Unit 41: Fluid Mechanics

Unit code: T/601/1445

QCF Level: 4

Credit value: 15

OUTCOME 3

TUTORIAL 4 - DRAG

3 Be able to determine the behavioural characteristics and parameters of real fluid flow

Head losses: head loss in pipes by Darcy's formula; Moody diagram; head loss due to sudden enlargement and contraction of pipe diameter; head loss at entrance to a pipe; head loss in valves; flow between reservoirs due to gravity; hydraulic gradient; siphons; hammerblow in pipes

Reynolds number: inertia and viscous resistance forces; laminar and turbulent flow; critical velocity

Viscous drag: dynamic pressure; form drag; skin friction drag; drag coefficient

Dimensional analysis: checking validity of equations such as those for pressure at depth; thrust on immersed surfaces and impact of a jet; forecasting the form of possible equations such as those for Darcy's formula and critical velocity in pipes

CONTENTS

This tutorial carries on from tutorial 3 and deals with how real fluids flow around bodies. When you have completed this tutorial you should be able to explain how fluids exert a drag force on a body.

1. DRAG

When a fluid flows around the outside of a body, it produces a force that tends to drag the body in the direction of the flow. The drag acting on a moving object such as a ship or an aeroplane must be overcome by the propulsion system. Drag takes two forms, skin friction drag and form drag.

1.1 SKIN FRICTION DRAG

Skin friction drag is due to the viscous shearing that takes place between the surface and the layer of fluid immediately above it. This occurs on surfaces of objects that are long in the direction of flow compared to their height. Such bodies are called ***STREAMLINED***. When a fluid flows over a solid surface, the layer next to the surface may become attached to it (it wets the surface). This is called the 'no slip condition'. The layers of fluid above the surface are moving so there must be shearing taking place between the layers of the fluid. The shear stress acting between the wall and the first moving layer next to it is called ***the wall shear stress*** and denoted τ_w.

The result is that the velocity of the fluid grows from zero at the surface to a maximum u_o at some distance δ above it. This layer is called the ***BOUNDARY LAYER*** and δ is the boundary layer thickness. Fig. 1 Shows how the velocity "u" varies with height "y" for a typical boundary layer.

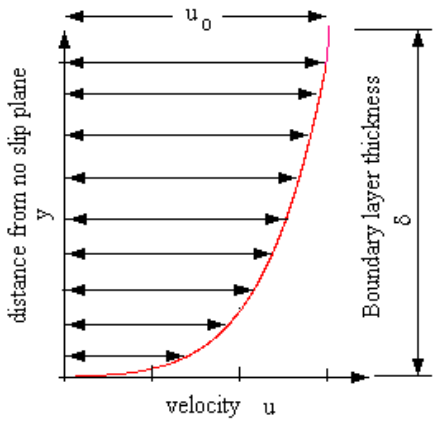

Fig. 1

In a pipe, this is the only form of drag and it results in a pressure and energy lost along the length. A thin flat plate is an example of a streamlined object. Consider a stream of fluid flowing with a uniform velocity u_o. When the stream is interrupted by the plate (fig. 2), the boundary layer forms on both sides. The diagram shows what happens on one side only.

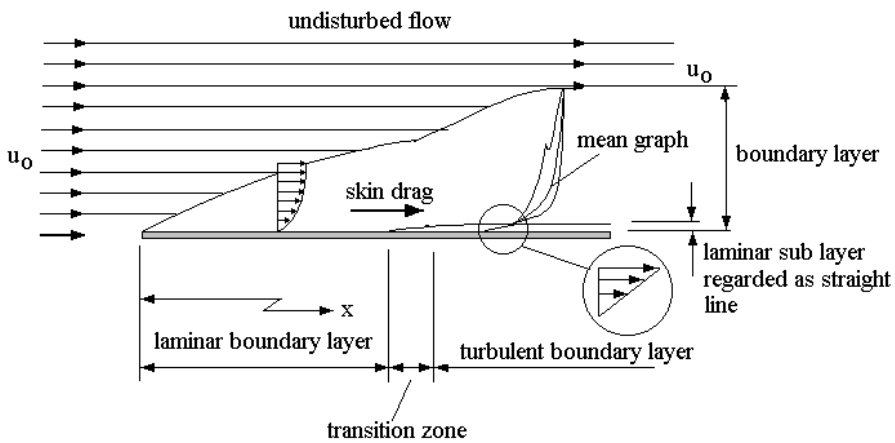

Fig. 2

The boundary layer thickness δ grows with distance from the leading edge. At some distance from the leading edge, it reaches a constant thickness. It is then called a ***FULLY DEVELOPED BOUNDARY LAYER***.

