Unit 41: Fluid Mechanics

Unit code: T/601/1445

QCF Level: 4

Credit value: 15

OUTCOME 4

TUTORIAL 7 – TURBINES AND PUMPS

4 **Understand the operating principles of hydraulic machines**

Impact of a jet: power of a jet; normal thrust on a moving flat vane; thrust on a moving hemispherical cup; velocity diagrams to determine thrust on moving curved vanes; fluid friction losses; system efficiency

Operating principles of turbines: operating principles, applications and typical system efficiencies of common turbo-machines including the Pelton wheel, Francis turbine and Kaplan turbine

Operating principles of pumps: operating principles and applications of reciprocating and centrifugal pumps; head losses; pumping power; power transmitted; system efficiency

This is another major outcome requiring a lot of study time and the tutorial probably contains more than required.

CONTENTS

Let’s start with forces due to changes in the pressure of the fluid.

1. TURBINES

A water turbine is a device for converting water (fluid) power into shaft (mechanical) power. A pump is a device for converting shaft power into water power.

Two basic categories of machines are the rotary type and the reciprocating type. Reciprocating motors are quite common in power hydraulics but the rotary principle is universally used for large power devices such as on hydroelectric systems.

Large pumps are usually of the rotary type but reciprocating pumps are used for smaller applications.

1.1 GENERAL PRINCIPLES OF TURBINES.

WATER POWER

This is the fluid power supplied to the machine in the form of pressure and volume.
Expressed in terms of pressure head the formula is W.P. = $mg\Delta H$
M is the mass flow rate in kg/s and ΔH is the pressure head difference over the turbine in metres.
Remember that $\Delta p = \rho g \Delta H$
Expressed in terms of pressure the formula is W.P. = $Q\Delta p$
Q is the volume flow rate in m^3/s. Δp is the pressure drop over the turbine in N/m^2 or Pascals.

SHAFT POWER

This is the mechanical, power output of the turbine shaft. The well known formula is
S.P. = $2\pi NT$ Where T is the torque in Nm and N is the speed of rotation in rev/s

DIAGRAM POWER

This is the power produced by the force of the water acting on the rotor. It is reduced by losses before appearing as shaft power. The formula for D.P. depends upon the design of the turbine and involves analysis of the velocity vector diagrams.

HYDRAULIC EFFICIENCY η_{hyd}

This is the efficiency with which water power is converted into diagram power and is given by

$$\eta_{hyd} = \text{D.P./W.P.}$$

MECHANICAL EFFICIENCY η_{mech}

This is the efficiency with which the diagram power is converted into shaft power. The difference is the mechanical power loss.

$$\eta_{mech} = \text{S.P./D.P.}$$

OVERALL EFFICIENCY $\eta_{o/a}$

This is the efficiency relating fluid power input to shaft power output.

$$\eta_{o/a} = \text{S.P./W.P.}$$

It is worth noting at this point that when we come to examine pumps, all the above expressions are inverted because the energy flow is reversed in direction.
The water power is converted into shaft power by the force produced when the vanes deflect the direction of the water. There are two basic principles in the process, ***IMPULSE and REACTION.***

IMPULSE occurs when the direction of the fluid is changed with no pressure change. It follows that the magnitude of the velocity remains unchanged.

REACTION occurs when the water is accelerated or decelerated over the vanes. A force is needed to do this and the reaction to this force acts on the vanes.
Impulsive and reaction forces are determined by examining the changes in velocity (magnitude and direction) when the water flows over the vane. The following is a typical analysis.
The vane is part of a rotor and rotates about some centre point. Depending on the geometrical layout, the inlet and outlet may or may not be moving at the same velocity and on the same circle. In order to do a general study, consider the case where the inlet and outlet rotate on two different diameters and hence have different velocities.

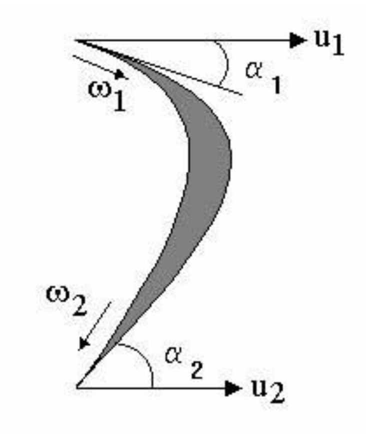


Fig. 1

u_1 is the velocity of the blade at inlet and u_2 is the velocity of the blade at outlet. Both have tangential directions.ω_1 is the relative velocity at inlet and ω_2 is the relative velocity at outlet.

The water on the blade has two velocity components. It is moving tangentially at velocity u and over the surface at velocity ω. The absolute velocity of the water is the vector sum of these two and is denoted v. At any point on the vane $v = \omega + u$

At inlet, this rule does not apply unless the direction of v_1 is made such that the vector addition is true. At any other angle, the velocities will not add up and the result is chaos with energy being lost as the water finds its way onto the vane surface. The perfect entry is called ***"SHOCKLESS ENTRY"*** and the entry angle β_1 must be correct. This angle is only correct for a given value of v_1.

