HYDROLOGY - TUTORIAL 1

UNIFORM FLOW IN CHANNELS

In this tutorial you will

- Derive formula for flow through notches.
- Solve problems involving flow through notches.
- Define uniform channel flow.
- Derive formulae relating channel dimensions and flow rate.
- Define the Froude Number.
- Define sub-critical and super critical flow.

The student is advised to study Tutorial 1 from the Fluid mechanics D203 section before starting this tutorial.

1. FLOW THROUGH NOTCHES

A notch is placed in a channel to measure the flow by restricting it. The flow rate is related to the depth of water behind the notch and a calibrated depth gauge is all that is needed to indicate the flow rate.

RECTANGULAR NOTCH

The velocity of water due to a pressure head only is $u = \sqrt{2gh}$. This assumes there is negligible velocity approaching the notch.

The flow through the elementary strip is
$dQ = u\,B\,dh$

$$Q = B\int_0^H u\,dh = B\sqrt{2g}\int_0^H h^{1/2}dh = \frac{2B}{3}\sqrt{2g}H^{3/2}$$

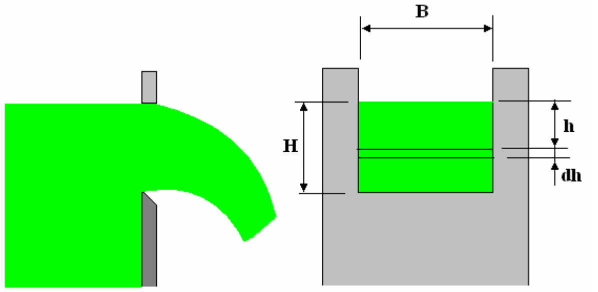


Figure 1

Where the flow approaches the edge of a notch, there is a contraction because the velocity at the edge is not normal to the plane of the notch. This produces a reduction in the cross section of flow and some friction in the flow. Depending on the design of the edges a coefficient of discharge C_d is needed to correct the formula.

$$Q = C_d \frac{2B}{3}\sqrt{2g}\,H^{3/2}$$

Further study will yield formula for C_d based on the various shapes of the edges.

SUBMERGED RECTANGULAR NOTCH and SLUICE GATE

If the notch is a rectangular hole, the integration must be between the two depths H_1 and H_2 yielding

$$Q = C_d \frac{2B}{3}\sqrt{2g}\left(H_2^{3/2} - H_1^{3/2}\right)$$

If the bottom of the notch is the floor of the downstream channel, we have a sluice gate and the same formula applies.

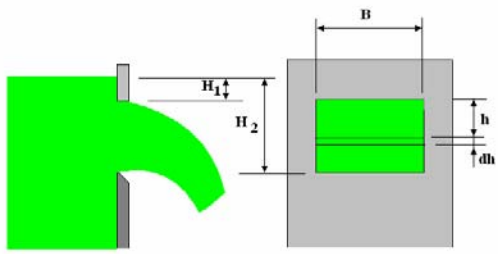


Figure 2

VEE NOTCH

The width of the elementary strip varies depth such that

$b = 2(H - h)\tan(\theta/2)$

$$Q = \int_0^H u\,b\,dh = 2\sqrt{2g}\tan\left(\frac{\theta}{2}\right)\int_0^H (H-h)h^{1/2}dh$$

$$Q = 2\sqrt{2g}\tan\left(\frac{\theta}{2}\right)\int_0^H \left(Hh^{1/2} - h^{3/2}\right)dh$$

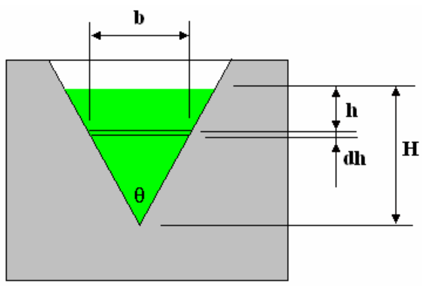


Figure 3

$$Q = 2\sqrt{2g}\tan\left(\frac{\theta}{2}\right)\left[\frac{2}{3}H^{5/2} - \frac{2}{5}H^{5/2}\right] = 2\sqrt{2g}\tan\left(\frac{\theta}{2}\right)\left[\frac{4}{15}H^{5/2}\right]$$ and introducing C_d we have

$$Q = C_d \frac{8}{15}\sqrt{2g}\tan\left(\frac{\theta}{2}\right)H^{5/2}$$

VELOCITY OF APPROACH

If the velocity approaching the notch is not negligible say u_1 then the velocity through the elementary strip is $u = \sqrt{\left(u_1^2 + 2gh\right)}$. If a notch is fitted into a channel not much bigger than the notch, the velocity of the water approaching the notch is not negligible and a correction needs to be made.