The Reynolds number for these cases is defined as:

$$(R_e)_x = \frac{\rho u_o x}{\mu}$$

x is the distance from the leading edge. At low Reynolds numbers, the boundary layer may be laminar throughout the entire thickness. At higher Reynolds numbers, it is turbulent. This means that at some distance from the leading edge the flow within the boundary layer becomes turbulent.

A turbulent boundary layer is very unsteady and the streamlines do not remain parallel. The boundary layer shape represents an average of the velocity at any height. There is a region between the laminar and turbulent section where transition takes place

The turbulent boundary layer exists on top of a thin laminar layer called the ***LAMINAR SUB LAYER***. The velocity gradient within this layer is linear as shown. A deeper analysis would reveal that for long surfaces, the boundary layer is turbulent over most of the length. Many equations have been developed to describe the shape of the laminar and turbulent boundary layers and these may be used to estimate the skin friction drag.

Note that for this ideal example, it is assumed that the velocity is the undisturbed velocity u_o everywhere outside the boundary layer and that there is no acceleration and hence no change in the static pressure acting on the surface. There is hence no drag force due to pressure changes.

CALCULATING SKIN DRAG

The skin drag is due to the wall shear stress τ_w and this acts on the surface area (wetted area).
The drag force is hence: **R = τ_w x wetted area.** The dynamic pressure is the pressure resulting from the conversion of the kinetic energy of the stream into pressure and is defined by the expression $\frac{\rho u_o^2}{2}$.The drag coefficient is defined as

$$C_{Df} = \frac{\text{Drag force}}{\text{dynamic pressure x wetted area}} = \frac{2R}{\rho u_0^2 \text{ x wetted area}} = \frac{2\tau_w}{\rho u_0^2}$$

Note that this is the same definition for the pipe friction coefficient C_f and it is in fact the same thing. It is used in the Darcy formula to calculate the pressure lost in pipes due to friction. For a smooth surface, it can be shown that $C_{Df} = 0.074\ (R_e)_x^{-1/5}$

$(Re)_l$ is the Reynolds number based on the length. $(R_e)_x = \frac{\rho u_o L}{\mu}$

WORKED EXAMPLE No.1

Calculate the drag force on each side of a thin smooth plate 2 m long and 1 m wide with the length parallel to a flow of fluid moving at 30 m/s. The density of the fluid is 800 kg/m^3 and the dynamic viscosity is 8 cP.

SOLUTION

$$(R_e)_x = \frac{\rho u_o L}{\mu} = \frac{800 \text{ x } 30 \text{ x } 2}{0.008} = 6 \text{ x } 10^6$$

$$C_{Df} = 0.074 \text{ x } (6 \text{ x} 10^6)^{-\frac{1}{5}} = 0.00326$$

$$\text{Dynamic pressure} = \frac{\rho u_0^2}{2} = \frac{800 \text{ x } 30^2}{2} = 360 \text{ kPa}$$

$$\tau_w = C_{Df} \text{ x dynamic pressure} = 0.00326 \text{ x } 360 \text{ x } 10^3 = 1173.6 \text{ Pa}$$

$$R = \tau_w \text{ x Wetted Area} = 1173.6 \text{ x } 2 \text{ x } 1 = 2347.2 \text{ N}$$

On a small area the drag is $dR = \tau_w\, dA$. If the body is not a thin plate and has an area inclined at an angle θ to the flow direction, the drag force in the direction of flow is $\tau_w\, dA \cos\theta$.

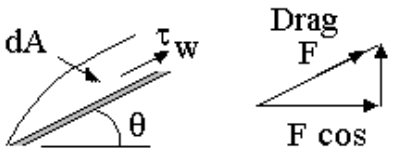

Fig.3

The drag force acting on the entire surface area is found by integrating over the entire area.

$$R = \oint \tau_w \cos\theta \, dA$$

Solving this equation requires more advanced studies concerning the boundary layer and students should refer to the classic textbooks on this subject.

