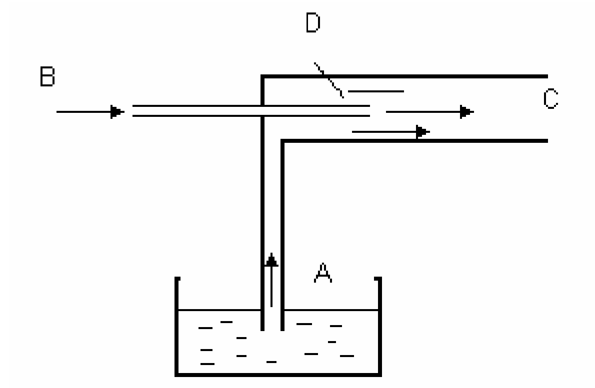
p_c - p_b = (997/2)(6.677^2 - 1.135^2) – 15311 = 6271 Pa



η = 15.31/21.581 = 71% Energy recovered = 6.27/21.58 = 29%

Force = π x 0.03^2/4 x 15310 = 10.8 N (on the control section)

7. The figure shows an ejector (or jet pump) which extracts 2 x 10^{-3} m^3/s of water from tank A which is situated 2.0 m below the centre-line of the ejector. The diameter of the outer pipe of the ejector is 40 mm and water is supplied from a reservoir to the thin-walled inner pipe which is of diameter 20 mm. The ejector discharges to atmosphere at section C.



Evaluate the pressure p at section D, just downstream of the end of pipe B, the velocity in pipe B and the required height of the free water level in the reservoir supplying pipe B. (-21.8 kPa gauge, 12.9 m/s, 6.3 m).

It may be assumed that both supply pipes are loss free.

$A_B = \pi \times 0.02^2/4 = 314.2 \times 10^{-6}\ m^2$ $\qquad$ $A_C = \pi \times 0.04^2/4 = 1256 \times 10^{-6}\ m^2$

$A_D = A_C - A_B = 942.48 \times 10^{-6}\ m^2$ $\qquad$ $u_D = Q_D/A_D = 0.002 \times 10^{-6}/0.94248 \times 10^{-6} = 2.122$ m/s

Apply Bernoulli from A to D $\qquad h_A + \frac{u_A^2}{2g} + z_A = h_D + \frac{u_D^2}{2g} + z_D$

$h_D = -\frac{u_D^2}{2g} - z_D = -\frac{2.122^2}{2g} - 2 = -2.23\,m$ $\qquad p_D = \rho g h_D = -21.8$ kPa

Next apply the conservation of momentum between the points where B and D join and the exit at C. This results in the following.

$$Q_B^2\left\{\frac{1}{A_B} - \frac{1}{A_C}\right\} - \frac{2Q_BQ_D}{A_C} + \frac{p_BA_C}{\rho} + Q_D^2\left\{\frac{1}{A_D} - \frac{1}{A_C}\right\} = 0$$

$$a = \left\{\frac{1}{A_B} - \frac{1}{A_C}\right\} = \left\{\frac{10^6}{314.2} - \frac{10^6}{1256}\right\} = 2386$$

$$b = \frac{2Q_D}{A_C} = \frac{2 \times 2\text{x}10^{-3}}{1.256 \times 10^{-3}} = 3.1847$$

$$c = \frac{p_BA_C}{\rho} + Q_D^2\left\{\frac{1}{A_D} - \frac{1}{A_C}\right\} = \frac{-21800 \times 1256 \times 10^{-6}}{1000} + \left(2 \times 10^{-3}\right)^2\left\{\frac{10^6}{942.48} - \frac{10^6}{1256}\right\}$$