WORKED EXAMPLE No.1

The depth of water above the sill of a rectangular notch is 0.23 m and the notch is 0.5 m wide. The coefficient of discharge is 0.6. Calculate the flow rate of water.

SOLUTION

$$Q = C_d \frac{2B}{3}\sqrt{2g}H^{3/2} = 0.6\left(\frac{2 \text{ x } 0.5}{3}\right)\sqrt{2g}\, 0.25^{3/2} = 0.1107 \text{ m}^3/\text{s}$$

WORKED EXAMPLE No.2

The depth of water above the sill of a vee notch is 0.4 m and has an included angle of 90^o. The coefficient of discharge is 0.65. Calculate the flow rate of water.

SOLUTION

$$Q = C_d \frac{8}{15}\sqrt{2g}\tan\left(\frac{\theta}{2}\right)H^{5/2} = 0.65 \text{ x } \frac{8}{15}\sqrt{2g}\tan 45^o \text{ x } 0.4^{5/2} = 0.155 \text{ m}^3/\text{s}$$

WORKED EXAMPLE No.3

The depth of water behind a sluice gate in a horizontal rectangular channel is 3 m and the sluice is 0.8 m high. The coefficient of discharge is 0.75. Calculate the flow rate of water in the channel downstream.

SOLUTION

$$Q = C_d \frac{2B}{3}\sqrt{2g}\left(H_2^{3/2} - H_2^{3/2}\right) = 0.75\frac{2 \text{ x } 3}{3}\sqrt{2g}\left(3^{3/2} - 2.2^{3/2}\right) = 12.84 \text{ m}^3$$

SELF ASSESSMENT EXERCISE No.1

1 The depth of water above the sill of a rectangular notch is 0.4 m and the notch is 0.75 m wide. The coefficient of discharge is 0.62. Calculate the flow rate of water. (0.347 m^3/s)

2 The depth of water above the sill of a Vee notch is 0.2 m and has an included angle of 60^o. The coefficient of discharge is 0.6. Calculate the flow rate of water. (0.228 m^3/s)

3. A sluice controls the flow in a rectangular channel 2.5 m wide. The depth behind the sluice is 2 m and the sluice is 0.5 m high. What is the discharge? Take C_d =0.8. (5.85 m^3/s)

2. UNIFORM FLOW IN CHANNEL

Channel flow is characterised by constant pressure (usually atmosphere) at all points on the surface. This means that flow can only be induced by gravity so the bed of the channel must slope downwards. There is no pressure gradient in the fluid pushing it along.

If the cross section is uniform and the depth is uniform then the flow rate is uniform at all points along the length. This can only occur if the change of potential height is balanced by the friction losses.
This is UNIFORM FLOW.

DEFINITIONS
Flow rate= Q (m^3/s)   Flow rate per unit width q m^2/s
Cross sectional area = A (m^2)
Wetted perimeter = P (m)
Mean velocity = u_o = Q/A (m/s)
Slope of bed = S which is otherwise called the energy gradient.

The hydraulic gradient is i and this is the friction head loss per unit length of the bed.
The hydraulic gradient is the same as the slope if the flow has a constant depth (uniform flow).

The hydraulic radius is defined as $R_h = A/P$ and this is also often called the hydraulic mean depth with symbol m.

The wetted area is $A_w = PL$

τ_w is the wall shear stress. This is the force per unit surface area resisting flow at the surface of contact between the fluid and the wall.

CHEZY FORMULA

Consider part of a flow of regular cross section A and length L.

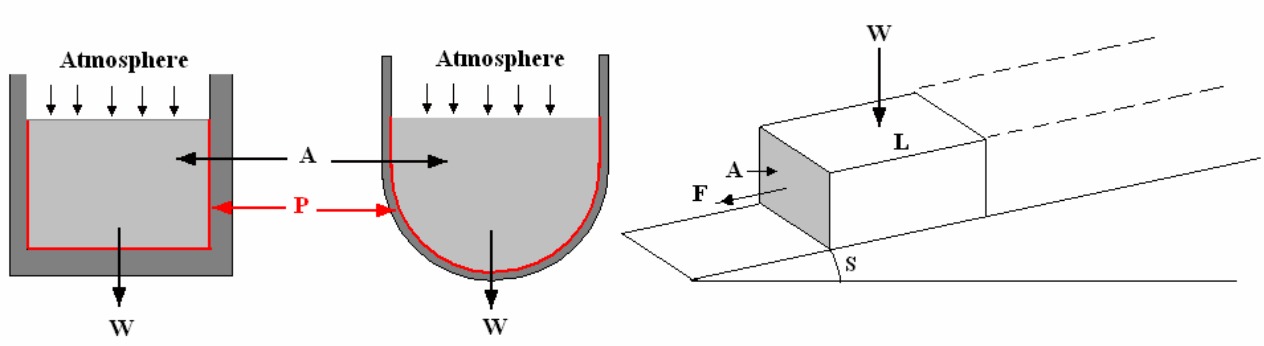


Figure 4

If the slope is small the weight of the section considered is $W = \rho gAL$
Resolving the weight parallel to the bed the force causing flow is $F = W \sin(S)$
If S is small sin S = S radians so $F = W S = \rho gALS$
If the flow is steady there is no inertia involved so the force resisting motion must be equal to this force.

