FLUID MECHANICS D203
SAE SOLUTIONS TUTORIAL 1 - FLUID FLOW THEORY

S.A.E. No. 1

1. Describe the principle of operation of the following types of viscometers.
a. Redwood Viscometers.
b. British Standard 188 glass U tube viscometer.
c. British Standard 188 Falling Sphere Viscometer.
d. Any form of Rotational Viscometer

The solutions are contained in part 1 of the tutorial.

S.A.E. No. 2

1. Oil flows in a pipe 80 mm bore diameter with a mean velocity of 0.4 m/s. The density is 890 kg/m^3 and the viscosity is 0.075 Ns/m^2. Show that the flow is laminar and hence deduce the pressure loss per metre length.

$$R_e = \frac{\rho u d}{\mu} = \frac{890 \text{ x } 0.4 \text{ x } 0.08}{0.075} = 379.7$$

Since this is less than 2000 flow is laminar so Poiseuille's equation applies.

$$\Delta p = \frac{32\mu 2\mu}{d^2} = \frac{32 \text{ x } 0.075 \text{ x } 1 \text{ x } 0.4}{0.08^2} = 150 \text{ Pa}$$

2. Oil flows in a pipe 100 mm bore diameter with a Reynolds' Number of 500. The density is 800 kg/m^3. Calculate the velocity of a streamline at a radius of 40 mm. The viscosity μ = 0.08 Ns/m^2.

$$R_e = 500 = \frac{\rho u_m d}{\mu}$$

$$u_m = \frac{500\mu}{\rho d} = \frac{500 \text{ x } 0.08}{800 \text{ x } 0.1} = 0.5 \text{ m/s}$$

Since R_e is less than 2000 flow is laminar so Poiseuille's equation applies.

$$\Delta p = \frac{32\mu 2\mu}{d^2} = \frac{32 \text{ x } 0.08 \text{ x L x } 0.5}{0.1^2} = 128\text{L Pa}$$

$$u = \frac{\Delta p\left(R^2 - r^2\right)}{4L\mu} = \frac{128L\left(0.05^2 - 0.04^2\right)}{4L \text{ x } 0.08} = 0.36 \text{ m/s}$$

3. A liquid of dynamic viscosity 5 x 10^{-3} Ns/m^2 flows through a capillary of diameter 3.0 mm under a pressure gradient of 1800 N/m^3. Evaluate the volumetric flow rate, the mean velocity, the centre line velocity and the radial position at which the velocity is equal to the mean velocity.

$$\frac{\Delta P}{L} = 1800 = \frac{32\,\mu\, u_m}{d^2} \qquad u_m = 0.10125 \text{ m/s}$$

u_{max} = 2 u_m = 0.2025 m/s

$$u = 0.10125 = \frac{\Delta p\left(R^2 - r^2\right)}{4L\mu} = \frac{1800\left(0.0015^2 - r^2\right)}{4 \text{ x } 0.005} \qquad r = 0.0010107 \text{ m or } 1.0107 \text{ mm}$$

4.

a. Explain the term Stokes flow and terminal velocity.

b. Show that a spherical particle with Stokes flow has a terminal velocity given by
$u = d^2 g(\rho_s - \rho_f)/18\mu$
Go on to show that $C_D = 24/R_e$

c. For spherical particles, a useful empirical formula relating the drag coefficient and the Reynold's number is

$$C_D = \frac{24}{R_e} + \frac{6}{1+\sqrt{R_e}} + 0.4$$

Given ρ_f = 1000 kg/m^3, μ= 1 cP and ρ_s= 2630 kg/m^3 determine the maximum size of spherical particles that will be lifted upwards by a vertical stream of water moving at 1 m/s.

d. If the water velocity is reduced to 0.5 m/s, show that particles with a diameter of less than 5.95 mm will fall downwards.

a) For R_e<0.2 the flow is called Stokes flow and Stokes showed that $R = 3\pi d\,\mu\, u$ hence

