FLUID MECHANICS D203
SAE SOLUTIONS TUTORIAL 2 – APPLICATIONS OF BERNOULLI

SELF ASSESSMENT EXERCISE 1

1. A pipe 100 mm bore diameter carries oil of density 900 kg/m^3 at a rate of 4 kg/s. The pipe reduces to 60 mm bore diameter and rises 120 m in altitude. The pressure at this point is atmospheric (zero gauge). Assuming no frictional losses, determine:
   i. The volume/s (4.44 dm^3/s)
   ii. The velocity at each section (0.566 m/s and 1.57 m/s)
   iii. The pressure at the lower end. (1.06 MPa)

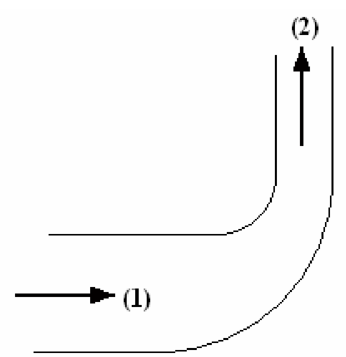
   $Q = m/\rho = 4/900 = 0.00444$ m^3/s
   $u_1 = Q/A_1 = 0.00444/(\pi \text{ x } 0.05^2) = 0.456$ m/s $\qquad$ $u_2 = Q/A_2 = 0.00444/(\pi \text{ x } 0.03^2) = 1.57$ m/s
   $h_1 + z_1 + u_1^2/2g = h_2 + z_2 + u_2^2/2g$ $\qquad$ $h_2 = 0 \quad z_1 = 0$
   $h_1 + 0 + 0.566^2/2g = 0 + 120 + 1.57^2/2g$
   $h_1 = 120.1$ m $\qquad$ $p = \rho gh = 900 \text{ x } 9.81 \text{ x } 120.1 = 1060$ kPa

2. A pipe 120 mm bore diameter carries water with a head of 3 m. The pipe descends 12 m in altitude and reduces to 80 mm bore diameter. The pressure head at this point is 13 m. The density is 1000 kg/m^3. Assuming no losses, determine

   i. The velocity in the small pipe (7 m/s) $\qquad$ ii. The volume flow rate. (35 dm^3/s)

   $h_1 + z_1 + u_1^2/2g = h_2 + z_2 + u_2^2/2g$ $\qquad$ $3 + 12 + u_1^2/2g = 13 + 0 + u_2^2/2g$
   $2 = (u_2^2 - u_1^2)/2g$ $\qquad$ $(u_2^2 - u_1^2) = 39.24$
   $u_1 A_1 = Q = u_2 A_2$ $\qquad$ $u_1 = u_2 (80/120)^2 = 0.444 u_2$
   $39.24 = u_2^2 - (0.444 u_2)^2 = 0.802 u_2^2$ $\qquad$ $u_2 = 6.99$ m/s $\qquad$ $u_1 = 3.1$ m/s
   $Q = u_2 A_2 = 6.99 \text{ x } \pi \text{ x } 0.04^2 = 0.035$ m^3/s or 35 dm^3/s

3. A horizontal nozzle reduces from 100 mm bore diameter at inlet to 50 mm at exit. It carries liquid of density 1000 kg/m^3 at a rate of 0.05 m^3/s. The pressure at the wide end is 500 kPa (gauge). Calculate the pressure at the narrow end neglecting friction. (196 kPa)

   $A_1 = \pi D_1^2/4 = \pi(0.1)^2/4 = 7.854 \text{ x } 10^{-3}$ m^2
   $A_2 = \pi D_2^2/4 = \pi(0.05)^2/4 = 1.9635 \text{ x } 10^{-3}$ m^2
   $u_1 = Q/A_1 = 0.05/7.854 \text{ x } 10^{-3} = 6.366$ m/s
   $u_2 = Q/A_2 = 0.05/1.9635 \text{ x } 10^{-3} = 25.46$ m/s
   $p_1 + \rho u_1^2/2 = p_2 + \rho u_2^2/2$
   $500 \text{ x } 10^3 + 100 \text{ x } (6.366)^2/2 = p_2 + 1000 \text{ x } (25.46)^2/2$ $\qquad$ $p_2 = 196$ kPa

4. A pipe carries oil of density 800 kg/m^3. At a given point (1) the pipe has a bore area of 0.005 m^2 and the oil flows with a mean velocity of 4 m/s with a gauge pressure of 800 kPa. Point (2) is further along the pipe and there the bore area is 0.002 m^2 and the level is 50 m above point (1). Calculate the pressure at this point (2). Neglect friction. (374 kPa)

   $800 \text{ x} 10^3 + 800\text{x } 4^2/2 + 0 = p_2 + 800\ 10^2/2 + 800 \text{ x } 9.81 \text{ x } 50$
   $p_2 = 374$ kPa

5. A horizontal nozzle has an inlet velocity u_1 and an outlet velocity u_2 and discharges into the atmosphere. Show that the velocity at exit is given by the following formulae.