The resisting force per unit surface area $= F/A_w = \tau_w = F/PL = \rho gALS/PL = \rho gAS/P = \rho g R_h S$

Chezy thought that $\tau_w \propto u_o^2$ and so $\tau_w = C_1 u_o^2$ Hence $C_1 u_o^2 = \rho g R_h S$

The Chezy formula is $u_o = \mathbf{C} (\mathbf{R_h S})^{1/2}$

$C = (\rho g/C_1)^{1/2}$ and C is the Chezy constant.

WORKED EXAMPLE No. 4

An open channel has a rectangular section 2 m wide. The flow rate is 0.05 m^3/s and the depth is 0.4 m. Calculate the slope of the channel using the Chezy formula for steady flow. Take the constant C = 50 $m^{½}$ /s

SOLUTION

$A = 2 \times 0.4 = 0.8\ m^2$
$P = 2 + 0.4 + 0.4 = 2.8$ m
$R_h = A/P = 0.2857$ m
$u_o = Q/A = 0.05/0.8 = 0.0625$ m/s
$u_o = 0.0625 = C (R_h S)^{½}$
$u_o = 0.0625 = 50 (0.2857 S)^{½}$
$S = 5.469 \times 10^{-6}$

SELF ASSESSMENT EXERCISE No.2

1. An open channel has a triangular section with sides at 45^o to the vertical. The flow rate is 0.0425 m^3/s and the depth is 0.225 m. Calculate the slope of the channel using the Chezy formula for steady flow. Take the constant C = 49 $m^{½}$ /s
(Answer 0.00369)

2. A channel with a section as shown carries 1.1 m^3/s of water with the depth as shown. The slope of the bed is 1/2000. Calculate the constant C in the Chezy formula.

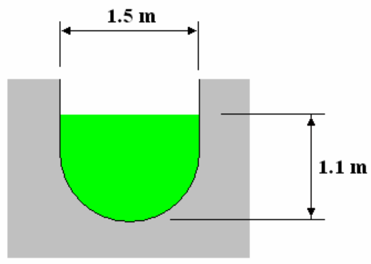


Figure 5

(Answer 51.44)

THE CHEZY - MANNING FORMULA

Manning extended Chezy's formula. Based on research he stated that $C = \frac{R_h^{1/6}}{n}$

n is a dimensionless constant based on the surface roughness of the channel. Substituting this into the Chezy formula yields

$$u_o = \frac{R_h^{2/3} S^{1/2}}{n} \text{ This is the Chezy - Manning formula.}$$

WORKED EXAMPLE No. 5

An open channel has a rectangular section 5 m wide. The flow rate is 1.2 m^3/s and the depth is 1.4 m. Calculate the slope of the channel using the Manning formula for steady flow. Take the constant n = 0.019 $m^{½}/s$

SOLUTION

$A = 5 \times 1.4 = 7\ m^2$
$P = 5 + 1.4 + 1.4 = 7.8$ m
$R_h = 7/7.8 = 0.897$ m
$u_o = Q/A = 0.171$ m/s

$$u_o = \frac{R_h^{2/3} S^{1/2}}{n} \text{ rearrange } \quad S = \left(\frac{nu_o}{R_h^{2/3}}\right)^2 = \left(\frac{0.019 \times 0.171}{0.897^{2/3}}\right)^2 = 12.256 \times 10^{-6}$$

SELF ASSESSMENT EXERCISE No.3

1. A rectangular channel is 2 m wide and runs 1.5 m deep. The slope of the bed is 1/4000. Using the Manning formula with n = 0.022, calculate the flow rate.
(Answer 1.534 m^3/s)

2. An open channel has a rectangular section 3 m wide. The flow rate is 1.4 m^3/s and the depth is 0.8 m. Calculate the slope of the channel using the Manning formula for steady flow. Take the constant n = 0.02 $m^{½}/s$
(Answer 292.5 x 10^{-6})

3. Water flows down a half full circular pipeline of diameter 1.4m. The pipeline is laid at a gradient if 1/250. If the constant n in the Manning formula is n = 0.015 what is the discharge.
(1.612 m^3/s)

DARCY FORMULA APPLIED TO CHANNELS

The Chezy formula may be related to the Darcy formula for flow in round pipes.

