# HYDROLOGY - TUTORIAL 2

## TRAPEZOIDAL CHANNELS

**I**n this tutorial you will

- Derive equations associated with flow in a trapezoidal channel.
- Derive equations for optimal dimensions.
- Solve slope of bed using Chezy and manning formulae.
- Solve questions from past papers.

This tutorial is a continuation of tutorial 1 which should be studied first.

## TRAPEZOIDAL SECTION

This topic occurs regularly in the Engineering Council Exam. The trapezoidal section is widely used in canals to accommodate the shape of boats and reduce the erosion of the sides.

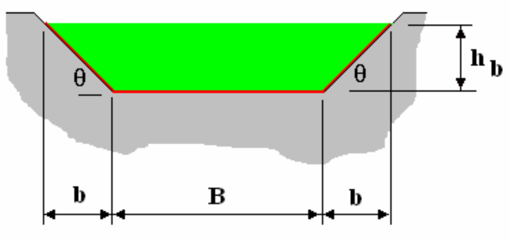


Figure 1

## BEST DIMENSION

The channel dimensions that give the maximum flow rate for a fixed cross sectional area is the one with the least amount of friction. This means that it must have the minimum wetted surface area and hence the minimum wetted perimeter P. If this value is then used in any formulae for the flow rate, we will have the maximum discharge possible. Using the notation shown on the diagram we proceed as follows.

Area $A = (B + b)\, h_b$ from which $B = (A/ h_b) - b = (A/ h_b) - h_b/\tan\theta$

Wetted Perimeter $P = B + 2h_b /\sin\theta$

Substitute for B $$P = \frac{A}{h_b} - \frac{h_b}{\tan\theta} + \frac{2h_b}{\sin\theta} = \frac{A}{h_b} + h_b\left(\frac{2}{\sin\theta} - \frac{1}{\tan\theta}\right)$$

For a given cross sectional area the minimum value of P occurs when $dp/dh_b = 0$

$$\frac{dP}{dh_b} = -\frac{A}{h_b^{\,2}} + \left(\frac{2}{\sin\theta} - \frac{1}{\tan\theta}\right)$$ Equate to zero and $A = h_b^{\,2}\left(\frac{2}{\sin\theta} - \frac{1}{\tan\theta}\right)$ and substitute for A

$$\left(B + \frac{h_b}{\tan\theta}\right)h_b = h_b^{\,2}\left(\frac{2}{\sin\theta} - \frac{1}{\tan\theta}\right) \qquad \left(B + \frac{h_b}{\tan\theta}\right) = h_b\left(\frac{2}{\sin\theta} - \frac{1}{\tan\theta}\right)$$

$$B = 2h_b\left(\frac{1}{\sin\theta} - \frac{1}{\tan\theta}\right) \text{ or } B = 2h_b K \text{ where } K = \left(\frac{1}{\sin\theta} - \frac{1}{\tan\theta}\right)$$

It can be shown that when this is the case, the bottom and sides are both tangents to a circle of radius $h_b$.

When $\theta = 90^o$ $K = 1$ and when $\theta = 45^o$ $K = \sqrt{2} - 1 = 0.414$ and in fact K is almost a linear function such that $K \approx \theta/90$

### WORKED EXAMPLE No.1

Calculate the dimensions of a trapezoidal channel with sides at $45^o$ if it must carry 2.5 $m^3/s$ of water with minimum friction given that C = 50 in the Chezy formula and the bed has a gradient of 1 in 1000

### SOLUTION

The Chezy formula is $u_o = C\,(R_hS)^{½}$ or $Q = A\,C\,(R_hS)^{½}$

$$B = 2h_b\left(\frac{1}{\sin 45} - \frac{1}{\tan 45}\right) = 0.828h_b \qquad b = h_b/\tan 45^o = h_b$$

$A = (B + b)\, h_b = (0.828\, h_b + h_b)\, h_b = 1.828\, h_b^2$

$$P = B + 2h_b\left(\frac{1}{\sin 45}\right) = 0.828h_b + 2.828h_b = 3.656h_b$$

$R_h = A/P = 0.5\, h_b$

$Q = 2.5 = 1.828\, h_b^2 \times 50(0.5\, h_b/1000)^{1/2}$

$0.000748 = h_b^4\,(0.5\, h_b/1000)$

$0.748 = 0.5\, h_b^5$

$h_b = 1.084$ m $\qquad$ $B = 0.828\, h_b = 0.897$ m

**SELF ASSESSMENT EXERCISE No.1**

1. Calculate the dimensions of a trapezoidal channel with sides at $60^o$ to the horizontal if it must carry 4 $m^3/s$ of water with minimum friction given that C = 55 in the Chezy formula and the bed has a gradient of 1 in 1200.
   ($h_b$ = 1.334 m   B = 1.541 m)