$u_2 = \{2\Delta p/\rho + u_1^2\}^{½}$ and $u_2 = \{2g\Delta h + u_1^2\}^{½}$

$p_1 + \rho u_1^2/2 + \rho g z_1 = p_2 + \rho u_2^2/2 + \rho g z_2$ $z_1 = z_2$
$p_1 + \rho u_1^2/2 = p_2 + \rho u_2^2/2$
$p_1 - p_2 = (\rho/2)(u_2^2 - u_1^2)$ $2(p_1 - p_2)/\rho = (u_2^2 - u_1^2)$
$u_2 = \sqrt{(2\Delta p/\rho + u_1^2)}$
Substitute $p = \rho g h$ and $u_2 = \sqrt{\{2g\Delta h + u_1^2\}^{½}}$

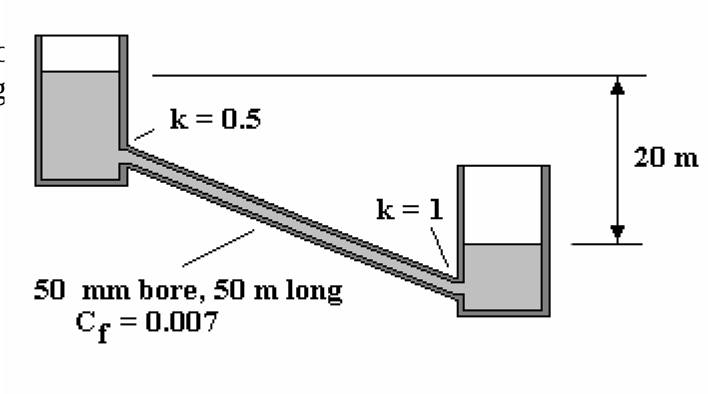
SELF ASSESSMENT EXERCISE 2

1. A pipe carries oil at a mean velocity of 6 m/s. The pipe is 5 km long and 1.5 m diameter. The surface roughness is 0.8 mm. The density is 890 kg/m^3 and the dynamic viscosity is 0.014 N s/m^2. Determine the friction coefficient from the Moody chart and go on to calculate the friction head h_f.

L = 5000 m d = 1.5 m k = 0.08 mm ρ = 890 kg/m^3 μ = 0.014 Ns/m^2 u = 6 m/s

$\varepsilon = k/D 0.8/1500 = 533 \times 10^{-6}$ $R_e = \rho u D/\mu = 890 \times 6 \times 1.5/0.014 = 572 \times 10^3$
From the Moody Chart $C_f = 0.0045$
$h_f = 4\, C_f\, Lu^2/(2\, g\, d) = 110$ m

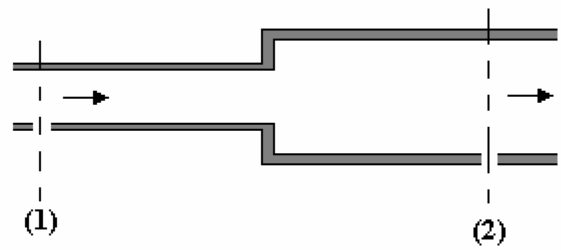
2. The diagram shows a tank draining into anc pressure is both zero on the surface on a larg the diagram. (Ans. 7.16 dm^3/s)



$h_1 + z_1 + u_1^2/2g = h_2 + z_2 + u_2^2/2g$
$0 + z_1 + 0 = 0 + 0 + 0 + h_L$
$h_L = 20$
$20 = 4\, C_f\, Lu^2/(2\, g\, d)$ + minor losses

$20 = \{4 \times 0.007 \times 50u^2/(2 \times 9.81 \times 0.05)\} + 0.5\, u^2/(2 \times 9.81) + u^2/(2 \times 9.81) = 29.5\, u^2/(2 \times 9.81)$
$u = 20(2 \times 9.81)/29.5 = 3.65$ m/s
$A = 0.00196$ m^2 $Q = Au = 0.00196 \times 3.65 = 0.00716$ m^3/s or 7.16 dm^3/s

3. Water flows through the sudden pipe expansion shown below at a flow rate of 3 dm^3/s. Upstream of the expansion the pipe diameter is 25 mm and downstream the diameter is 40 mm. There are pressure tappings at section (1), about half a diameter upstream, and at section (2), about 5 diameters downstream. At section (1) the gauge pressure is 0.3 bar.