The Darcy formula (not derived here) is $h_f = \dfrac{4C_f L u_o^2}{2gd}$

Sometimes this is stated as $h_f = \dfrac{f\,L u_o^2}{2gd}$ where $4C_f = f$

h_f is the friction head and C_f is the friction coefficient which is related to the Reynolds's number and the relative surface roughness.

If a round pipe runs full but with constant pressure along the length, then the Chezy and Darcy formulae may be equated.

From the Darcy formula we have $u_o^2 = \dfrac{2gdh_f}{4C_f L}$

For constant pressure, $h_f/L = S$ $u_o^2 = \dfrac{2gdS}{4C_f}$

From the Chezy formula we have $u_o^2 = C^2 R_h S$

For a round pipe diameter d running full $R_h = d/4$ $u_o^2 = C^2Sd/4$

Equating we have $\dfrac{C^2Sd}{4} = \dfrac{2gdS}{4C_f}$

$$C_f = \frac{2g}{C^2}$$

From the Chezy equation we have $u_o^2 = \dfrac{C^2Rh_f}{L}$

$$h_f = \frac{Lu_o^2}{C^2R_h} = \frac{C_f L u_o^2}{2gR_h}$$

This version of the Darcy formula may be used for pipes and channels of any shape with no pressure gradient. Discussion of the Darcy formula show that C_f is related to the surface roughness and this compares with Manning's work.

In the case of LAMINAR FLOW Poiseuille's equation is also relevant and this gives the friction head as

$$h_f = \frac{32\,\mu\,L\,u_o}{\rho g d^2}$$

Equating this to the Darcy formula gives:

$$\frac{32\,\mu\,L\,u_o}{\rho g d^2} = \frac{4C_f L u_o^2}{2gd} \text{ hence } C_f = \frac{16\mu}{\rho u_o d} = \frac{16}{R_e}$$

The complete relationship between the Reynolds' number R_e and the relative surface roughness is given on the Moody Chart. The chart has several regions, laminar flow, turbulent flow and a region between where it is in transition. The turbulent flow varies between smooth surfaces and fully rough surfaces that produce fully developed turbulent flow. ***Relative surface roughness*** is defined as $\varepsilon = k/D$ where k is the mean surface roughness and D the bore diameter. The chart is a plot of C_f vertically against R_e horizontally for various values of ε. In order to use this chart you must know two of the three co-ordinates in order to pick out the point on the chart and hence pick out the unknown third co-ordinate.

For the laminar region $C_f = \dfrac{16}{R_e}$

For smooth pipes, (the bottom curve on the diagram), various formulae have been derived such as those by Blasius and Lee.

BLASIUS $\quad C_f = 0.0791\ R_e^{0.25}$

LEE $\quad C_f = 0.0018 + 0.152\ R_e^{0.35}$.

The Moody diagram shows that the friction coefficient reduces with Reynolds number but at a certain point, it becomes constant. When this point is reached, the flow is said to be fully developed turbulent flow. This point occurs at lower Reynolds numbers for rough pipes.

A formula that gives an approximate answer for any surface roughness is that given by Haaland.

$$\frac{1}{\sqrt{C_f}} = -3.6\log_{10}\left\{\frac{6.9}{R_e} + \left(\frac{\varepsilon}{3.71}\right)^{1.11}\right\}$$