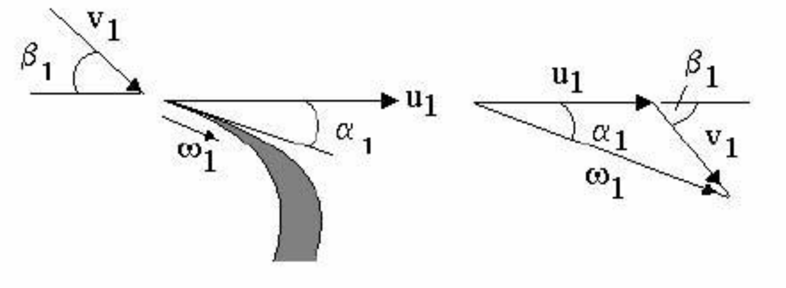


Fig.2

INLET DIAGRAM

For a given or fixed value of u_1 and v_1, shockless entry will occur only if the vane angle α_1 is correct or the delivery angle β_1 is correct. In order to solve momentum forces on the vane and deduce the flow rates, we are interested in two components of v_1. These are the components in the direction of the vane movement denoted v_w (meaning velocity of whirl) and the direction at right angles to it v_R (meaning radial velocity but it is not always radial in direction depending on the wheel design). The suffix (1) indicates the entry point. A typical vector triangle is shown.

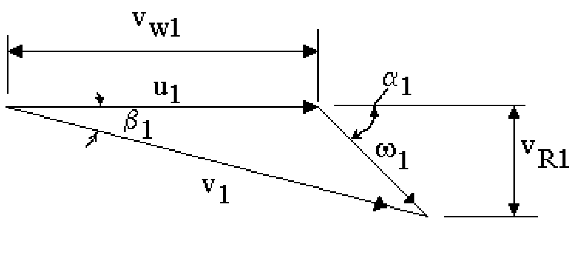


Fig.3

OUTLET DIAGRAM

At outlet, the absolute velocity of the water has to be the vector resultant of u and ω and the direction is unconstrained so it must come off the wheel at the angle resulting. Suffix (2) refers to the outlet point. A typical vector triangle is shown.

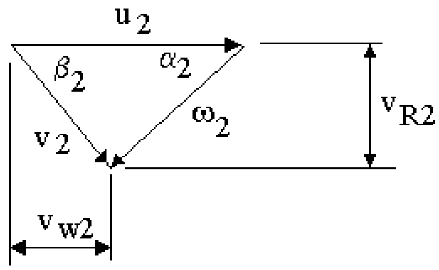


Fig. 4

DIAGRAM POWER

Diagram power is the theoretical power of the wheel based on momentum changes in the fluid. The force on the vane due to the change in velocity of the fluid is $F = m\Delta v$ and these forces are vector quantities. m is the mass flow rate. The force that propels the wheel is the force developed in the direction of movement (whirl direction). In order to deduce this force, we should only consider the velocity changes in the whirl direction (direction of rotation) Δv_w. The power of the force is always the product of force and velocity. The velocity of the force is the velocity of the vane (u). If this velocity is different at inlet and outlet it can be shown that the resulting power is given by

$$D.P. = m\ \Delta v_w = m\ (u_1 v_{w1} - u_2\ v_{w2})$$

1.2 PELTON WHEEL

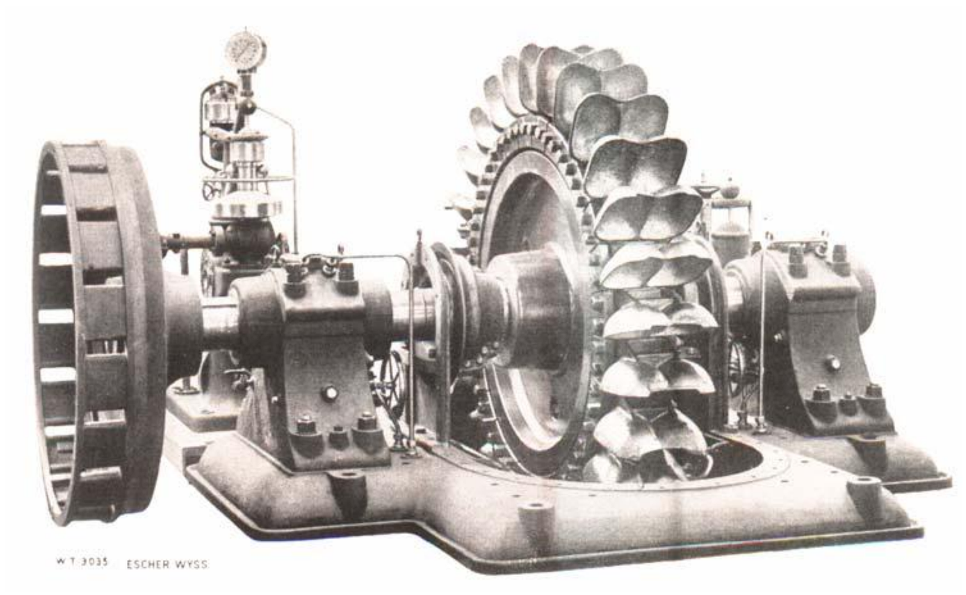


Fig. 5 Pelton wheel with the casing removed

Pelton wheels are mainly used with high pressure heads such as in mountain hydroelectric schemes. The diagram shows a layout for a Pelton wheel with two nozzles.