Evaluate the following.

(i) The gauge pressure at section (2) (0.387 bar)

(ii) The total force exerted by the fluid on the expansion. (-23 N)

$u_1 = Q/A_1 = 0.003/(\pi \times 0.0125^2) = 6.11$ m/s $\quad u_2 = Q/A_2 = 0.003/(\pi \times 0.02^2) = 2.387$ m/s

h_L (sudden expansion) $= (u_1^2 - u_2^2)/2g = 0.7067$ m

$u_1^2/2g + h_1 = u_2^2/2g + h_2 + h_L$

$h_1 - h_2 = 2.387^2/2g – 6.11^2/2 + 0.7067 = -0.9065$

$p_1 - p_2 = \rho g(h_1 - h_2) = 997 \times 9.81 \times (-0.9065) = -8866$ kPa

$p_1 = 0.3$ bar $\qquad p_2 = 0.3886$ bar

$p_1 A_1 - + \rho Q u_1 = p_2 A_2 - + \rho Q u_2 + F$

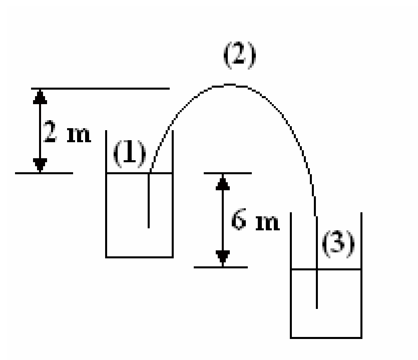
$0.3 \times 10^5 \times 0.491 \times 10^{-3} + 997 \times 0.003 \times 6.11 = 0.38866 \times 10^5 \times 1.257 \times 10^{-3} + 997 \times 0.003 \times 2.387 + F$

$F = -23$ N

If smooth $h_L = 0 \quad h_1 - h_2 = -1.613$ and $p_2 = 0.45778$ bar

4. A tank of water empties by gravity through a siphon into a lower tank. The difference in levels is 6 m and the highest point of the siphon is 2 m above the top surface level. The length of pipe from the inlet to the highest point is 3 m. The pipe has a bore of 30 mm and length 11 m. The friction coefficient for the pipe is 0.006.The inlet loss coefficient K is 0.6.

   Calculate the volume flow rate and the pressure at the highest point in the pipe.



Total length = 11 m $C_f = 0.006$

Bernoulli between (1) and (3)

$h_1 + u_1^2/2g + z_1 = h_3 + u_3^2/2g + z_3 + h_L$

$0 + 6 + 0 = 0 + 0 + 0 + h_L$ $\qquad h_L = 6$

h_L= Inlet + Exit + pipe

$6 = 0.6\ u^2/2g + u^2/2g + (4 \times 0.006 \times 11/0.03)\ u^2/2g$

$6 = 0.6\ u^2/2g + u^2/2g + 8.8\ u^2/2g = 10.4\ u^2/2g$

$u = 3.364$ m/s

$Q = A\ u = (\pi \times 0.03^2/4) \times 3.364 \quad Q = 0.002378\ m^3/s$

Bernoulli between (1) and (2)

$h_1 + u_1^2/2g + z_1 = h_2 + u_2^2/2g + z_2 + h_L$

$0 + 0 + 0 = h_2 + 2 + u^2/2g + h_L$

h_L= Inlet + pipe $= 0.6\ u^2/2g + (3/11) \times 8.8\ u^2/2g$

$h_L = 0.6 \times 3.364^2/2g + (3/11) \times 8.8 \times 3.364/2g$

$h_L = 1.73$ m

$0 = h_2 + 2 + 3.364^2/2g + 1.73$

$h_2 = -4.31$ m

5. (Q5 1989)

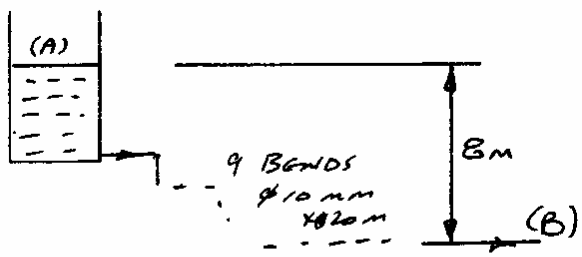
A domestic water supply consists of a large tank with a loss free-inlet to a 10 mm diameter pipe of length 20 m, that contains 9 right angles bends. The pipe discharges to atmosphere 8.0 m below the free surface level of the water in the tank.