SELF ASSESSMENT EXERCISE No.1

1. A smooth thin plate 5 m long and 1 m wide is placed in an air stream moving at 3 m/s with its length parallel with the flow. Calculate the drag force on each side of the plate. The density of the air is 1.2 kg/m^3 and the kinematic viscosity is 1.6 x 10^{-5} m^2/s. (0.128 N)

2. A pipe bore diameter D and length L has fully developed laminar flow throughout the entire length with a centre line velocity u_o. Given that the drag coefficient is given as $C_{Df} = 16/Re$ where $Re = \frac{\rho u_o D}{\mu}$, show that the drag force on the inside of the pipe is given as $R = 8\pi\mu u_o L$ and hence the pressure loss in the pipe due to skin friction is $p_L = 32\mu u_o L/D^2$

1.2 FORM DRAG and WAKES

Form or pressure drag applies to bodies that are tall in comparison to the length in the direction of flow. Such bodies are called ***BLUFF BODIES***.

Consider the case below that could for example, be the pier of a bridge in a river. The water speeds up around the leading edges and the boundary layer quickly breaks away from the surface. Water is sucked in from behind the pier in the opposite direction.

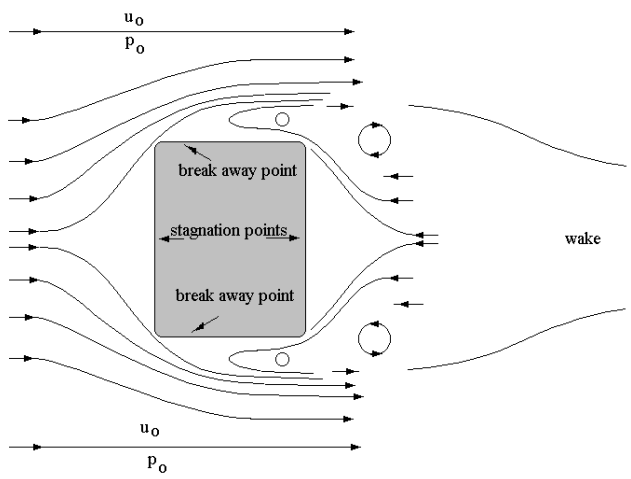

Fig. 4

The total effect is to produce eddy currents or whirl pools that are shed in the wake. There is a build up of positive pressure on the front and a negative pressure at the back. The pressure force resulting is the form drag. When the breakaway or separation point is at the front corner, the drag is almost entirely due to this effect but if the separation point moves along the side towards the back, then a boundary layer forms and skin friction drag is also produced. In reality, the drag is always a combination of skin friction and form drag. The degree of each depends upon the shape of the body.

The next diagram typifies what happens when fluid flows around a bluff object. The fluid speeds up around the front edge. Remember that the closer the streamlines, the faster the velocity. The line representing the maximum velocity is shown but also remember that this is the maximum at any point and this maximum value also increases as the stream lines get closer together.

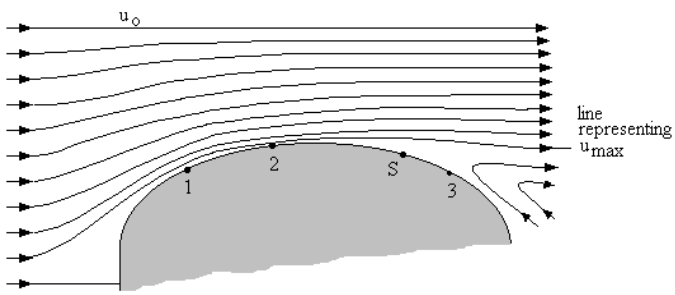

Fig. 5

Two important effects affect the drag.

Outside the boundary layer, the velocity increases up to point 2 so the pressure acting on the surface goes down. The boundary layer thickness δ gets smaller until at point S it is reduced to zero and the flow separates from the surface. At point 3, the pressure is negative. ***This change in pressure is responsible for the form drag.***

Inside the boundary layer, the velocity is reduced from u_{max} to zero and skin friction drag results.