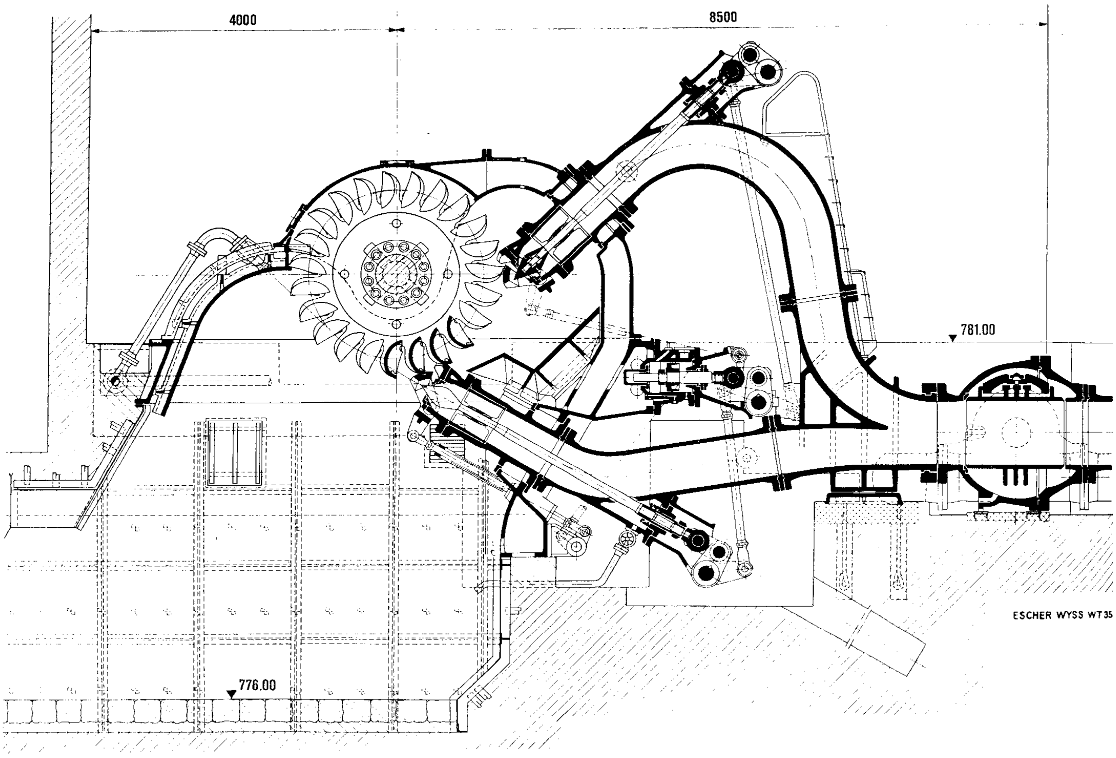


Fig. 6 Typical Layout

The Pelton Wheel is an impulse turbine. The fluid power is converted into kinetic energy in the nozzles. The total pressure drop occurs in the nozzle. The resulting jet of water is directed tangentially at buckets on the wheel producing impulsive force on them. The buckets are small compared to the wheel and so they have a single velocity $u = \pi ND$ D is the mean diameter of rotation for the buckets.

The theoretical velocity issuing from the nozzle is given by $v_1 = (2gH)^{1/2}$ or $v_1 = (2p/\rho)^{1/2}$

Allowing for friction in the nozzle this becomes $v_1 = C_v(2gH)^{1/2}$ or $v_1 = C_v(2p/\rho)^{1/2}$
H is the gauge pressure head behind the nozzle, p the gauge pressure and c_v the coefficient of velocity and this is usually close to unity.

The mass flow rate from the nozzle is $m = C_c\,\rho A v_1 = C_c \rho A C_v (2gH)^{1/2} = C_d\,\rho A(2gH)^{1/2}$

C_c is the coefficient of contraction (normally unity because the nozzles are designed not to have a contraction).

C_d is the coefficient of discharge and
$C_d = C_c\, C_v$

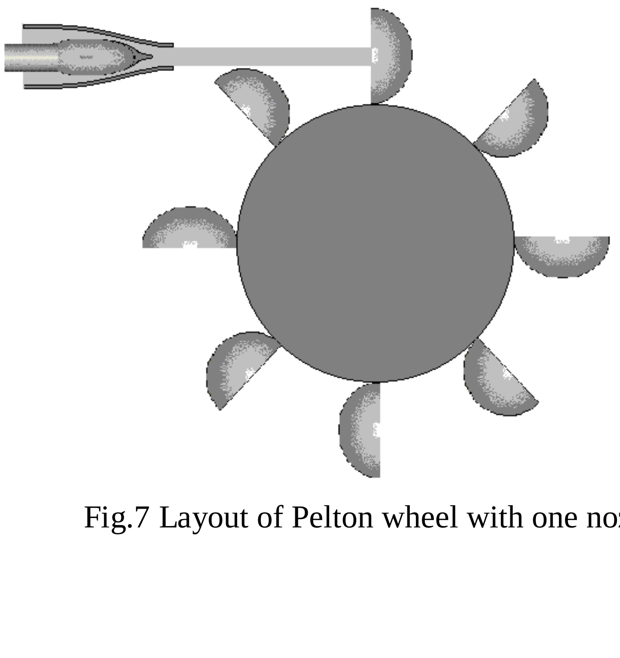
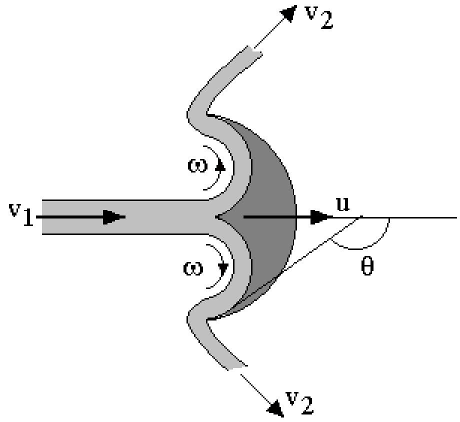