Evaluate the flow rate of water assuming that there is a loss of 0.75 velocity heads in each bend and that friction in the pipe is given by the Blasius equation $C_f=0.079(Re)^{-0.25}$

The dynamic viscosity is 0.89×10^{-3} and the density is 997 kg/m^3.

($0.118\ dm^3/s$).

Q5 (Q5 1989)



$C_f = 0.079\ Re^{-.25}$

$Re = \rho u D/\mu$

$\rho = 997\ kg/m^3$

$\mu = 0.89 \times 10^{-3}\ Ns/m^2$

$Re = 997 \times u \times 0.01 / 0.89 \times 10^{-3} = 11202\ u$

$C_f = 0.079\ (11202u)^{-.25} = 7.679 \times 10^{-3}\ u^{-.25}$

$h_f = \frac{4 C_f}{2g}\frac{L u^2}{d} = \frac{4 \times 7.679 \times 10^{-3}\ u^{-.25} \times 20\ u^2}{2g \times 0.01}$

$h_f = 3.1311\ u^{1.75}$

LOSS IN BENDS $= 9 \times \frac{.75 u^2}{2g} = 0.344\ u^2$

BERNOULLI (A) → (B)

$h_A + z_A + \frac{u_A^2}{2g} = h_B + z_B + \frac{u_B^2}{2g} + h_L$

$0 + 8 + 0 = 0 + 0 + \frac{u^2}{2g} + .344u^2 + 3.1311u^{1.75}$

$8 = .395u^2 + 3.1311u^{1.75}$

SOLVE BY GUESSING OR NEWTON'S METHOD

$u = 1.5\ m/s$

$Q = Au$

$Q = 117.8 \times 10^{-6}\ m^3/s$

6. A pump A whose characteristics are given in table 1, is used to pump water from an open tank through 40 m of 70 mm diameter pipe of friction factor C_f=0.005 to another open tank in which the surface level of the water is 5.0 m above that in the supply tank.

Determine the flow rate when the pump is operated at 1450 rev/min. (7.8 dm^3/s)

It is desired to increase the flow rate and 3 possibilities are under investigation.

(i) To install a second identical pump in series with pump A.

(ii) To install a second identical pump in parallel with pump A.

(iii) To increase the speed of the pump by 10%.

Predict the flow rate that would occur in each of these situations.

Head-Flow Characteristics of pump A when operating at 1450 rev/min

Head/m		**9.75**	**8.83**	**7.73**	**6.90**	**5.50**	**3.83**
Flow Rate/(l/s)	**4.73**	**6.22**	**7.57**	**8.36**	**9.55**	**10.75**	

Q6 (Q10 1989)

PIPE 40m × ϕ70 mm $C_f = 0.005$

$$h_f = \frac{4 \times 0.005 \times 40 u^2}{2g \times 0.07} = 0.582 u^2$$

HEAD REQUIRED = h = LIFT + h_f + EXIT LOSS

$$h = 5 + 0.582u^2 + \frac{u^2}{2g} = 5 + 0.632u^2$$

PLOT PUMP HEAD H AGAINST Q

PLOT SYSTEM HEAD h AGAINST Q

FIND Q WHERE H = h

$h = 5 + 4.2672.2\,Q^2$

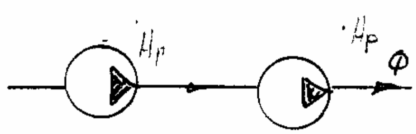
TABLE

H	9.75	8.83	7.73	6.9	5.5	3.83	m
Q	4.73	6.22	7.57	8.36	9.55	10.75	dm^3/s
u	1.23	1.62	1.97	2.17	2.48	2.79	m/s
h	5.88	6.52	7.25	7.75	8.89	9.93	m

