FLUID MECHANICS D203
SAE SOLUTIONS TUTORIAL 1 - FLUID FLOW THEORY

ASSIGNMENT 3

1. A pipe is 25 km long and 80 mm bore diameter. The mean surface roughness is 0.03 mm. It carries oil of density 825 kg/m^3 at a rate of 10 kg/s. The dynamic viscosity is 0.025 N s/m^2.

   Determine the friction coefficient using the Moody Chart and calculate the friction head. (Ans. 3075 m.)

   $Q = m/\rho = 10/825 = 0.01212$ m^3/s
   $u_m = Q/A = 0.01212/(\pi \times 0.04^2) = 2.411$ m/s
   $Re = \rho ud/\mu = 825 \times 2.4114 \times 0.08/0.025 = 6366$
   $k/D = 0.03/80 = 0.000375$
   From the Moody chart $C_f = 0.0083$
   $h_f = 4\ C_f\ L\ u^2/2gd = 4 \times 0.0083 \times 25000 \times 2.4114^2/(2 \times 9.91 \times 0.08) = 3075$ m

2. Water flows in a pipe at 0.015 m^3/s. The pipe is 50 mm bore diameter. The pressure drop is 13 420 Pa per metre length. The density is 1000 kg/m^3 and the dynamic viscosity is 0.001 N s/m^2.

   Determine
   i. the wall shear stress (167.75 Pa)
   ii. the dynamic pressure (29180 Pa).
   iii. the friction coefficient (0.00575)
   iv. the mean surface roughness (0.0875 mm)

   $\tau_o = \Delta p\ D/4L = 13420 \times 0.05/4 = 167.75$ Pa

   $u_m = Q/A = 0.015/(\pi \times 0.025^2) = 7.64$ m/s

   Dynamic Pressure $= \rho u^2/2 = 1000 \times 7.64^2/2 = 29180$ Pa

   $C_f = \tau_o/\text{Dyn Press} = 167.75/29180 = 0.00575$

   From the Moody Chart we can deduce that $\varepsilon = 0.0017 = k/D$ $k = 0.0017 \times 50 = 0.085$ mm

3. Explain briefly what is meant by fully developed laminar flow. The velocity u at any radius r in fully developed laminar flow through a straight horizontal pipe of internal radius r_o is given by

$$u = (1/4\mu)(r_o^2 - r^2)dp/dx$$

dp/dx is the pressure gradient in the direction of flow and μ is the dynamic viscosity.
Show that the pressure drop over a length L is given by the following formula.

$$\Delta p = 32\mu L u_m/D^2$$

The wall skin friction coefficient is defined as $C_f = 2\tau_o/(\rho u_m^2)$.
Show that $C_f = 16/R_e$ where $R_e = \rho u_m D/\mu$ and ρ is the density, u_m is the mean velocity and τ_o is the wall shear stress.

THE BOUNDARY LAYER IS ENTIRELY LAMINAR

3) AND CONSTANT THICKNESS

$$u = \frac{1}{4\mu}(r_o^2 - r^2)\frac{dp}{dx} \qquad \text{Assume } \frac{dp}{dx} = \frac{\Delta p}{L}$$

$$d\varphi = u \times 2\pi r\, dr \quad \varphi = \frac{1}{4\mu}\frac{\Delta p}{L} \times 2\pi \int_0^{r_o} (r r_o^2 - r^3)\, dr$$

$$\varphi = \frac{\Delta p \pi}{2\mu L}\left[\frac{r^2}{2}r_o^2 - \frac{r^4}{4}\right]_0^{r_o} = \frac{\Delta p \pi}{2\mu L}\left[\frac{r_o^4}{2} - \frac{r_o^4}{4}\right]$$

$$\varphi = \frac{\Delta p \pi}{2\mu L}\frac{r_o^4}{4} = \frac{\Delta p \pi r_o^4}{8\mu L} \qquad r_o = D/2$$

$$\varphi = \frac{\Delta p \pi D^4}{128\mu L} \qquad \varphi = u_m \times A = u_m \frac{\pi D^2}{4}$$

$$u_m = \frac{\Delta p D^2}{32\mu L} \qquad \Delta p = \frac{32\mu L u_m}{D^2}$$

$$C_f = \frac{2\tau_o}{\rho u_m^2} \qquad \tau_o \times \cancel{\pi} \cancel{D} L = \Delta p \times \frac{\cancel{\pi} D^{\cancel{2}}}{4}$$

$$\tau_o = \frac{\Delta p}{L}\frac{D}{4}$$

$$C_f = \frac{2\Delta p D}{\rho u_m^2 4L}$$

$$C_f = \frac{\Delta p D}{2\rho u_m^2 L} = \frac{\cancel{\Delta p}\cancel{D}}{2\rho \cancel{L} u_m} \times \frac{32\mu \cancel{L}}{\cancel{\Delta p} D^{\cancel{2}}} = \frac{32\mu}{2\rho D u_m}$$

$$C_f = \frac{16\mu}{\rho u_m D} = \frac{16}{R_e}$$

3. Oil with viscosity 2 x 10^{-2} Ns/m^2 and density 850 kg/m^3 is pumped along a straight horizontal pipe with a flow rate of 5 dm^3/s. The static pressure difference between two tapping points 10 m apart is 80 N/m^2. Assuming laminar flow determine the following.