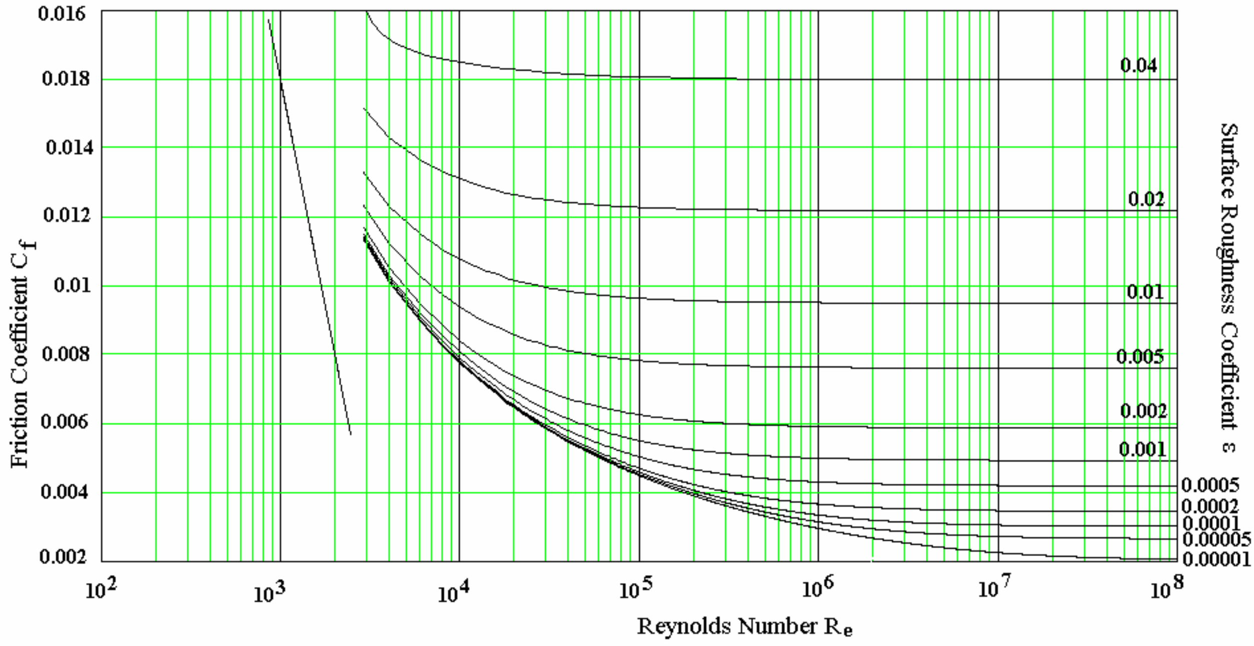


Figure 6

SELF ASSESSMENT EXERCISE No. 4

1. The Darcy - Weisbach formula for a round pipe running full states that $h_f = 4\ C_f L u^2/2gd$ where L is the length, d the diameter and u the mean velocity.

a. Show that for laminar flow $C_f = 16/R_e$

b. Relate the Chezy formula $u = C\ (RS)^{1/2}$ and the Manning formula $u = (R^{2/3}\ S^{1/2})/n$ to the Darcy - Weisbach formula and list the ranges of applicability of all three formula.

c. Sketch the relationship between C_f and R_e for the range $R_e = 10^0$ to $R_e = 10^6$ in a pipe of circular cross section for typical values of surface roughness k.

d. If ageing causes the surface roughness of a pipe to increase, what affect would this have on the flow carrying capacity of the pipe?

3. CRITICAL FLOW

SPECIFIC ENERGY HEAD - h_s

At any point in the length of the channel the fluid has three forms of energy relative to the bed, kinetic, gravitational (potential) and flow (pressure) energy.

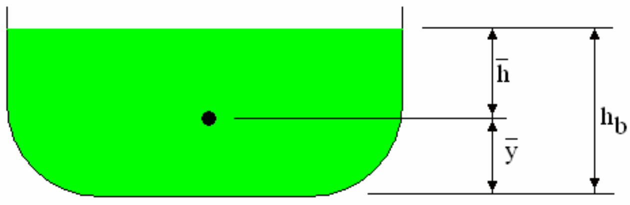


Figure 7

Strictly, all energy terms should be the mean values. The mean depth is $\bar{h}$ and the mean gravitational (potential) head is $\bar{y}$ (the distance to the centroid). The depth at the bottom is $h_b = \bar{h} + \bar{y}$ and the mean velocity is u_o

From the Bernoulli Equation $h_s = h + \bar{y} + \frac{u_o^2}{2g} = h_b + \frac{u_o^2}{2g}$

Text books jump straight to this formula wrongly giving h_b as the pressure head.

Rearrange the formula and $u_o = \{2g(h_s - h_b)\}^{1/2}$

Consider a channel with an unspecified cross section of area A.

$Q = Au_o$ $\qquad Q = A\{2g(h_s - h_b)\}^{1/2}$

CRITICAL DEPTH – h_C

It will be shown that for a given value of h_s there is a depth h_c that produces maximum flow rate but the value of h_c depends on the shape of the channel since the width is a function of depth and hence the area is a function of depth. Let's examine a rectangular cross section.

RECTANGULAR SECTION

$$Q = A\{2g(h_s - h_b)\}^{1/2} = Bh_b\{2g(h_s - h_b)\}^{1/2}$$

$$Q = B\sqrt{2g}\left\{\left(h_b{}^2 h_s - h_b{}^3\right)\right\}^{1/2}$$

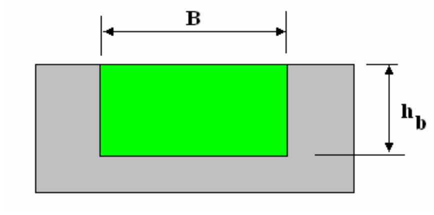
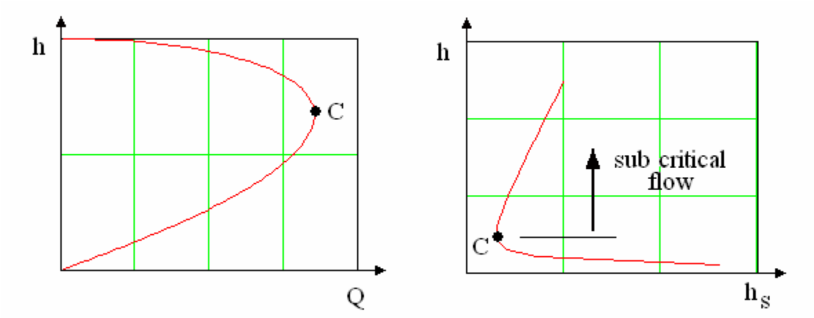


Figure 8

If we plot h – Q for a given value of B and h_s we get figure 9a and if we plot h – h_s for a given value of B and Q we get figure 9b.