FROM THE GRAPH A MATCHING POINT IS

$Q = 7.8\ dm^3/s$ $h = 7.4m$

WITH TWO PUMPS IN SERIES, EACH PUMP HAS SAME Q BUT HALF VALUE OF h



PLOT $2H_p - Q$

$\cancel{h} h - Q$ ← REMAINS SAME

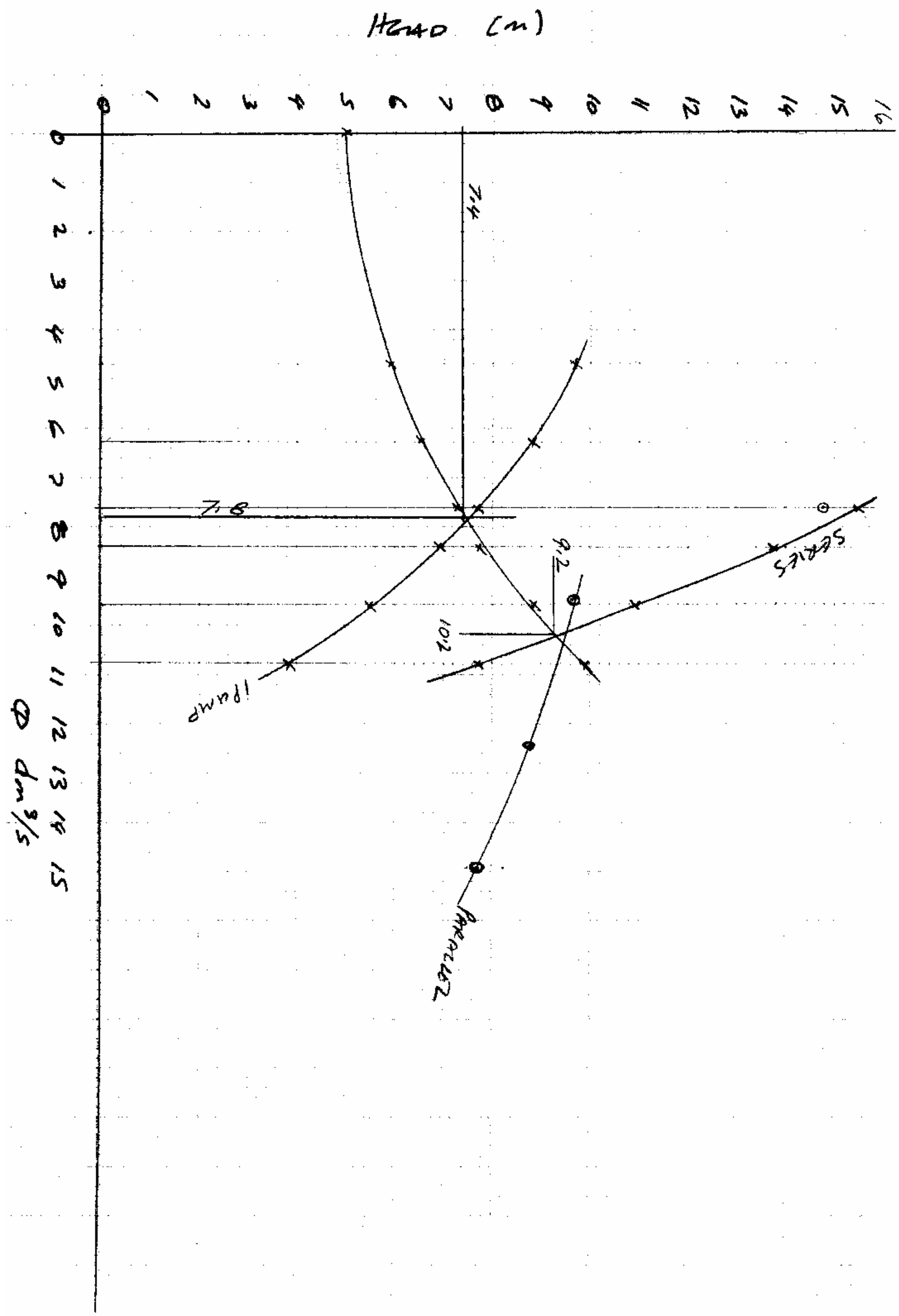
MATCHING POINT IS $h = 9.8m$ $Q = 10.3\ dm^3/s$