   i. The pipe diameter.
   ii. The Reynolds number.

   Comment on the validity of the assumption that the flow is laminar

4.
$\mu = 2 \times 10^{-2}\ Ns/m^2$
$\rho = 850\ kg/m^3$
$Q = 5\ dm^3/s$
$\Delta p = 80\ N/m^2$
$L = 10\ m$

POISEUILLE'S EQUATION

$$\Delta p = \frac{32 \mu L u_m}{D^2}$$

$$D^2 = \frac{32 \mu L u_m}{\Delta p} = \frac{32 \times 2 \times 10^{-2} \times 10 \times u_m}{80}$$

$$D^2 = 0.08\ u_m \qquad u_m = \frac{4Q}{\pi D^2} = \frac{4 \times 5 \times 10^{-3}}{\pi D^2}$$

$$D^2 = \frac{.08 \times .006366}{D^2} \qquad u_m = \frac{.006366}{D^2}$$

$$D^4 = 509 \times 10^{-6} \qquad D = 0.150\ m$$

$$Re = \frac{\rho u D}{\mu} = \frac{850 \times u_m \times .15}{2 \times 10^{-2}}$$

$$u_m = .006366 / .15^2 = .2829\ m/s$$

$$Re = \frac{850 \times .2829 \times .15}{2 \times 10^{-2}} = 1803.7$$

$Re < 2000$

JUST LAMINAR

ASSIGNMENT 4

1. Research has shown that tomato ketchup has the following viscous properties at 25°C.

   Consistency coefficient K = 18.7 Pa s^n
   Power n = 0.27
   Shear yield stress = 32 Pa

   Calculate the apparent viscosity when the rate of shear is 1, 10, 100 and 1000 s^{-1} and conclude on the effect of the shear rate on the apparent viscosity.

   This fluid should obey the Herchel-Bulkeley equation so $\mu_{app} = \frac{\tau_y}{\dot{\gamma}} + K\dot{\gamma}^{n-1} = \frac{32}{\dot{\gamma}} + 18.7\dot{\gamma}^{0.27-1}$

   put γ = 1 and μ_{app} = 50.7
   put γ = 10 and μ_{app} = 6.682
   put γ = 100 and μ_{app} = 0.968
   put γ = 1000 and μ_{app} = 0.153
   The apparent viscosity reduces as the shear rate increases.

2. A Bingham plastic fluid has a viscosity of 0.05 N s/m^2 and yield stress of 0.6 N/m^2. It flows in a tube 15 mm bore diameter and 3 m long.

(i) Evaluate the minimum pressure drop required to produce flow.

The actual pressure drop is twice the minimum value. Sketch the velocity profile and calculate the following.

(ii) The radius of the solid core.
(iii) The velocity of the core.
(iv) The volumetric flow rate.

$\tau = \tau_Y + \mu \frac{du}{dy}$ The minimum value of τ is τ_y

Balancing forces on the plug τ_y x $2\pi rL = \Delta p \pi r^2$

$\Delta p = \tau_Y \frac{2L}{r}$ and the minimum Δp- is at r = R $\Delta p = 0.6 \frac{2 \text{ x } 3}{0.0075} = 480 \text{ Pa}$

b Δp = 2 x 480 = 960 Pa From the force balance $\Delta p = \tau_Y \frac{2L}{r}$

$r = \tau_Y \frac{2L}{\Delta p} = 0.6 \frac{2 \text{ x } 3}{960} = 0.00375 \text{ m or } 3.75 \text{ mm}$

The profile is follows Poiseuille's equation

$u = \frac{\Delta p}{4 \mu L}\left(R^2 - r^2\right) = \frac{960}{4 \text{ x } 0.05 \text{ x } 3}\left(0.0075^2 - 0.00375^2\right) = 0.0675 \text{ m/s}$