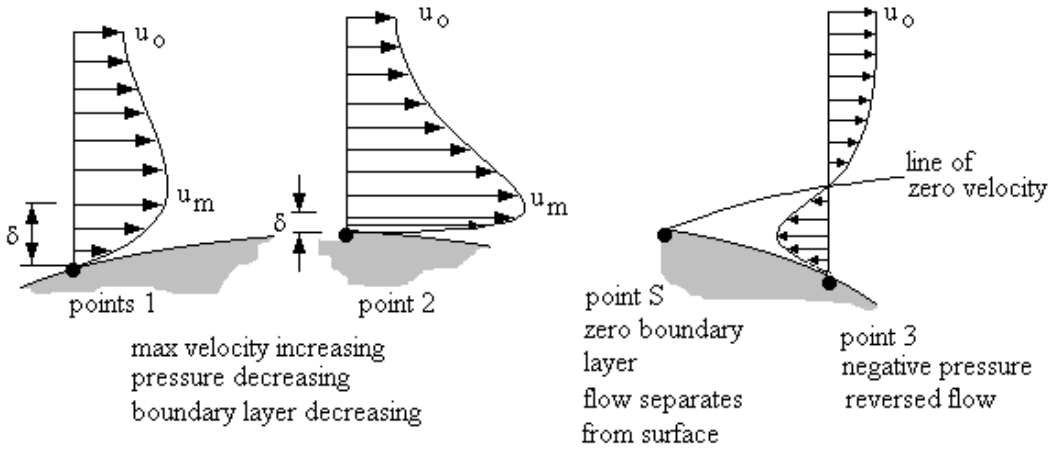

Fig. 6

In problems involving liquids with a free surface, a negative pressure shows up as a drop in level and the pressure build up on the front shows as a rise in level. If the object is totally immersed, the pressure on the front rises and a vacuum is formed at the back. This results in a pressure force opposing movement (form drag). The swirling flow forms ***vortices*** and the wake is an area of great turbulence behind the object that takes some distance to settle down and revert to the normal flow condition.

Here is an outline of the mathematical approach needed to solve the form drag.

Form drag is due to pressure changes only. The drag coefficient due to pressure only is denoted C_{Dp} and defined as

$$C_{Dp} = \frac{\text{Drag force}}{\text{dynamic pressure x projected area}} = \frac{2R}{\rho u_0^2 \text{ x projected area}}$$

The projected area is the area of the outline of the shape projected at right angles to the flow. The pressure acting at any point on the surface is p. The force exerted by the pressure on a small surface area is p dA. If the surface is inclined at an angle θ to the general direction of flow, the force is p cosθ dA. The total force is found by integrating all over the surface.

$$R = \oint p\cos\theta \, dA$$

The pressure distribution over the surface is often expressed in the form of a pressure coefficient defined as follows.

$$C_p = \frac{2(p - p_o)}{\rho u_o^2}$$

p_o is the static pressure of the undisturbed fluid, u_o is the velocity of the undisturbed fluid and $\frac{\rho u_o^2}{2}$ is the dynamic pressure of the stream.

Consider any streamline that is affected by the surface. Applying Bernoulli between an undisturbed point and another point on the surface, we have the following.

$$p_o + \frac{\rho u_o^2}{2} = p + \frac{\rho u^2}{2} \qquad p - p_o = \frac{\rho}{2}(u_o^2 - u^2)$$

$$C_p = \frac{2(p - p_o)}{\rho u_o^2} = \frac{2\left(\frac{\rho}{2}(u_o^2 - u^2)\right)}{\rho u_o^2} = \frac{(u_o^2 - u^2)}{u_o^2} = 1 - \frac{u^2}{u_o^2}$$

In order to calculate the drag force, further knowledge about the velocity distribution over the object would be needed and students are again recommended to study the classic textbooks on this subject. The equation shows that if $u<u_o$ then the pressure is positive and if $u>u_o$ the pressure is negative.

1.3 TOTAL DRAG

It has been explained that a body usually experiences both skin friction drag and form drag. The total drag is the sum of both. This applies to aeroplanes and ships as well as bluff objects such as cylinders and spheres. The drag force on a body is very hard to predict by purely theoretical methods. Much of the data about drag forces is based on experimental data and the concept of a drag coefficient is widely used.

The ***DRAG COEFFICIENT*** is denoted $\mathbf{C_D}$ and is defined by the following expression.

$$C_D = \frac{\text{Resistance force}}{\text{Dynamic pressure x projected Area}} = \frac{2R}{\rho u_o^2 \text{ x projected Area}}$$

WORKED EXAMPLE No.2

A cylinder 80 mm diameter and 200 mm long is placed in a stream of fluid moving at 0.5 m/s. The axis of the cylinder is normal to the direction of flow. The density of the fluid is 800 kg/m^3. The drag force is measured and found to be 30 N.

Calculate the drag coefficient.

At a point on the surface, the pressure is measured as 96 Pa above ambient.

Calculate the velocity at this point.