Fig.7 Layout of Pelton wheel with one nozzle

In order to produce no axial force on the wheel, the flow is divided equally by the shape of the bucket. This produces a zero net change in momentum in the axial direction. The water is deflected over each half of the bucket by an angle of θ degrees. Since the change in momentum is the same for both halves of the flow, we need only consider the vector diagram for one half. The initial velocity is v1 and the bucket velocity u1 is in the same direction. The relative velocity of the water at inlet (in the middle) is ω_1 and is also in the same direction so the vector diagram is a straight line.



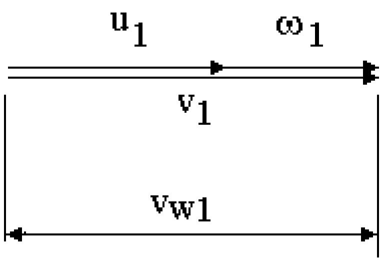


Fig. 9 Vector Diagram

Fig.8 Cross section through bucket

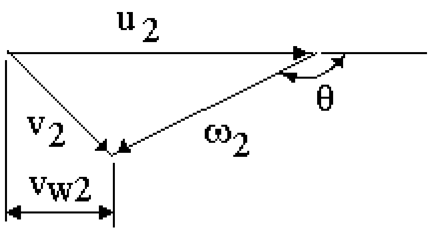
If the water is not slowed down as it passes over the bucket surface, the relative velocity ω_2 will be the same as ω_1. In reality friction slows it down slightly and we define a blade friction coefficient as

$$k = \omega_2/\omega_1$$

The exact angle at which the water leaves the sides of the bucket depends upon the other velocities but as always the vectors must add up so that $v_2 = u + \omega_2$

Note that $u_2 = u_1 = u$ since the bucket has a uniform velocity everywhere.

It is normal to use ω_1 and u as common to both diagrams and combine them as shown.



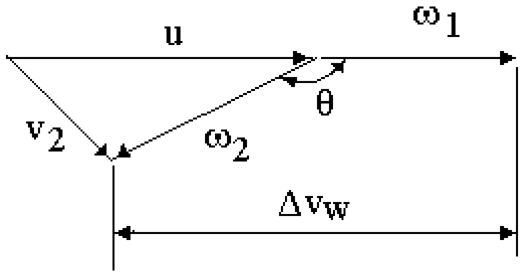


Fig. 10 Inlet Vector Diagram

Fig. 11 Combined Vector Diagarm

Since $u_2 = u_1 = u$ the diagram power becomes D.P. $= mu\Delta v_W$

Examining the combined vector diagram shows that $\Delta v_W = \omega_1 - \omega_2\cos\theta$

Hence D.P.$= mu(\omega_1 - \omega_2\cos\theta)$ but $\omega_2 = k\omega_1$

D.P.$= mu\omega_1(1 - k\cos\theta)$ but $\omega_1 = v_1 - u$

D.P.$= mu(v_1 - u)(1 - k\cos\theta)$

WORKED EXAMPLE No. 1

A Pelton wheel is supplied with 1.2 kg/s of water at 20 m/s. The buckets rotate on a mean diameter of 250 mm at 800 rev/min. The deflection angle is 165^o and friction is negligible. Determine the diagram power. Draw the vector diagram to scale and determine □v_W.

SOLUTION

$u = \pi ND/60 = \pi \times 800 \times 0.25/60 = 10.47$ m/s
D.P $= mu(v_1 - u)(1 - k\cos\theta)$
D.P $= 1.2 \times 10.47 \times (20 - 10.47)(1 - \cos 165) = 235$ Watts

You should now draw the vector diagram to scale and show that $\Delta v_W = 18.5$ m/s

SELF ASSESSMENT EXERCISE No. 1

1. The buckets of a Pelton wheel revolve on a mean diameter of 1.5 m at 1500 rev/min. The jet velocity is 1.8 times the bucket velocity. Calculate the water flow rate required to produce a power output of 2MW. The mechanical efficiency is 80% and the blade friction coefficient is 0.97. The deflection angle is 165^o.
(Ans. 116.3 kg/s)

2. Calculate the diagram power for a Pelton Wheel 2m mean diameter revolving at 3000 rev/min with a deflection angle of 170^o under the action of two nozzles , each supplying 10 kg/s of water with a velocity twice the bucket velocity. The blade friction coefficient is 0.98. (Ans. 3.88 MW)