2. Calculate the dimensions of a trapezoidal channel with sides at $30^o$ to the horizontal if it must carry 2 $m^3/s$ of water with minimum friction given that C = 49 in the Chezy formula and the bed has a gradient of 1 in 2000.
   ($h_b$ = 1.053 m   B = 0.564 m)

**CRITICAL DEPTH**

It requires a lot of Algebra to get to the critical values. Start as before $h_s = h_b + u_o^2/2g$

Rearrange to make u the subject $u_o^2 = \{2g(h_s - h_b)\}$

$Q = Au_o$ $\qquad Q^2 = A^2u_o^2$

$A = (B + b)h_b$ $\qquad Q^2 = (B + b)^2h_b^2\, u_o^2$

Substitute for $u_o$ $\qquad \dfrac{Q^2}{2g} = (h_s - h_b)(B + b)^2 h_b{}^2$

We cannot differentiate this expression because b is a function of h so we make a substitution first.

$$b = h_b/\tan\theta$$

$$\frac{Q^2}{2g} = (h_s - h_b)\left(B + \frac{h_b}{\tan\theta}\right)^2 h_b{}^2 = (h_s - h_b)\left(Bh_b + \frac{h_b^2}{\tan\theta}\right)^2$$ Now we need to multiply out.

$$\frac{Q^2}{2g} = (h_s - h_b)\left(B^2h_b^2 + \frac{h_b^4}{\tan^2\theta} + \frac{2Bh_b^3}{\tan\theta}\right) = \left(B^2h_b^2h_s + \frac{h_b^4h_s}{\tan^2\theta} + \frac{2Bh_b^3h_s}{\tan\theta}\right) - \left(B^2h_b^3 + \frac{h_b^5}{\tan^2\theta} + \frac{2Bh_b^4}{\tan\theta}\right)$$

Now differentiate with respect to $h_b$ to find the maximum flow rate for a given specific energy head.

$$\frac{2QdQ}{2gdh_b} = 2B^2h_bh_s + \frac{4h_b^3h_s}{\tan^2\theta} + \frac{6Bh_b^2h_s}{\tan\theta} - 3B^2h_b^2 - \frac{5h_b^4}{\tan^2\theta} - \frac{8Bh_b^3}{\tan\theta}$$

For maximum Flow rate equate $dQ/dh_b$ to zero.

$$0 = 2B^2h_bh_s + \frac{4h_b^3h_s}{\tan^2\theta} + \frac{6Bh_b^2h_s}{\tan\theta} - 3B^2h_b^2 - \frac{5h_b^4}{\tan^2\theta} - \frac{8Bh_b^3}{\tan\theta}$$

We can simplify by substituting back $h_b/\tan\theta = b$

$$0 = 2B^2h_bh_s + 4b^2h_bh_s + 6Bbh_bh_s - 3B^2h_b^2 - 5b^2h_b^2 - 8Bbh_b^2$$

$$0 = h_b\left(2B^2h_s + 4b^2hh_s + 6Bbh_s\right) - h_b^2\left(3B^2 + 5b^2 + 8Bb\right)$$

$$0 = \left(2B^2h_s + 4b^2hh_s + 6Bbh_s\right) - h_b\left(3B^2 + 5b^2 + 8Bb\right)$$

Rearrange to get the critical depth $\quad h_b = h_c = \dfrac{(2B^2 + 4b^2 + 6Bb)}{(3B^2 + 5b^2 + 8Bb)}h_s = Ch_s$

$$C = \frac{(2B^2 + 4b^2 + 6Bb)}{(3B^2 + 5b^2 + 8Bb)} = \frac{(2B + 4b)(B + b)}{(3B + 5b)(B + b)} = \frac{(2B + 4b)}{(3B + 5b)}$$

$$h_c = \frac{(2B+4b)}{(3B+5b)}h_s \text{ or } h_s = \frac{(3B+5b)}{(2B+4b)}h_c$$

If $B = 0$ we have a Vee section $h_b = h_c = \frac{4h_s}{5}$ as before.

If $b = 0$ we have a rectangular section $h_b = h_c = \frac{2h_s}{3}$ as before.