∴ FLOW IS INCREASED

WITH TWO PUMPS IN PARALLEL, FLOW IS HALVED BUT SAME h

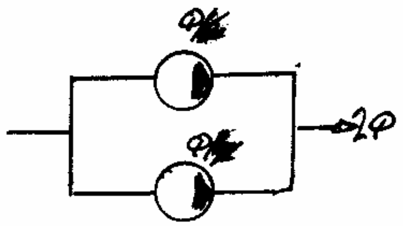
PLOT $H - Q$ MATCHING POINT IS

$h - Q/2$ $h = 9.6m$ $Q = 10\ dm^3/s$



HEAD (m)
Q dm³/s
PUMP
SERIES
PARALLEL
7.4
7.8
9.2
10.1

PUMPS IN PARALLEL



SAME H – Q
FOR PUMP PLOT 2Q – H

H	9.75	8.83	7.73	6.9	5.5	3.83
2Q	9.46	12.44	15.14	16.72	19.1	21.5

FROM GRAPH, THE MATCHING POINT IS ALMOST THE SAME AS FOR SERIES PUMPS.

SPEED CONTROL

$$Q_2 = Q_1 \left(N_1 / N_2 \right)^2$$

FOR 10% INCREASE

$N_2 = 1.1 N_1$

$Q_2 = .826\, Q_1$

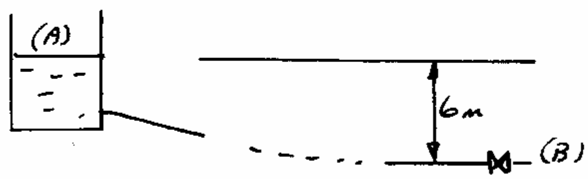
REDUCED FLOW SAME HEAD

7. A steel pipe of 0.075 m inside diameter and length 120 m is connected to a large reservoir. Water is discharged to atmosphere through a gate valve at the free end, which is 6 m below the surface level in the reservoir. There are four right angle bends in the pipe line. Find the rate of discharge when the valve is fully open. (ans. 8.3 dm^3/s).The kinematic viscosity of the water may be taken to be 1.14 x 10^{-6} m^2/s. Use a value of the friction factor C_f taken from table 2 which gives C_f as a function of the Reynolds number Re and allow for other losses as follows.

at entry to the pipe 0.5 velocity heads.
at each right angle bend 0.9 velocity heads.
for a fully open gate valve 0.2 velocity heads.

Re x 10^5	0.987	1.184	1.382
C_f	0.00448	0.00432	0.00419

Q7 (Q12 1985)



BERNOULLI (A) → (B)

$$h_A + z_A + \frac{u_A^2}{2g} = h_B + z_B + \frac{u_B^2}{2g} + h_L$$

$$0 + 6 + 0 = 0 + 0 + \frac{u_B^2}{2g} + h_L$$

$$h_L = 6 - \frac{u_B^2}{2g}$$

$$h_L = \underbrace{\frac{4C_f L u^2}{2gd}}_{\text{pipe}} + \underbrace{\frac{0.5u^2}{2g}}_{\text{entry}} + \underbrace{\frac{4\times.9u^2}{2g}}_{\text{Bends}} + \underbrace{\frac{.2u^2}{2g}}_{\text{gate}}$$

$$h_L = \frac{u^2}{2g}\left\{\frac{4C_f L}{d} + 4.3\right\} = 6 - \frac{u_B^2}{2g}$$

$$6 = \frac{u^2}{2g}\left\{\frac{4C_f L}{d} + 5.3\right\} = \frac{u^2}{2g}\left\{\frac{4\,C_f \times 120}{0.075} + 5.3\right\}$$

$$6 = \frac{u^2}{2g}\left\{6400 C_f + 5.3\right\}$$

$$117.72 = u^2\left\{6400 C_f + 5.3\right\}$$

$$Re = \frac{ud}{\nu} = \frac{u \times 0.075}{1.14\times10^{-6}} = 65789.5\,u$$

$$u = \frac{Re}{65789.5}$$

$$117.72 = \frac{Re^2}{4.3283 \times 10^9} \left\{ 6400 C_f + 5.3 \right\}$$

$$509.522 \times 10^9 = Re^2 \left\{ 6400 C_f + 5.3 \right\}$$

SOLVE BY TRIAL & ERROR OR PLOTTING DATA
A SUITABLE GRAPH

A PAIR OF POINTS THAT FIT ARE

$$Re = 1.24 \times 10^5 \qquad C_f = 0.00428$$

$$1.24 \times 10^5 = 65789.5\, u$$

$$u = 1.885 \text{ m/s}$$

DISCHARGE $\quad Q = Au = \frac{\pi \times .075^2}{4} \times 1.885$

$$Q = 8.33 \text{ dm}^3/\text{s}$$

8. (i) Sketch diagrams showing the relationship between Reynolds number, Re, and friction factor, C_f , for the head lost when oil flows through pipes of varying degrees of roughness. Discuss the importance of the information given in the diagrams when specifying the pipework for a particular system.