   If the coefficient of velocity is 0.97, calculate the pressure behind the nozzles. (Ans 209.8 MPa)

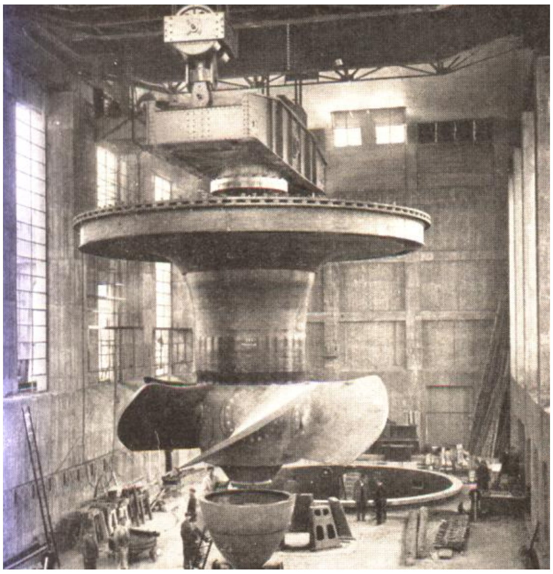
3. A Pelton Wheel is 1.7 m mean diameter and runs at maximum power. It is supplied from two nozzles. The gauge pressure head behind each nozzle is 180 metres of water. Other data for the wheel is :

   Coefficient of Discharge $C_d = 0.99$
   Coefficient of velocity $C_v = 0.995$
   Deflection angle = 165^o.
   Blade friction coefficient = 0.98
   Mechanical efficiency = 87%
   Nozzle diameters = 30 mm

   Calculate the following.

a) The jet velocity (59.13 m/s)
b) The mass flow rate (41.586 kg/s)
c) The water power (73.432 kW)
d) The diagram power (70.759 kW)
e) The diagram efficiency (96.36%)
f) The overall efficiency (83.8%)
g) The wheel speed in rev/min (332 rev/min)

1.3 KAPLAN TURBINE



The Kaplan turbine is a pure reaction turbine. The main point concerning this is that all the flow energy and pressure is expended over the rotor and not in the supply nozzles. The picture shows the rotor of a large Kaplan turbine. They are most suited to low pressure heads and large flow rates such as on dams and tidal barrage schemes.

The diagram below shows the layout of a large hydroelectric generator in a dam.