Figures 9a Figure 9 b

The plots reveal some interesting things. Point C is called the critical point and this gives the minimum energy head for a given flow rate or a maximum flow rate for a given energy head.

For a flow rate other than the critical value, there are two possible depths of flow. This is logical since for a given amount of energy the flow can be slow and deep or fast and shallow. Flow at the shallow depth is super-critical and flow at the larger depth is sub-critical. The critical depth is denoted h_c.

To find the critical depth we use max and min theory. At point C $dQ/dh_b = 0$
Differentiate and we get:

$$\frac{dQ}{dh_b} = \left\{(2g)^{1/2}\left(h_s - \frac{3h_b}{2}\right)\right\}^{1/2} \frac{B}{(h_s - h_b)^{1/2}} \qquad 0 = \left(h_s - \frac{3h_b}{2}\right) \qquad h_b = \frac{2h_s}{3} = h_c$$

Since $u_o = \{2g(h_s - h_b)\}^{1/2}$ then substituting for h_s will produce the critical velocity.

$$u_c = \sqrt{2g\left(\frac{3h_c}{2} - h_c\right)} = \sqrt{2g\left(\frac{h_c}{2}\right)} = \sqrt{gh_c}$$

It follows that the critical flow rate is $Q_c = Au_c = B\sqrt{g}\, h_c^{3/2}$

Here is an alternative derivation for the rectangular channel.

$A = B\, h_b \qquad u_o = Q/(A) = Q/(B\, h_b)$

$$h_s = h_b + \frac{Q^2}{2g(B^2 h_b^2)}$$ For a given flow rate the minimum value of h_s is found by differentiating.

$$\frac{dh_s}{h_b} = 1 - \frac{2Q^2}{2g(B^2 h_b^3)} = 1 - \frac{Q^2}{g(B^2 h_b^3)}$$ For a minimum value equate to zero.

$$0 = 1 - \frac{Q^2}{g(B^2 h_b^3)} \qquad Q = \sqrt{g(B^2 h_b^3)} \text{ or } h_b = \left(\frac{Q^2}{gB^2}\right)^{1/3}$$

These are the critical values so it follows that $Q_c = B\sqrt{g}\, h_c^{3/2}$ or $h_c = \left(\frac{Q_c^2}{gB^2}\right)^{1/3}$

$$u_c = \frac{Q_c}{Bh_c} = \frac{B\sqrt{g}h_c^{3/2}}{Bh_c} = \sqrt{gh_c} \text{ or } h_c = \frac{u_c^2}{g}$$

$$h_s = h_b + \frac{u_o^2}{2g} = h_c + \frac{u_c^2}{2g} = \frac{u_c^2}{g} + \frac{u_c^2}{2g} = \frac{3u_c^2}{2}$$

$$h_s = h_c + \frac{u_c^2}{2g} = h_c + \frac{gh_c}{2g} = \frac{3}{2}h_c \qquad h_c = \frac{2}{3}h_s$$

The critical flow in terms of h_s is $Q_c = B\sqrt{g}\, h_c^{3/2} = B\sqrt{g}\left(\frac{2}{3}h_s\right)^{3/2} = B\sqrt{g\left(\frac{8}{27}\right)}h_s^{3/2}$

The critical velocity in terms of h_s is $u_c = \sqrt{gh_c} = \sqrt{\frac{2gh_s}{3}}$

FROUDE NUMBER

You may have studied this in dimensional analysis. The Froude Number is a dimensionless number important to channel flow as well as to surface waves. It is defined as :

$F_r = \frac{u}{\sqrt{gh}}$ For critical flow $F_r = \frac{u_c}{\sqrt{gh_c}}$ Substitute $u_c = \sqrt{gh_c}$ into this and $F_r = \frac{\sqrt{gh_c}}{\sqrt{gh_c}} = 1$

The Froude number is always 1 when the flow is critical in a RECTANGULAR CHANNEL but not for other shapes. Another name for super-critical flow is SHOOTING or RAPID FLOW and sub–critical is called TRANQUIL FLOW.

