D204 Q5 2005

(a) A pipeline of diameter D = 0.5 m has a length L = 200 m, and the value of the Darcy friction λ may be assumed to have a constant value of 0.024. The pipeline contains two fully open valves, the local head loss at each of which is $0.2v^2/2g$, and three bends at each of which the head loss is $0.5v^2/2g$ where V is the velocity of water in the pipe. Calculate the value of K in the expression $h = K Q^2$ relating the total head loss h to the flow Q through the pipeline.

SOLUTION part (a)

Note $\lambda = 4C_f$

Straight pipe $h_f = \frac{4C_f L v^2}{2gD} = \frac{(0.024)(200)v^2}{2g(0.5)} = \frac{9.6v^2}{2g}$

Total for pipe line $h_f = \frac{9.6v^2}{2g} + \frac{2(0.2)v^2}{2g} + \frac{3(0.5)v^2}{2g} = \frac{11.5v^2}{2g}$

$v = \frac{Q}{A} = \frac{4Q}{\pi(.5)^2} = 5.093Q$

$h_f = \frac{11.5(5.093)Q^2}{2g} = 2.985Q^2$ hence $K = 2.985\ s^2/m^5$

(b) For a network of pipelines, such as that described in part (a), show that the flow correction term in an iterative head balance calculation is given by

$$\Delta Q = \frac{-\sum h_f}{2\sum(h_f/Q)}$$

SOLUTION part (b)

Starting with $h_f = K Q^n$ Normally n = 2 so $h_f = K Q^2$

Differentiate to get $dh_f = 2KQdQ = \frac{2KQ^2 dQ}{Q}$ and since $K Q^2 = h_f$

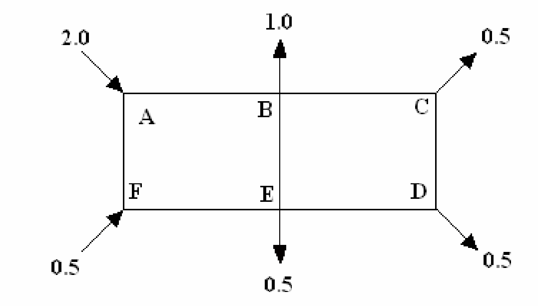
$$dh_f = \frac{2h_f dQ}{Q} \text{ or } dQ = \frac{Q dh_f}{2h_f}$$

If this relationship holds approximately true for finite changes then $\delta h_f = \frac{2h_f \delta Q}{Q}$ or $\delta Q = \frac{Q\, \delta h_f}{2h_f}$

In a balance of heads, the flow is corrected until $\Delta\theta = 0$ so the correction factor to be used for each pipe is $\delta Q = \frac{-Q\, \delta h_f}{2h_f} = \frac{-\delta h_f}{2\left(\frac{h_f}{Q}\right)}$ (The correction must be to reduce the flow rate).

For a network we must total all the terms to give the total correction factor of $\Delta Q = \frac{-\sum h_f}{2\sum(h_f/Q)}$

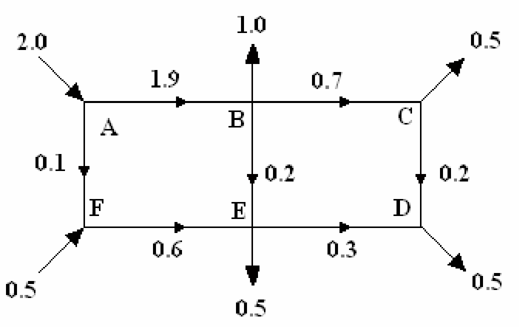
(c) The diagram shows two loops of a horizontal network with inflows and outflows in m^3/s. The K values of the seven pipes are given in the table. The pressure head at node A is 25 m. Calculate the flow rate through each pipe and the pressure head at each node. No more than two rounds of iteration are required, and final values of pressure heads may be rounded to the nearest metre.



Pipe	AB	BC	CD	DE	BE	EF	AF
K (s^2/m^5)	2	2	20	20	10	10	10

SOLUTION part (c)

The problem must be solved as two loops with a common pipe BE. First make a guess at the flow rates. Bear in mind that the net flow is zero at all nodes.



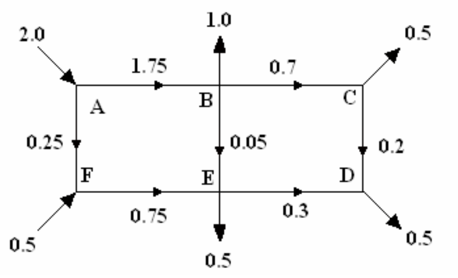
Data shown for initial guess

Start with loop ABEFA

PIPE	K	Q	h_f	h_f/Q
AB	2	1.9	7.22	3.8
BE	10	0.2	0.4	2
EF	10	-0.6	-3.6	6
FA	10	-0.1	-0.1	1
		Totals	3.92	12.8

$$\Delta Q = -\frac{\sum h_f}{2\sum h_f/Q} = -\frac{3.92}{2 \text{ x } 12.8} = -0.153$$

Correct all flows in this loop by adding -0.153



First loop correction

Now do loop BCDEB

PIPE	K	Q	h_f	h_f/Q
BC	2	0.7	0.98	1.4
CD	20	0.2	0.8	4
DE	20	-0.3	-1.8	6
BE	10	0.04688	0.021973	0.46875
		Totals	0.001973	11.86875

$$\Delta Q = -\frac{\sum h_f}{2\sum h_f/Q} = -\frac{0.002}{2 \times 11.869} = -0.000083$$

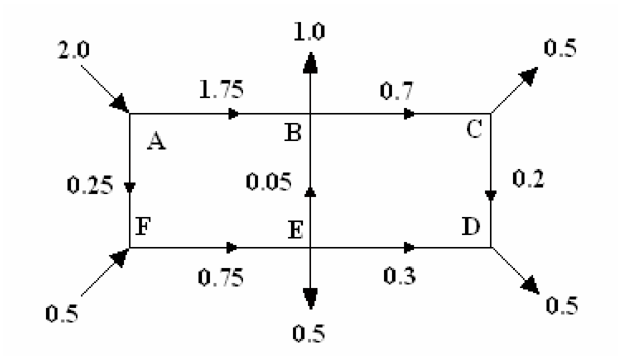
Correct loop 2 The initial guess was so good that the correction is minor

Second iteration of loop 1 is:

PIPE	K	Q	h_f	h_f/Q
AB	2	1.74688	6.10314	3.49375
BE	10	-0.0468	-0.02189	0.467919
EF	10	-0.7531	-5.67197	7.53125
FA	10	-0.2531	-0.64072	2.53125
			-0.23145	14.02417

$$\Delta Q = -\frac{\sum h_f}{2\sum h_f/Q} = -\frac{0.231}{2 \times 14.02} = -0.00825$$

Final solution is



Pipe	AB	BC	CD	DE	BE	EF	AF
Q	1.75	0.7	0.2	-0.3	±0.05	-0.75	-0.25

Head at A is 25 m and rounding off the h_f values
Head B is 25 - 6 = 19 m
Head at C is 19 – 1 = 18 m
Head at D = 18 - 1 = 17 m
Head at E = 17 + 2 = 19 m or 19 - 0 = 19 m
Head at F = 19 + 6 = 25 m or 25 -1 = 24 m error due to rounding off values.