Flow rate of plug =Au = $\pi(0.00375^2)$x0.0675 = 2.982 x 10^{-6} m^3/s

$$dQ = u\,(2\pi r\,dr) = \frac{\Delta p(2\,\pi\,r)}{4\,\mu\,L}\left(R^2 - r^2\right)$$

$$Q = \int_r^R \frac{\Delta p(2\,\pi)}{4\,\mu\,L}\left(rR^2 - r^3\right) \qquad Q = \frac{\Delta p(2\,\pi)}{4\,\mu\,L}\left[\frac{R^2 r^2}{2} - \frac{r^4}{2}\right]_r^R$$

$$Q = \frac{\Delta p(2\,\pi)}{4\,\mu\,L}\left\{\left[\frac{R^4}{2} - \frac{R^4}{4}\right] - \left[\frac{R^2 r^2}{2} - \frac{r^4}{4}\right]\right\}$$

$$Q = \frac{\Delta p\,\pi}{2\,\mu\,L}\left[\frac{R^4}{4} - \frac{R^2 r^2}{2} - \frac{r^4}{4}\right]$$

$$Q = \frac{960\,\pi}{2\,x0.05\,x\,3}\left[\frac{0.0075^4}{4} - \frac{0.0075^2\ x\ 0.00375^2}{2} - \frac{0.00375^4}{4}\right] = 4.473x10^{-6}\,m^3/s$$

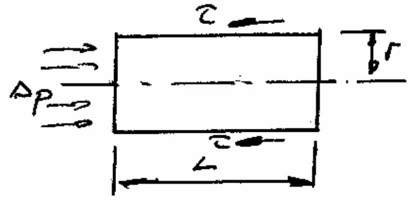
Total Q = (4.473 + 2.982) x 10^{-6} = 7.46 x 10^{-6} m^3/s

3. A non-Newtonian fluid is modelled by the equation $\tau = K\left(\frac{du}{dr}\right)^n$ where n = 0.8 and K = 0.05 N $s^{0.8}/m^2$. It flows through a tube 6 mm bore diameter under the influence of a pressure drop of 6400 N/m^2 per metre length. Obtain an expression for the velocity profile and evaluate the following.
   (i) The centre line velocity. (0.953 m/s)
   (ii) The mean velocity. (0.5 m/s)

$$\tau = K\left(\frac{du}{dy}\right)^n$$

TUBE ϕ6mm $\quad \frac{\Delta p}{L} = 6400$ Pa/m

CYLINDER OF FLUID



SHEAR FORCE $F_s = \tau \times$ SURFACE AREA
$= \tau \times 2\pi r L$

PRESSURE FORCE $= \Delta p \times \pi r^2$

TOTAL IS ZERO $\quad \Delta p \, \pi r^2 + \tau \, 2\pi r L = 0$

$$\frac{\Delta p r}{2L} = \tau = -K\left(\frac{du}{dy}\right)^n \qquad \text{note } dy = -dr$$

$$\frac{\Delta p r}{2LK} = \left(\frac{du}{dr}\right)^n$$

$$du = \left(\frac{\Delta p r}{2LK}\right)^{\frac{1}{n}} \times dr \qquad \text{INTEGRATE}$$

$$u = \int \left(\frac{\Delta p r}{2LK}\right)^{1/n} dr = \left(\frac{\Delta p}{2LK}\right)^{1/n} \int_R^r r^{1/n} \, dr$$

$$u = \left(\frac{\Delta p}{2LK}\right)^{1/n} \left[\frac{r^{1+1/n}}{1+1/n}\right]_R^r = \left(\frac{\Delta p}{2LK}\right)^{1/n} \left[\frac{nR^{\frac{n+1}{n}}}{n+1} - \frac{n r^{\frac{n+1}{n}}}{n+1}\right]$$

AT THE CENTRE LINE $r = 0$

$$u = \left(\frac{\Delta P}{2LK}\right)^{1/n} \frac{n}{n+1} \left[R^{\frac{n+1}{n}} - 0 \right]$$

$$\frac{\Delta P}{L} = 6400 \quad R = .003\,m \quad K = \mu = 0.05\,Ns/m^2$$
$$n = 0.8$$

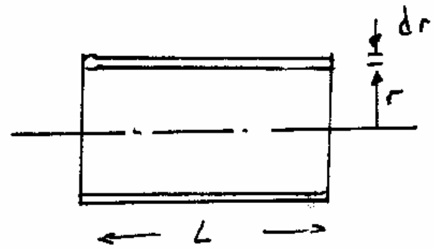
$$u = \frac{6400}{2 \times 0.05} \times \frac{0.8}{1.8} \left[0.003^{1.8/.8} \right]$$

$$u = 1.0179 \times 10^6 \times \frac{.8}{1.8} \times .003^{2.25}$$

$$u = 452.4 \times 10^3 \times .003^{2.25}$$

$$u = 0.953\ m/s$$

<u>FLOW RATE</u>



CONSIDER A THIN WALL CYLINDER MOVING AT VELOCITY u

$$\text{FLOW} = d\varphi = \text{CROSS SECTIONAL AREA} \times u$$
$$= 2\pi r\, dr\, u$$

$$d\varphi = \left(\frac{\Delta P}{2LK}\right)^{1/n} \frac{n}{n+1} \; 2\pi \left[R^{\frac{n+1}{n}} r - r^{1+1+1/n} \right] dr$$