Summary for a rectangular channel

The critical depth is $h_c = \frac{2}{3}h_s$ The critical velocity is $u_c = \sqrt{gh_c} = \sqrt{\frac{2gh_s}{3}}$

The critical flow is $Q_c = B\sqrt{g}\, h_c^{3/2} = B\sqrt{g\left(\frac{8}{27}\right)}\, h_s^{3/2}$ Froude Number $F_r = 1$

WORKED EXAMPLE No. 6

A rectangular channel 1.6 m wide must carry water at depth of 1 m. What would be the maximum possible flow rate and what would be the mean velocity?

SOLUTION

For maximum flow rate the depth must be the critical depth so $h_c = 1$m.
The critical velocity is $u_c = (g\ h_c)^{1/2} = (9.81 \times 1)^{1/2} = 3.132$ m/s
The critical flow is $Q_c = A\ u_c = 1.6 \times 1 \times 3.132 = 5.01\ m^3/s$

Check the Froude number $F_r = \frac{u_c}{\sqrt{gh_c}} = \frac{3.132}{\sqrt{g \times 1}} = 1$

If the constant n in the Manning formula is 0.019 $m^{1/2}$ /s what must the slope of the bed be for constant depth at maximum flow rate?

$A = 1.6 \times 1 = 1.6\ m^2$ $P = 1.6 + 1 + 1 = 3.6$ m $R_h = 1.6/3.6 = 0.444$ m
$u_o = u_c = 3.132$ m/s

$u_o = \frac{R_h^{2/3} S^{1/2}}{n}$ rearrange $S = \left(\frac{n u_o}{R_h^{2/3}}\right)^2 = \left(\frac{0.019 \times 3.132}{0.444^{2/3}}\right)^2 = 0.0104$

WORKED EXAMPLE No. 7

Water flows in a rectangular channel 3 m wide with a mean velocity of 1.5 m/s and a depth of 1.2 m. Determine whether the flow is tranquil or shooting. Calculate the following.

The actual flow rate
The specific energy head
The critical depth
The maximum flow possible

SOLUTION

$F_r = \frac{u_o}{\sqrt{gh}} = \frac{1.5}{\sqrt{g \times 1.2}} = 0.437$ It follows that the flow is tranquil.
Actual flow rate = $A\ u_o = (3 \times 1.2) \times 1.5 = 5.5\ m^3/s$

Energy Head $h_s = h + u_o^2/2g = 1.2 + 1.5^2/2g = 1.315$ m

$h_c = 2h_s/3 = 2 \times 1.315/3 = 0.876$ m

For maximum flow rate $F_r = 1$

$F_r = 1 = \frac{u_c}{\sqrt{gh_c}}$ $u_c = \sqrt{gh_c} = \sqrt{9.81 \times 0.876} = 2.931$ m/s
$A = 3 \times 0.876 = 2.629\ m^2$
$Q_c = Au_c = 2.629 \times 2.931 = 7.71\ m^3/s$
If the depth changed to the critical depth, the flow rate would increase.

SELF ASSESSMENT EXERCISE No.5

1. A rectangular channel is 3.2 m wide and must carry 5 m^3/s of water with the minimum specific head. What would the depth and mean velocity be? (1.563 m and 3.915 m/s)
2. If the channel in question 1 must carry flow at a constant depth and n in the manning formula is 0.022, what is the slope of the bed? (0.013)
3. The flow in a horizontal, rectangular channel, 6m wide is controlled by a sluice gate. The depths of flow upstream and downstream of the gate are 1.5m and 0.300 m respectively. Determine:
   (a) the discharge (b) the specific energy of the flow (c) the critical depth.

VEE OR TRIANGULAR SECTION

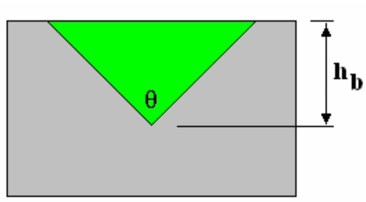


Figure 10

$$Q = A\{2g(h_s - h_b)\}^{1/2} \qquad A = \tfrac{1}{2} h_b \times 2 h_b \tan(\theta/2)$$

$$Q = h_b^2 \tan(\theta/2)\{2g(h_s - h_b)\}^{1/2}$$

$$Q = \tan(\theta/2)\{2g(h_b^4 h_s - h_b^5)\}^{1/2}$$

$$\frac{dQ}{dh_b} = \tan(\theta/2)(2g)^{1/2}\left(4h_b^3 h_s - 5h_b^4\right)^{1/2}$$

For maximum $\left(4h_b^3 h_s = 5h_b^4\right)$ $\quad (4h_s = 5h_b)$ $\quad h_c = \dfrac{4h_s}{5}$

Since $u_o = \{2g(h_s - h_b)\}^{1/2}$ then substituting for h_s will produce the critical velocity.