Fig. 12 Picture of a Kaplan Turbine Rotor

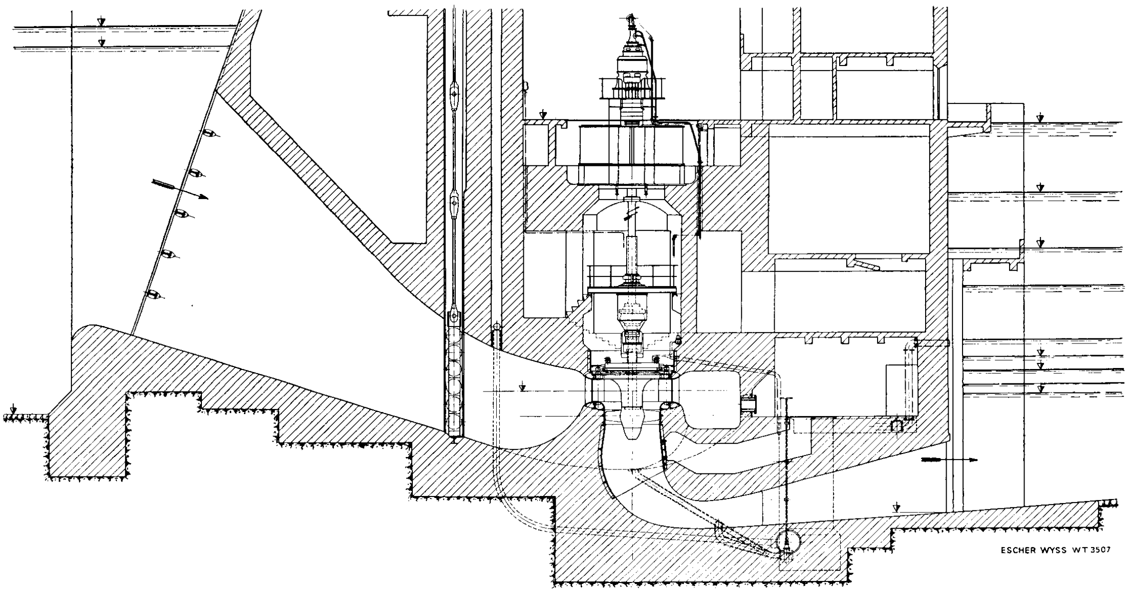


Fig. 13 Typical Layout

1.4 **FRANCIS WHEEL**

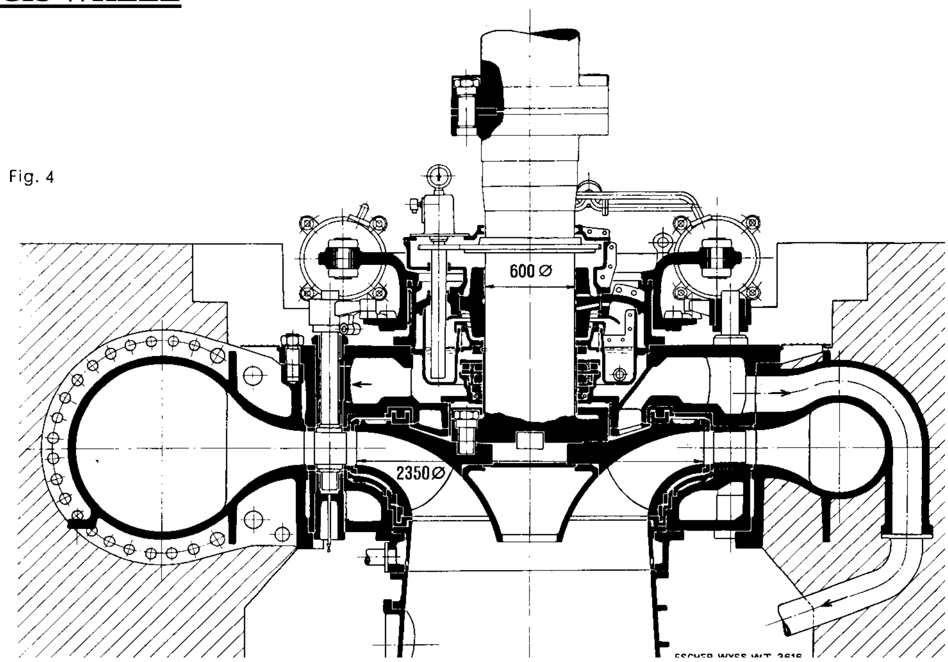


Fig. 14

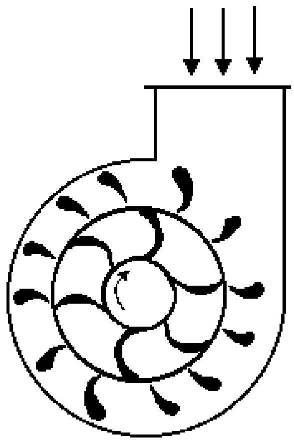


Fig. 15

The Francis wheel is an example of a mixed impulse and reaction turbine. They are adaptable to varying heads and flows and may be run in reverse as a pump such as on a pumped storage scheme. The diagram shows the layout of a vertical axis Francis wheel.

The Francis Wheel is an inward flow device with the water entering around the periphery and moving to the centre before exhausting. The rotor is contained in a casing that spreads the flow and pressure evenly around the periphery.

The impulse part comes about because guide vanes are used to produce an initial velocity v_1 that is directed at the rotor. Pressure drop occurs in the guide vanes and the velocity is $v_1 = k\ (\Delta H)^{½}$ where ΔH is the head drop in the guide vanes.

The angle of the guide vanes is adjustable so that the inlet angle β_1 is correct for shockless entry.

The shape of the rotor is such that the vanes are taller at the centre than at the ends. This gives control over the radial velocity component and usually this is constant from inlet to outlet. The volume flow rate is usually expressed in terms of radial velocity and circumferential area.

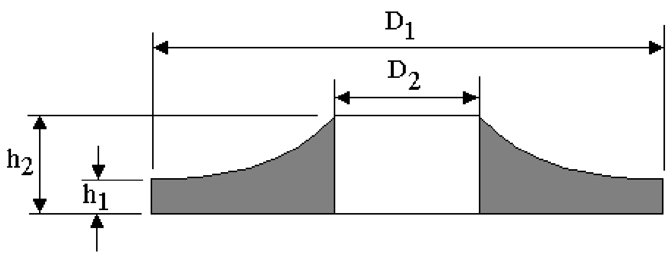


Fig. 16

v_R = radial velocity A = circumferential area = D h k
$Q = v_R\ \pi D\ h\ k$ h = height of the vane.

k is a factor which allows for the area taken up by the thickness of the vanes on the circumference. If v_R is constant then since Q is the same at all circumferences, $D_1h_1=D_2h_2$.

VECTOR DIAGRAMS

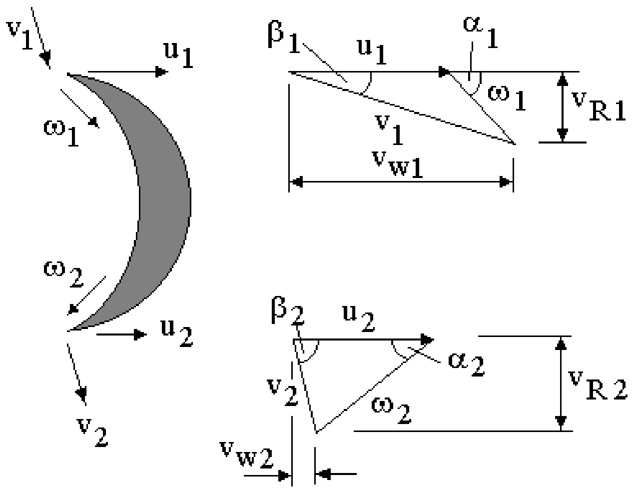


Fig. 17

The diagram shows how the vector diagrams are constructed for the inlet and outlet. Remember the rule is that the vectors add up so that $u + v = \omega$

If u is drawn horizontal as shown, then V_W is the horizontal component of v and v_R is the radial component (vertical).