TOTAL FLOW

$$\varphi = \left(\frac{\Delta P}{2LK}\right)^{1/n} \frac{n}{n+1} \; 2\pi \int_0^R \left[R^{\frac{n+1}{n}} r - r^{2+1/n} \right] dr$$

$$Q = \left(\frac{\Delta P}{2LK}\right)^{1/n} \frac{n}{n+1} \quad 2\pi \left[\frac{R^{1+1/n} r^2}{2} - \frac{r^{2+1/n+1}}{2+1/n+1} \right]_0^R$$

$$Q = \left(\frac{\Delta P}{2LK}\right)^{1/n} \frac{n}{n+1} \quad 2\pi \left[\frac{R^{3+1/n}}{2} - \frac{R^{3+1/n}}{3+1/n} \right]$$

$$Q = \left(\frac{\Delta P}{2LK}\right)^{1/n} \frac{n}{n+1} \quad 2\pi \quad R^{3+1/n} \left[\frac{1}{2} - \frac{1}{3+1/n} \right]$$

$$Q = \left(\frac{\Delta P}{2LK}\right)^{1/n} \frac{n}{n+1} \quad 2\pi \left[\frac{n+1}{6n+2} \right] R^{3+1/n}$$

$$Q = \left(\frac{\Delta P}{2LK}\right)^{1/n} \pi R^{3+1/n} \left(\frac{n}{3n+1}\right)$$

$$Q = \left(\frac{6400}{2 \times .05}\right)^{1/0.8} \pi \times 0.003^{3+1/.8} \times \frac{0.8}{(3 \times 0.8+1)}$$

$$Q = 64000^{1.25} \times \pi \times .003^{4.25} \times \frac{1}{4.25}$$

$$Q = 14.26 \times 10^{-6} \ m^3/s$$

MEAN VELOCITY

$$u_m = Q/A = \frac{14.26 \times 10^{-6}}{\pi \times .003^2} = \underline{\underline{0.504 \ m/s}}$$

NOTE IF $n=1$ ALL EQUATIONS BECOME THE SAME AS FOR NEWTONIAN FLOW

ADDITIONAL PROOF

$$u = \left(\frac{\Delta P}{2L\mu}\right)^{1/n}\left[\frac{nR^{1+1/n}}{n+1} - \frac{nr^{1+1/n}}{n+1}\right]$$

If $n = 1$ (NEWTONIAN)

$$u = \frac{\Delta P}{2L\mu} \times \frac{1}{2}\left[R^2 - r^2\right]$$

$$dQ = u \times 2\pi r\, dr$$

$$= \left(\frac{\Delta P}{2L\mu}\right)^{1/n}\left(\frac{n}{n+1}\right)\left[R^{1+1/n} - r^{1+1/n}\right] \times 2\pi r\, dr$$

$$Q = \left(\frac{\Delta P}{2L\mu}\right)^{1/n}\frac{2\pi n}{(n+1)}\int_0^R \left(R^{1+1/n}r - r^{2+1/n}\right) dr$$

$$= \left(\frac{\Delta P}{2L\mu}\right)^{1/n}\frac{2\pi n}{n+1}\left[R^{1+1/n}\frac{r^2}{2} - \frac{r^{3+1/n}}{3+1/n}\right]_0^R$$

$$= \left(\frac{\Delta P}{2L\mu}\right)^{1/n}\frac{2\pi n}{n+1}\left[\frac{R^{3+1/n}}{2} - \frac{R^{3+1/n}}{3+1/n}\right]$$

$$= \left(\frac{\Delta P}{2L\mu}\right)^{1/n}\frac{2\pi n}{n+1} R^{3+1/n}\left[\frac{1}{2} - \frac{1}{3+1/n}\right]$$

If $n = 1$ $$Q = \frac{\Delta P}{2L\mu} \times \frac{2\pi}{2} R^4\left[\frac{1}{2} - \frac{1}{4}\right]$$

$$Q = \frac{\Delta P\, \pi R^4}{2L\mu} \times \frac{1}{4}$$ — Correct