R = W = volume x density difference x gravity

$$R = W = \frac{\pi d^3 g(\rho_s - \rho_f)}{6} = 3\pi d\,\mu\, u$$

ρ_s = density of the sphere material ρ_f = density of fluid   d = sphere diameter

$$u = \frac{\pi d^3 g(\rho_s - \rho_f)}{18\,\pi\, d\,\mu} = \frac{d^2 g(\rho_s - \rho_f)}{18\,\mu}$$

b) C_D = R/(projected area x $\rho u^2/2$) $C_D = \frac{\pi d^3 g(\rho_s - \rho_f)}{(\rho\, u^2/2) 6\,\pi\, d^2/4} = \frac{4dg(\rho_s - \rho_f)}{3\rho\, u^2}$

$$C_D = \frac{4 \text{ x } 9.81 \text{x } (1630 - 998)\text{d}}{3 \text{x } 998 \text{ x } u^2} = 21.389\frac{d}{u^2}$$

$$C_D = \frac{24}{R_e} + \frac{6}{1+\sqrt{R_e}} + 0.4 = 21.389\frac{d}{u^2}$$

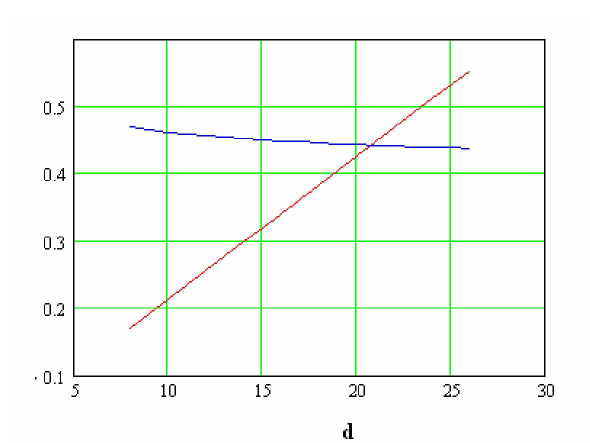
$$21.389\frac{d}{u^2} - \frac{24}{R_e} - \frac{6}{1+\sqrt{R_e}} = 0.4 \qquad \text{let } 21.389\frac{d}{u^2} - \frac{24}{R_e} - \frac{6}{1+\sqrt{R_e}} = x$$

Re = $\rho ud/\mu$ = 998 x 1 x d/0.89 x 10^{-3} = 1.1213 x 10^6 d

Make a table

D	0.001	0.003	0.01	0.02	0.03
Re	1121.3	3363.9	11213	22426	33639
x	-0.174	-0.045	0.156	0.387	0.608

Plot and find that when d = 0.0205 m (20.5 mm) x = 0.4



c) u = 0.5m/s d = 5.95mm

Re = ρud/μ = 998 x 0.5 x 0.00595/0.89 x 10^{-3} = 3336

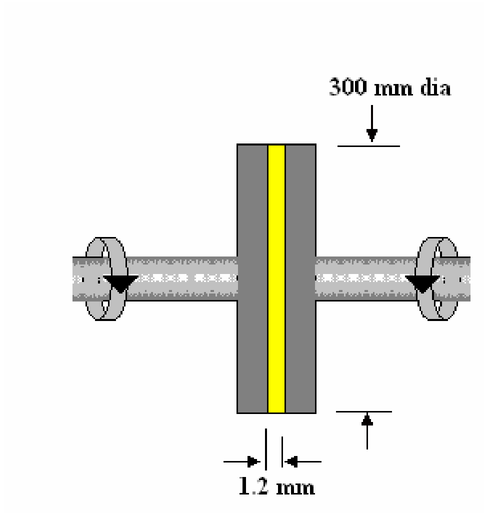
$$C_D = 21.389\frac{d}{u^2} = 0.509$$

$$C_D = \frac{24}{3336} + \frac{6}{1+\sqrt{3336}} + 0.4 = 0.509$$

Since C_D is the same, larger ones will fall.

5. Similar to Q5 1998

A simple fluid coupling consists of two parallel round discs of radius R separated by a a gap h. One disc is connected to the input shaft and rotates at ω_1 rad/s. The other disc is connected to the output shaft and rotates at ω_2 rad/s. The discs are separated by oil of dynamic viscosity μ and it may be assumed that the velocity gradient is linear at all radii.