$$u_o = \left\{2g\left(\frac{5}{4}h_c - h_c\right)\right\}^{1/2} = (2g)^{1/2}\left(\frac{h_c}{4}\right)^{1/2} = \left(\frac{gh_c}{2}\right)^{1/2} \qquad u_c = \left(\frac{gh_c}{2}\right)^{1/2}$$

$$Q_c = Au_c = h_c^2 \tan(\theta/2)\left(\frac{gh_c}{2}\right)^{1/2} \qquad Q_c = \left(\frac{g}{2}\right)^{1/2} \tan(\theta/2) h_c^{5/2}$$

FROUDE NUMBER

$F_r = \dfrac{u}{\sqrt{gh}}$ For critical flow $F_r = \dfrac{u_c}{\sqrt{gh_c}}$ Substitute for u_c $F_r = \dfrac{\sqrt{\dfrac{gh_c}{2}}}{\sqrt{gh_c}} = \dfrac{1}{\sqrt{2}} = 0.707$

In terms of h_s

$$Q_c = \left(\frac{g}{2}\right)^{1/2} \tan(\theta/2) h_c^{5/2} = \left(\frac{g}{2}\right)^{1/2} \tan(\theta/2)\left(\frac{4h_s}{5}\right)^{5/2} = \sqrt{\left(\frac{g}{2}\right)}\left(\frac{4h_s}{5}\right)^{5/2} \tan(\theta/2)$$

$$u_c = \left(\frac{gh_c}{2}\right)^{1/2} = \left(\frac{g}{2} \times \frac{4}{5} h_s\right)^{1/2} = \sqrt{\frac{2g}{5} h_s}$$

Summary for triangular section

The critical depth is $\qquad h_c = \dfrac{4h_s}{5}$

The critical velocity is $\qquad u_c = \sqrt{\dfrac{gh_c}{2}} = \sqrt{\dfrac{2g}{5} h_s}$

The critical flow is $\qquad Q_c = \left(\dfrac{g}{2}\right)^{1/2} \tan(\theta/2) h_c^{5/2} = \sqrt{\left(\dfrac{g}{2}\right)}\left(\dfrac{4h_s}{5}\right)^{5/2} \tan(\theta/2)$

Froude Number $\qquad F_r = \dfrac{1}{\sqrt{2}} = 0.707$

WORKED EXAMPLE No. 8

A triangular channel 3 m wide with an included angle of 90^o must carry water with a depth of 3 m. What would be the maximum possible flow rate the mean velocity at this flow rate?

SOLUTION

For maximum flow rate the depth must be the critical depth so $h_c = 3$ m.

The critical velocity is $u_c = \left(\frac{gh_c}{2}\right)^{1/2} = \left(\frac{3g}{2}\right)^{1/2} = 3.836 \text{ m/s}$

$A = h_c^2 \tan(\theta/2) = 3^2 \tan(45) = 9 \text{ m}^2$

The critical flow is $Q_c = A\,u_c = 9 \times 3.836 = 34.524 \text{ m}^3/\text{s}$

Check the Froude number $F_r = \frac{u_c}{\sqrt{gh_c}} = \frac{3.836}{\sqrt{g \times 3}} = 0.707$

If the flow must remain at constant depth and n in the manning formula is 0.025, calculate the slope of the bed.

$P = 2h_c/\cos(\theta/2) = 8.485$ $R_h = A/P = 1.061$ $S = \left(\frac{nu_o}{R_h^{2/3}}\right)^2 = \left(\frac{0.025 \times 3.836}{1.061^{2/3}}\right)^2 = 0.0085$

WORKED EXAMPLE No. 9

A triangular channel 3 m wide with an included angle of 120^o must carry 0.75 m^3/s with the minimum specific head. What would be the maximum flow rate the mean velocity?

SOLUTION

For minimum specific head, the flow rate and velocity must be the critical values.

$Q_c = \left(\frac{g}{2}\right)^{1/2} \tan(\theta/2) h_c^{5/2}$ rearranging

$$\frac{Q_c^{2/5}}{\left(\left(\frac{g}{2}\right)^{1/2} \tan(\theta/2)\right)^{2/5}} = h_c = \frac{0.75^{2/5}}{\left(\left(\frac{g}{2}\right)^{1/2} \tan(60)\right)^{2/5}} = 0.521 \text{ m}$$

$A = h_c^2 \tan\theta = 0.469 \text{ m}^2$ $u_c = Q_c/A = 1.6$ m

SELF ASSESSMENT EXERCISE No.6

1. A uniform channel has a vee cross section with a symmetrical included angle of 100^o. If it carries 1.25 m^3/s of water with minimum specific energy head, what would be the depth and mean velocity.
(0.742 m and 1.907 m/s)

2. The same channel described in question 1 must carry the flow at a constant depth. If n in the Manning formula is 0.022, what must be the slope of the bed.
(0.00943)