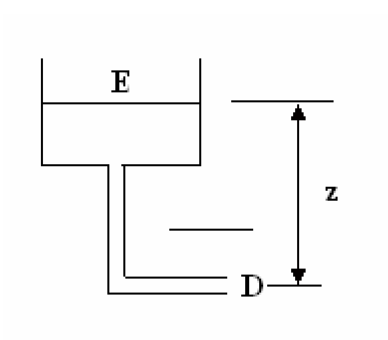
$$c = -27.38\text{x}10^{-3} + 1.06 \times 10^{-3} = -26.32 \times 10^{-3}$$

$$aQ_B^2 + bQ_B + c = 0 \qquad Q_B = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$Q_B = \frac{-3.1847 \pm \sqrt{3.1847^2 + 4 \times 2386 \times 0.02632}}{2 \times 2386}$$

$$Q_B = \frac{-3.1847 \pm 16.17}{2 \times 2386} = -0.00272 \text{ or } 0.00405\,m^3/s$$

$u_B = Q_B/A_B = 12.922$ m/s



Apply Bernoulli between E and point D

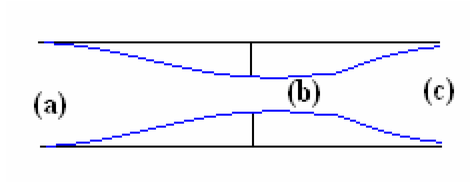
$z = h_B + u_B^2/2g = 6.282$ m

8. Discuss the use of orifice plates and venturi-meters for the measurement of flow rates in pipes.

Water flows with a mean velocity of 0.6 m/s in a 50 mm diameter pipe fitted with a sharp edged orifice of diameter 30 mm. Assuming the contraction coefficient is 0.64, find the pressure difference between tappings at the vena contracta and a few diameters upstream of the orifice, and hence evaluate the discharge coefficient.
Estimate also the overall pressure loss caused by the orifice plate.
It may be assumed that there is no loss of energy upstream of the vena contracta.

d_o = 30 mm d_j = jet diameter = d_b Cc = 0.64 = $(A_b/A_o) = (d_b/30)^2$ d_b = 24 mm

$Q = 0.6 \times \pi \times 0.05^2/4 = 0.001178\ m^3/s$



No Friction between (a) and (b)
so Cv = 1.0 Cd = Cc Cv = Cc

$$Q = A_o C_d \sqrt{\frac{2\Delta p}{\rho\left(1 - Cc^2\beta^4\right)}} \qquad \beta = 30/50 = 0.6$$

$$0.001178 = \frac{\pi \times 0.03^2}{4} \times 0.64 \sqrt{\frac{2\Delta p}{997\left(1 - 0.64^2 \times 0.6^4\right)}}$$

$$2.06 = \sqrt{\frac{\Delta p}{472}} \qquad \Delta p = p_a - p_b = 3200\ \text{Pa}$$

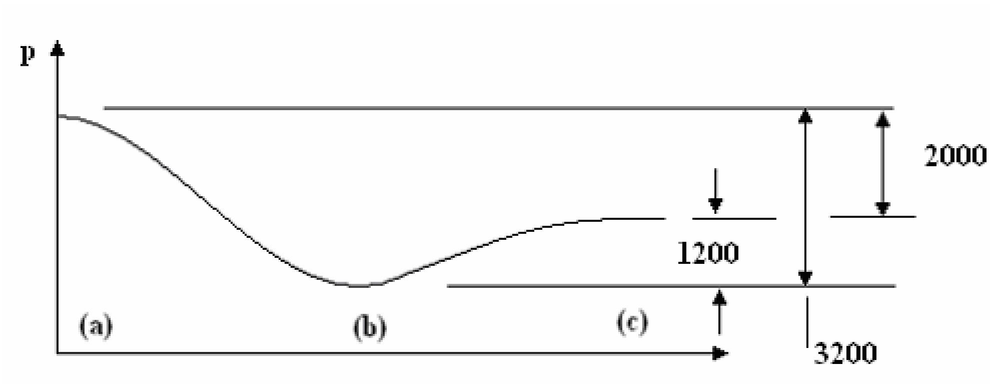
Note the same answer may be obtained by applying Bernoulli's equation between (a) and (b)
Now apply Bernoulli's equation between (b) and (c)

$p_b + \rho u_b^2/2 = p_c + \rho u_c^2/2 + \text{loss}$ $\text{loss} = \rho (u_b - u_c)^2/2$