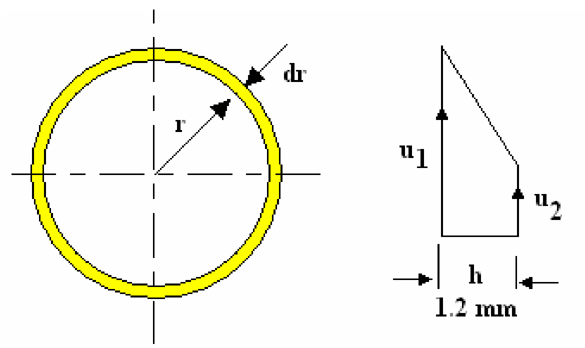
Show that the Torque at the input shaft is given by

$$T = \frac{\pi D^4 \mu(\omega_1 - \omega_2)}{32h}$$

The input shaft rotates at 900 rev/min and transmits 500W of power. Calculate the output speed, torque and power. (747 rev/min, 5.3 Nm and 414 W)

Show by application of max/min theory that the output speed is half the input speed when maximum output power is obtained.

SOLUTION



Assume the velocity varies linearly from u_1 to u_2 over the gap at any radius. Gap is h = 1.2 mm

T = μ du/dy = μ $(u_1 - u_2)$/h

For an elementary ring radius r and width dr the shear force is

Force = τ dA = τ 2πr dr

$$dF = \mu\frac{u_1 - u_2}{h} x 2\pi r\ dr$$

Torque due to this force is $$dT = rdF = \mu\frac{u_1 - u_2}{h} x\ 2\pi\ r^2\ dr$$

Substitute u = ωr $$dT = rdF = \mu\frac{(\omega_1 - \omega_2)}{h} x\ 2\pi\ r^3\ dr$$

Integrate $$T = \mu\frac{(\omega_1 - \omega_2)}{h} x\ 2\pi\int_0^R r^3\ dr\ = \mu\frac{(\omega_1 - \omega_2)}{h} x\ 2\pi\frac{R^4}{4}$$

Rearrange and substitute R = D/2 $$T = \mu\frac{(\omega_1 - \omega_2)}{h} x\ \pi\frac{D^4}{32}$$

Put D = 0.3 m, μ = 0.5 N s/m^2, h = 0.012 m $T = 0.5\frac{(\omega_1 - \omega_2)}{0.012} x\ \pi\frac{0.3^4}{32} = 0.33(\omega_1 - \omega_2)$

N = 900 rev/min P = 500 W Power = $2\pi NT/60$ $T = \frac{60P}{2\pi\ N} = \frac{60 \text{ x } 500}{2\pi \text{ x } 900} = 5.305$ Nm

The torque input and output must be the same. $\omega_1 = 2\pi N_1 /60 = 94.25$ rad/s

$5.305 = 0.33(94.25_1 - \omega_2)$ hence ω_2 = 78.22 rad/s and N_2 = 747 rev/min

$P_2 = 2\pi N_2 T/60 = \omega_2 T$ = 78.22 x 5.305 = 414 W (Power out)

For maximum power output $dp_2/d\omega_2 = 0$ $P_2 = \omega_2 T = 0.33(\omega_1\omega_2 - \omega_2^2)$

Differentiate $\frac{dP_2}{d\omega_2} = 0.33(\omega_1 - 2\omega_2)$

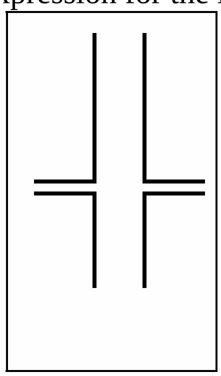
Equate to zero and it follows that for maximum power output $\omega_1 = 2\ \omega_2$

And it follows $N_1 = 2\ N_2$ so N_2 = 450 rev/min

6. Show that for fully developed laminar flow of a fluid of viscosity μ between horizontal parallel plates a distance h apart, the mean velocity u_m is related to the pressure gradient dp/dx by

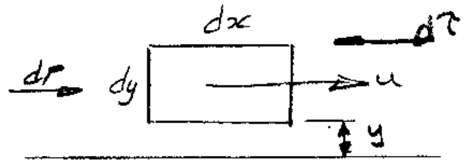
$u_m = -(h^2/12\mu)(dp/dx)$

A flanged pipe joint of internal diameter d_i containing viscous fluid of viscosity μ at gauge pressure p. The flange has an outer diameter d_o and is imperfectly tightened so that there is a narrow gap of thickness h. Obtain an expression for the leakage rate of the fluid through the flange.