There are computer programs for making the calculations such as the one at
http://www.lmnoeng.com/Channels/trapezoid.htm

To find the critical velocity flow rate substitute $h_s = \frac{(3B+5b)}{(2B+4b)}h_c$ into $u_o^2 = u_c^2 = \{2g(h_s - h_c)\}$

$$u_o^2 = u_c^2 = \left\{2g\left(\frac{(3B+5b)}{(2B+4b)}h_c - h_c\right)\right\} = \left\{2gh_c\left(\frac{(3B+5b)}{(2B+4b)} - 1\right)\right\}$$

$$u_c = \sqrt{\left\{2gh_c\left(\frac{(3B+5b)}{(2B+4b)} - 1\right)\right\}} = \sqrt{\left\{2gh_c\left(\frac{(3B+5b)-(2B+4b)}{(2B+4b)}\right)\right\}}$$

$$u_c = \sqrt{\left\{2gh_c\left(\frac{B+b}{(2B+4b)}\right)\right\}}$$

If $B = 0$ we have a Vee section $u_c = \sqrt{\left\{\frac{gh_c}{2}\right\}}$ as before.

If $b = 0$ we have a rectangular section we have $u_c = \sqrt{\{gh_c\}}$ as before.

To find the critical flow rate substitute use $Q_c = A\,u_c$ $\quad A = (B + b)h_c$

$$Q_c = (B+b)h_c\sqrt{\left\{2gh_c\left(\frac{B+b}{(2B+4b)}\right)\right\}} \qquad Q_c = (B+b)h_c^{3/2}\sqrt{\left\{2g\left(\frac{B+b}{(2B+4b)}\right)\right\}}$$

If $B = 0$ we have a Vee section $Q_c = bh_c^{3/2}\sqrt{\left\{\frac{g}{2}\right\}}$ as before in a slightly different form

If $b = 0$ we have a rectangular section we have $Q_c = Bh_c^{3/2}\sqrt{g}$ as before.

**Summary for trapezoidal section**

The critical depth is $\qquad h_c = \frac{(2B+4b)}{(3B+5b)}h_s$

The critical velocity is $\qquad u_c = \sqrt{\left\{2gh_c\left(\frac{B+b}{(2B+4b)}\right)\right\}}$

The critical flow is $\qquad Q_c = (B+b)h_c^{3/2}\sqrt{\left\{2g\left(\frac{B+b}{(2B+4b)}\right)\right\}}$

The major problem exists that solving with these formulae requires a value for b and this depends on the answer itself.

## WORKED EXAMPLE No. 2

A canal has a trapezoidal section with a base 5 m wide and sides inclined at 50° to the horizontal. It is required to have a depth of 2 m, what would the flow rate be if the specific energy head is a minimum? Calculate the depth, flow rate and mean velocity for this condition. What is the Froude Number?

## SOLUTION

For minimum specific energy, the flow and depth must be critical so $h_c = 2$ m.
$b = 2/\tan 50^\circ = 1.678$   $B = 5$

$$Q_c = (B+b)h_c^{3/2}\sqrt{\left\{2g\left(\frac{B+b}{(2B+4b)}\right)\right\}} = (6.678)2^{3/2}\sqrt{\left\{2g\left(\frac{6.678}{16.713}\right)\right\}} = 52.89\ \text{m}^3/\text{s}$$

$A = (B + b)h_c = 6.678 \times 2 = 13.356\ \text{m}^2$   $u_c = Q_c/A = 3.96$ m   $Fr = u_c/\sqrt{(gh_c)} = 0.89$

## WORKED EXAMPLE No. 3

A channel has a trapezoidal section with a base 0.5 m wide and sides inclined at 45° to the horizontal. It must carry 0.3 m³/s of water at the critical depth. Calculate the depth and mean velocity.

## SOLUTION

There is no simple way to solve this problem because of the complexity of the formula.

$$Q_c = (B+b)h_c^{3/2}\sqrt{\left\{2g\left(\frac{B+b}{(2B+4b)}\right)\right\}} \quad \text{where } b = h_c/\tan\theta$$

Evaluate and plot $Q_c$ for various values of $h_c$ and we get the following graphs.