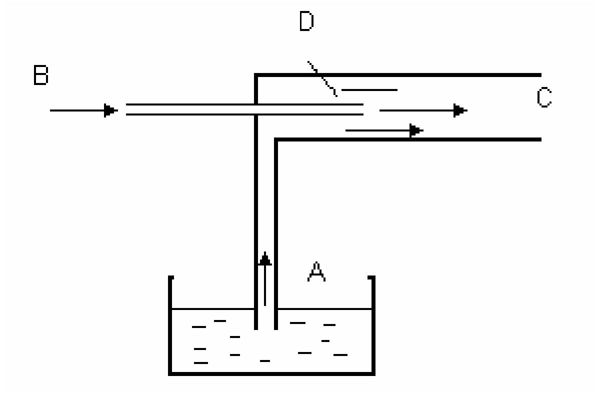
$$u_b = \frac{Q}{A_b} = \frac{0.001178}{\pi \times 0.024^2/4} = 2.6\ \text{m/s}$$

$$u_c = \frac{q}{A_c} = \frac{0.001178}{\pi \times 0.035^2/4} = 0.6\ \text{m/s}$$

loss = 997 (2.6 – 0.6)^2/2 = 2000 Pa



9. The figure shows an ejector pump BDC designed to lift 2 x 10^{-3} m^3/s of water from an open tank A, 3.0 m below the level of the centre-line of the pump. The pump discharges to atmosphere at C.



The diameter of thin-walled inner pipe 12 mm and the internal diameter of the outer pipe of the is 25 mm. Assuming that there is no energy loss in pipe AD and there is no shear stress on the wall of pipe DC, calculate the pressure at point D and the required velocity of the water in pipe BD.

Derive all the equations used and state your assumptions.

$A_B = \pi \times 0.012^2/4 = 113.1 \times 10^{-6}\ m^2$

$A_C = \pi \times 0.025^2/4 = 491 \times 10^{-6}\ m^2$

$A_D = A_C - A_B = 377.8 \times 10^{-6}\ m^2$

$u_D = Q_D/A_D = 0.002 \times 10^{-6}/377.8 \times 10^{-6} = 5.294$ m/s

Apply Bernoulli from A to D $\quad h_A + \frac{u_A^2}{2g} + z_A = h_D + \frac{u_D^2}{2g} + z_D$

$$h_D = -\frac{u_D^2}{2g} - z_D = -\frac{5.294^2}{2g} - 3 = -4.429\ \text{m} \qquad p_D = \rho g h_D = -43.4\ \text{kPa}$$

Next apply the conservation of momentum between the points where B and D join and the exit at C. This results in the following.

$$Q_B^2\left\{\frac{1}{A_B} - \frac{1}{A_C}\right\} - \frac{2Q_BQ_D}{A_C} + \frac{p_BA_C}{\rho} + Q_D^2\left\{\frac{1}{A_D} - \frac{1}{A_C}\right\} = 0$$

$$a = \left\{\frac{1}{A_B} - \frac{1}{A_C}\right\} = \left\{\frac{10^6}{113.1} - \frac{10^6}{491}\right\} = 6805$$

$$b = \frac{-2Q_D}{A_C} = \frac{2 \times 2 \times 10^{-3}}{491 \times 10^{-6}} = -8.149$$

$$c = \frac{p_BA_C}{\rho} + Q_D^2\left\{\frac{1}{A_D} - \frac{1}{A_C}\right\} = \frac{-43400 \times 491 \times 10^{-6}}{1000} + \left(2 \times 10^{-3}\right)^2\left\{\frac{10^6}{377.8} - \frac{10^6}{491}\right\}$$

$$c = -18.886 \times 10^{-3}$$

$$aQ_B^2 + bQ_B + c = 0 \qquad Q_B = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$Q_B = \frac{8.149 \pm \sqrt{8.149^2 + 4 \times 6805 \times 0.0188}}{2 \times 6805}$$

$$Q_B = -0.001172 \text{ or } 0.002369\ m^3/s$$

$u_B = Q_B/A_B = 20.95$ m/s