MORE DETAILED EXAMINATION OF VECTOR DIAGRAM

Applying the sine rule to the inlet triangle we find

$$\frac{v_1}{\sin(180-\alpha_1)} = \frac{u_1}{\sin\{180-\beta_1-(180-\alpha_1)\}}$$

$$\frac{v_1}{\sin(\alpha_1)} = \frac{u_1}{\sin(\alpha_1-\beta_1)} \qquad v_1 = \frac{u_1\sin(\alpha_1)}{\sin(\alpha_1-\beta_1)}\text{.................(1)}$$

$$v_1 = \frac{v_{r1}}{\sin(\beta_1)}\text{...........(2)} \qquad v_{r1} = v_{w1}\tan\beta_1\text{................(3)} \qquad \text{equate (1) and (2)}$$

$$\frac{u_1\sin(\alpha_1)}{\sin(\alpha_1-\beta_1)} = \frac{v_{r1}}{\sin(\beta_1)} \qquad v_{r1} = \frac{u_1\sin(\alpha_1)\sin(\beta_1)}{\sin(\alpha_1-\beta_1)}\text{...........(4)} \qquad \text{equate (3) and (4)}$$

$$v_{w1} = \frac{u_1\sin(\alpha_1)\sin(\beta_1)}{\sin(\alpha_1-\beta_1)\tan\beta_1}\text{.............(5)}$$

If all the angles are known, then v_{w1} may be found as a fraction of u_1.

DIAGRAM POWER

Because u is different at inlet and outlet we express the diagram power as :

$$\text{D.P.} = m\,\Delta(uv_w) = m\,(u_1v_{w1} - u_2v_{w2})$$

The kinetic energy represented by v_2 is energy lost in the exhausted water. For maximum efficiency, this should be reduced to a minimum and this occurs when the water leaves radially with no whirl so that $v_{w2} = 0$. This is produced by designing the exit angle to suit the speed of the wheel. The water would leave down the centre hole with some swirl in it. The direction of the swirl depends upon the direction of v_2 but if the flow leaves radially, there is no swirl and less kinetic energy. Ideally then,

$$\text{D.P.} = m\,u_1\,v_{w1}$$

WATER POWER

The water power supplied to the wheel is $mg\Delta H$ where ΔH is the head difference between inlet and outlet.

HYDRAULIC EFFICIENCY

The maximum value with no swirl at exit is $\eta_{hyd} = D.P./W.P. = u_1 v_{w1}/g\rho H$

OVERALL EFFICIENCY

$$\eta_{o/a} = \text{Shaft Power/Water Power}$$
$$\eta_{o/a} = 2\pi NT/mg\Delta H$$

LOSSES

The hydraulic losses are the difference between the water power and diagram power.

Loss = $mg\Delta H - mu_1 v_{w1} = mgh_L$

$h_L = \Delta H - u_1 v_{w1}/g$

$\Delta H - h_L = u_1 v_{w1}/g$

SELF ASSESSMENT EXERCISE No. 2

You have studied the basic principles of Pelton , Kaplan and Francis turbines.

Hydroelectric schemes may have very high pressures (e.g. mountain lakes). They may have very low pressures (e.g. dammed lakes). The pressure head may vary (e.g. tidal barrage schemes). They may have access to large or small quantities of water. Sometimes they are used as pumps (e.g. pumped storage schemes).

Find out what each turbine is best suited to. Explain what it is in their design that suit to them to their application.

2. CENTRIFUGAL PUMPS

2.1 GENERAL THEORY

A Centrifugal pump is a Francis turbine running backwards. The water between the rotor vanes experiences centrifugal force and flows radially outwards from the middle to the outside. As it flows, it gains kinetic energy and when thrown off the outer edge of the rotor, the kinetic energy must be converted into flow energy. The use of vanes similar to those in the Francis wheel helps. The correct design of the casing is also vital to ensure efficient low friction conversion from velocity to pressure. The water enters the middle of the rotor without swirling so we know v_{w1} is always zero for a c.f. pump. Note that in all the following work, the inlet is suffix 1 and is at the inside of the rotor. The outlet is suffix 2 and is the outer edge of the rotor.

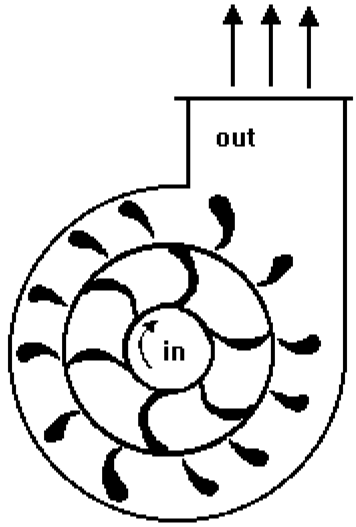


Fig. 18 Basic Design

The increase in momentum through the rotor is found as always by drawing the vector diagrams. At inlet v_1 is radial and equal to v_{r1} and so v_{w1} is zero. This is so regardless of the vane angle but there is only one angle which produces shockless entry and this must be used at the design speed.