From the graph we see that when $Q_c = 0.3\ \text{m}^3/\text{s}$, $h_c = 0.27$
$A = (0.5 + 0.275)(0.275) = 0.213\ \text{m}^2$   $u_c = Q_c/A = 1.$

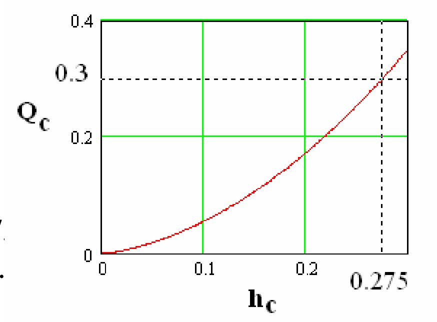

Figure 2

## SELF ASSESSMENT EXERCISE No.2

1. A channel has a trapezoidal section with a base 2 m wide and sides inclined at 60° to the horizontal. It must carry 0.4 m³/s of water with the minimum specific energy head. Calculate the depth and mean velocity for this condition.
(0.157 m and 1.22 m/s)

2. A canal has a trapezoidal section with a base 4 m wide and sides inclined at 40° to the horizontal. It is required to have a depth of 1.5 m, what would the flow rate be if the specific energy head is a minimum? Calculate the flow rate and mean velocity for this condition.
(29.1 m³/s and 3.353 m/s)

**WORKED EXAMPLE No. 4**

An open channel has a trapezoidal cross section with sides inclined at 45° to the vertical. The channel must carry 21 $m^3/s$ with a velocity of 3 m/s with minimum friction. Determine the smallest slope of the bed for these conditions and the corresponding depth and dimensions of the channel. The constant n in the Manning formula is 0.012. Show that this is a sub critical flow.

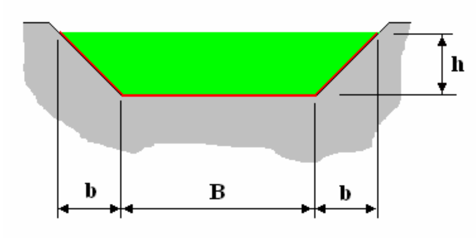


Figure 3

**SOLUTION**

$Q = 21\ m^3/s$ $\qquad u_o = 3\ m/s$ $\qquad A = Q/u = 7\ m^2$

For minimum friction the optimal value of B is $B = 2h_b\left(\frac{1}{\sin\theta} - \frac{1}{\tan\theta}\right)$

$$B = 2h_b\left(\frac{1}{\sin 45} - \frac{1}{\tan 45}\right) = 0.8284h_b \quad b = h_b$$

$A = (B + b)h_b \qquad 7 = (0.8284h_b + h_b)h_b = 1.\ 8284h_b^2$

$h_b = \sqrt{}(7/1.\ 8284) = 1.957$ m

B = 1.621 m $\qquad$ b = 1.957 m

P = B + 2b/sin 45 = 1.621 + 2 x 1.957/sin 45 = 7.155

A = 7

$R_h = 7/7.155 = 0.978$ m (Note that for 45° $R_h = 0.5\ h_b$)

Manning formula $\qquad u = \frac{R^{2/3}S^{1/2}}{n} \qquad 3 = \frac{0.978^{2/3}S^{1/2}}{0.012}$

S = 0.001333

The specific energy head is $h_s = 1.957 + 3^2/2g = 2.416$ m

The critical depth is $\qquad h_c = \frac{(2B + 4b)}{(3B + 5b)}h_s = \frac{2 \text{ x } 1.621 + 4 \text{ x } 1.957}{3 \text{ x } 1.621 + 5 \text{ x } 1.957} = \frac{11.07}{14.648} = 0.756\ m$

Since the actual depth is larger the flow is sub critical.

**SELF ASSESSMENT EXERCISE No. 3**

These are exam standard questions.

1. An open channel has a trapezoidal section with sides inclined at $45^o$ to the vertical. The channel must carry 20 $m^3/s$ of water with a mean velocity of 2.5 m/s. Determine the smallest slope of the bed possible and the corresponding depth and dimensions of the channel. The constant n in the Manning formula is 0.012. Show that this is a sub critical flow.

   $u = (R^{2/3}S^{1/2})/n$

   (Answer S = 0.000845, h = 2.1, B = 2.1 m and b = 2.1 m.)

2. A channel has a trapezoidal section 5 m wide at the bottom. The sides slope at 1 metre up for each 2 horizontal. The bed has a slope of 1/3600 and n in the manning formula is 0.024.

   Calculate the flow rates corresponding to mean velocities of 0.3 and 0.6 m/s.

   (Ans. 0.549 $m^3/s$ and 4.81 $m^3/s$)