Note that this is a radial flow problem and B in the notes becomes $2\pi r$ and dp/dx becomes -dp/dr. An integration between inner and outer radii will be required to give flow rate Q in terms of pressure drop p.

The answer is $Q = (2\pi h^3 p/12\mu)/\{\ln(d_o/d_i)\}$



$d\tau$ acts on dx $\quad dp$ acts dy

Balancing forces $\quad d\tau\, dx = dp\, dy$

$$dp/dx = d\tau/dy$$

But $\tau = \mu \frac{du}{dy}$ for Newtonian fluids

$$dp/dx = \mu\, d\left(du/dy\right)/dy$$

Assume dp/dx is constant

Integrate $\quad \left(\frac{dp}{dx}\right) y = \mu \frac{du}{dy} + A$

Integrate $\quad \left(\frac{dp}{dx}\right) \frac{y^2}{2} = \mu u + Ay + B \quad$ —— (1)

Boundary conditions $\quad y = 0$ @ $u = 0 \quad \therefore B = 0$

$y = h \quad u = 0$ (upper solid surface)

$$\left(\frac{dp}{dx}\right)\frac{h^2}{2} = \mu(0) + Ah \qquad A = \left(\frac{dp}{dx}\right)\frac{h}{2}$$

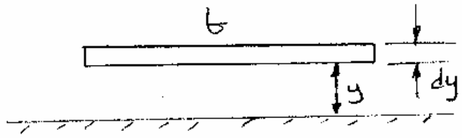
Substitute into (1)

$$\left(\frac{dp}{dx}\right)\frac{y^2}{2} = \mu u + \left(\frac{dp}{dx}\right)\frac{h}{2}y$$

Rearrange

$$u = \left(\frac{dp}{dx}\right)\frac{1}{2\mu}\left[y^2 - hy\right]$$

CONSIDER FLOW THROUGH A SMALL RECTANGULAR SLIT. (OUT OF PAGE)



$$dQ = u\, b\, dy = b\left(\frac{dP}{dx}\right)\frac{1}{2\mu}\left[y^2 - hy\right]dy$$

INTEGRATE

$$Q = b\left(\frac{dP}{dx}\right)\frac{1}{2\mu}\int_0^h (y^2 - hy)\, dy$$

$$Q = b\left(\frac{dP}{dx}\right)\frac{1}{2\mu}\left[y^3/3 - hy^2/2\right]_0^h$$

$$Q = b\left(\frac{dP}{dx}\right)\frac{1}{2\mu}\left[h^3/3 - h^3/2\right]$$

$$Q = -b\left(\frac{dP}{dx}\right)\frac{h^3}{12\mu}$$

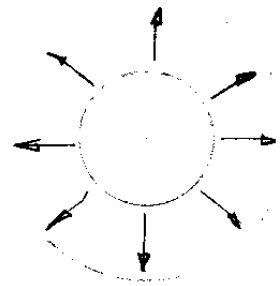
Mean velocity = Q/A $\quad A = bh$

$$u_m = \frac{-b\left(\frac{dP}{dx}\right)\frac{h^3}{12\mu}}{bh} = -\left(\frac{dP}{dx}\right)\frac{h^2}{12\mu}$$

PART B

$$Q = -b\left(\frac{dp}{dx}\right)\frac{h^3}{12\mu}$$

Flow between flanges is radial



b = circumference $= 2\pi r$

x = radius $= r$

h = GAP

$$Q - 2\pi r \frac{dp}{dr}\frac{h^3}{12\mu}$$

$$\frac{dr}{r} = -\frac{2\pi h^3}{12\mu Q}\, dp$$

Q is constant at all radii

$$\int_r^R \frac{dr}{r} = \frac{-2\pi h^3}{12\mu Q}\int_p^0 dp$$

$$\ln r/R = \frac{-2\pi h^3}{12\mu Q}\, p$$

$$Q = -\frac{2\pi h^3 p}{12\mu \ln r/R} = +\frac{2\pi h^3 p}{12\mu \ln(R/r)}$$

$$Q = \frac{2\pi h^3 p}{12\mu \ln(d_o/d_i)}$$