At outlet, the shape of the vector diagram is greatly affected by the vane angle. The diagram below shows a typical vector diagram when the vane is swept backwards (referred to the vane velocity u).

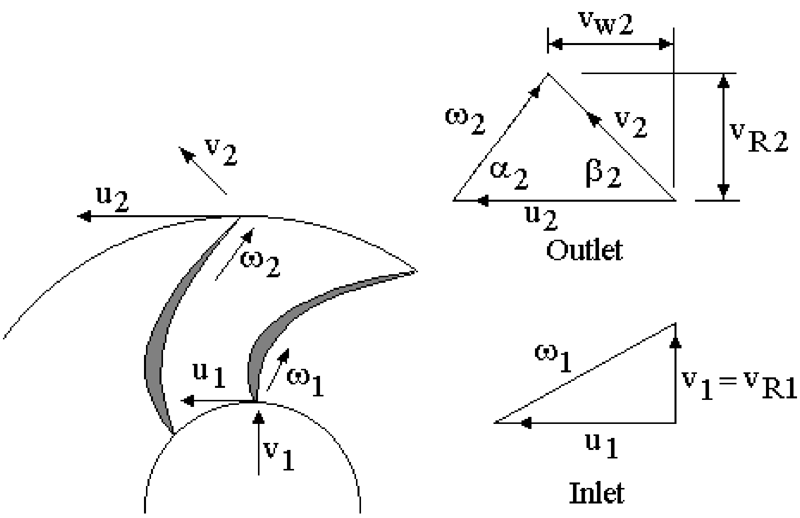


Fig. 19

v_{w2} may be found by scaling from the diagram. We can also apply trigonometry to the diagram as follows.

$$v_{w2} = u_2 - v_{r2}/\tan\alpha_2$$

$$v_{r2} = Q/\text{circumferential area} = Q/(\pi D_2 h_2 k)$$

$$u_2 = \pi N D_2$$

hence $$v_{w2} = u_2 - Q/(\pi D_2 h_2 k \tan \alpha_2) = u_2 - v_{r2}/\tan \alpha_2)$$

DIAGRAM POWER

$$D.P. = m\Delta u v_W$$

Usually v_{w1} is zero this becomes D.P.= mu_2v_{w2}

WATER POWER

$$W.P. = mg\Delta h$$

MANOMETRIC HEAD Δh_m

This is the head that would result if all the energy given to the water is converted into pressure head. It is found by equating the diagram power and water power.

$mu_2v_{w2} = mg\Delta h_m$ $\quad$ $\Delta h_m = u_2v_{w2}/g$

MANOMETRIC EFFICIENCY η_m

$$\eta_m = W.P./D.P. = mg\Delta h/ mu_2v_{w2} = mg\Delta h/ mg\Delta h_m$$
$$\eta_m = \Delta h/\Delta h_m$$

SHAFT POWER

$$S.P. = 2\pi NT$$

OVERALL EFFICIENCY

$$\eta_{o/a} = W.P./S.P.$$

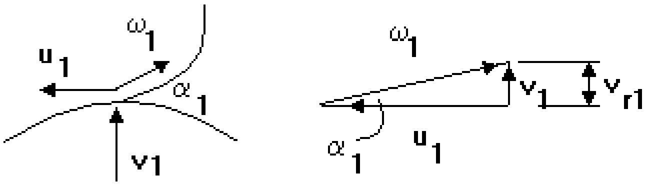
KINETIC ENERGY AT ROTOR OUTLET

$$K.E. = mv_2^2/2$$

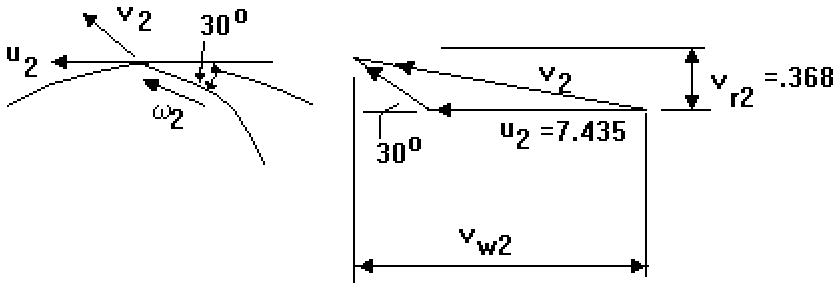
Note the energy lost is mainly in the casing and is usually expressed as a fraction of the K.E. at exit.

SELF ASSESSMENT EXERCISE No. 3

Below is the vector diagram for a centrifugal pump.



Inlet vector diagram



OUTLET VECTOR DIAGRAM

The important data follows.

The flow enters radially without shock.

Rotor outlet diameter	D_2 = 100 mm
Flow rate	Q = 0.0022 m^3/s
Density of water	1000 kg/m^3

The developed head is 5 m and the power input to the shaft is 170 Watts.
$v_{r1} = v_{r2} = 0.35$ m/s
$u_1 = 3$ m/s
$u_2 = 7.5$ m/s

Draw the vector diagrams to scale and determine the following.

a. The inlet vane angle α_1
b. The change in the velocity of whirl Δ_{vw}
c. The speed N
d. The diagram power
e. The manometric head Δh_m
f. The manometric efficiency η_m
g. The overall efficiency $\eta_{o/a}$