(ii) The connection between the supply tank and the suction side of a pump consists of 0.4 m of horizontal pipe , a gate valve one elbow of equivalent pipe length 0.7 m and a vertical pipe down to the tank.

If the diameter of the pipes is 25 mm and the flow rate is 30 l/min, estimate the maximum distance at which the supply tank may be placed below the pump inlet in order that the pressure there is no less than 0.8 bar absolute. (Ans. 1.78 m)

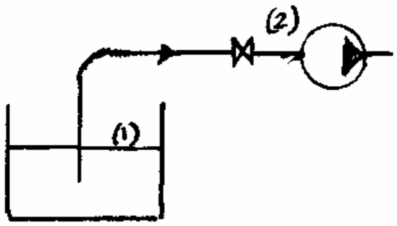
The fluid has kinematic viscosity 40×10^{-6} m^2/s and density 870 kg/m^3.

Assume
(a) for laminar flow $C_f = 16/(Re)$ and for turbulent flow $C_f = 0.08/(Re)^{0.25}$.

(b) head loss due to friction is $4C_f V^2 L/2gD$ and due to fittings is $KV^2/2g$.

where K=0.72 for an elbow and K=0.25 for a gate valve.

What would be a suitable diameter for the delivery pipe ?



$\nu = 40 \times 10^{-6}\ m^2/s$

$Q = 30/60 = 0.5\ dm^3/s$

$u = Q/A = \dfrac{0.5 \times 10^{-3}}{\pi \times 0.0125^2} = 1.0186\ m/s$

$Re = \dfrac{\rho u D}{\mu} = \dfrac{uD}{\nu} = \dfrac{1.0186 \times 0.025}{40 \times 10^{-6}}$

$Re = 636.6$

LAMINAR FLOW $C_f = 16/Re = 0.0251$

BERNOULLI

$$h_1 + z_1 + \frac{u_1^2}{2g} = h_2 + z_2 + \frac{u_2^2}{2g} + loss$$

$$h_1 + 0 + 0 = h_2 + z_2 + \frac{u_2^2}{2g} + loss$$

$$h_1 = p_1/\rho g = \frac{1.013 \times 10^5}{997 \times 9.81} = 11.869\,m$$

$$h_2 = p_2/\rho g = \frac{0.8 \times 10^5}{997 \times 9.81} = 9.373\,m$$

LOSS = h_f + BEND + GATE + INLET

$$h_f = \frac{4 C_f L u^2}{2 g d} \qquad L = .4 + .7 + Z \qquad L = 1.10 + Z$$

$$h_f = \frac{4 \times .0251 \times (1.1 + Z) \times 1.0186^2}{2g \times 0.025} = 0.2124(1.1 + Z)$$

$$h_L = h_f + \frac{.72 \times 1.0186^2}{2g} + \frac{.25 u^2}{2g} + ?$$ NO DATA FOR INLET IGNORE

$$h_L = h_f + 0.0381 + 0.0132 = h_f + 0.05129$$

$$h_L = .2124(1.1 + Z) + 0.05129$$

$$h_L = 0.2124 Z + 0.2336 + 0.05129$$

$$h_L = 0.2124 Z + 0.285$$

$$h_1 = h_2 + z_2 + \frac{u_2^2}{2g} + h_L \qquad z_2 \equiv Z$$

$$11.869 = 9.373 + Z + \frac{1.0186^2}{2g} + .2124 Z + .285$$

$$11.869 = 9.373 + Z + 0.0523 + .2124 Z + .285$$

$$2.159 = Z + .2124 Z$$

$$2.159 = 1.2124 Z$$

$$Z = 1.78 \text{ m}$$

CONVENTION $D_1 = \frac{3}{4} D_2$

D_1 = SUCTION PIPE

D_2 = DELIVERY PIPE

HELPS PREVENT CAVITATION