SOLUTION

Projected area = 0.08 x 0.2 = 0.016 m^2

R = 30 N, u_o = 0.5 m/s ρ = 800 kg/m^3

Dynamic pressure = $\rho u^2/2$ = 800 x 0.5^2/2 = 100 Pa

$$C_D = \frac{\text{Resistance force}}{\text{Dynamic pressure x projected Area}} = \frac{30}{100 \text{ x } 0.016} = 18.75$$

$$p - p_o = \frac{\rho}{2}(u_o^2 - u^2) \qquad 96 = \frac{800}{2}(0.5^2 - u^2) \qquad \frac{96 \text{ x } 2}{800} = (0.5^2 - u^2)$$

$$0.24 = 0.25 - u^2 \qquad u^2 = 0.01 \qquad u = 0.1 \text{ m/s}$$

1.4 APPLICATION TO A CYLINDER

The drag coefficient is defined as : $C_D = \dfrac{2R}{\rho u_o^2 \text{ x projected Area}}$ The projected Area is LD where L is the length and D the diameter. The drag around long cylinders is more predictable than for short cylinders and the following applies to long cylinders. Much research has been carried out into the relationship between drag and Reynolds number. $Re = \dfrac{\rho u_o d}{\mu}$ and d is the diameter of the cylinder.

At very small velocities, (Re <0.5) the fluid sticks to the cylinder all the way round and never separates from the cylinder. This produces a streamline pattern similar to that of an ideal fluid. The drag coefficient is at its highest and is mainly due to skin friction. The pressure distribution shows that the dynamic pressure is achieved at the front stagnation point and vacuum equal to three dynamic pressures exists at the top and bottom where the velocity is at its greatest.

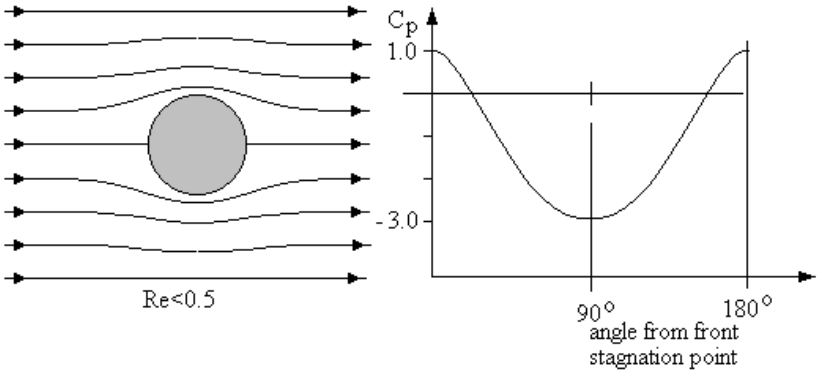

Fig. 7

As the velocity increases the boundary layer breaks away and eddies are formed behind. The drag becomes increasingly due to the pressure build up at the front and pressure drop at the back.

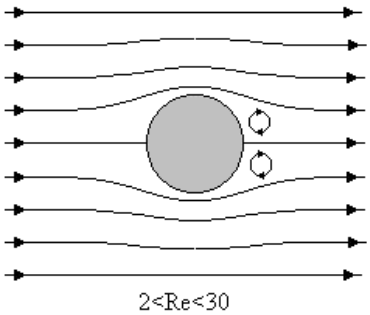

Fig. 8

Further increases in the velocity cause the eddies to elongate and the drag coefficient becomes nearly constant. The pressure distribution shows that ambient pressure exists at the rear of the cylinder.

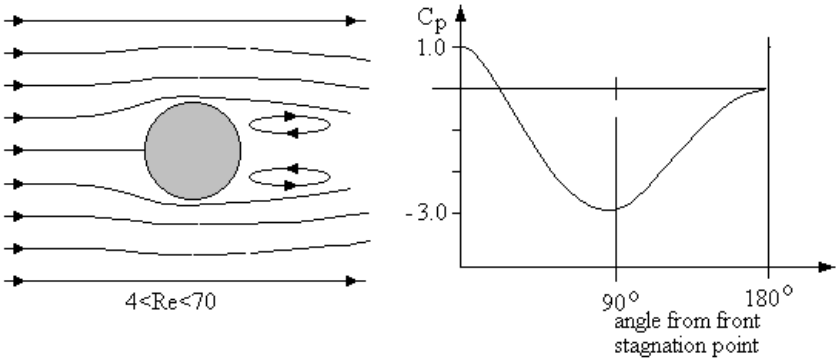

Fig. 9

At a Reynolds number of around 90 the vortices break away alternatively from the top and bottom of the cylinder producing a vortex street in the wake. The pressure distribution shows a vacuum at the rear.

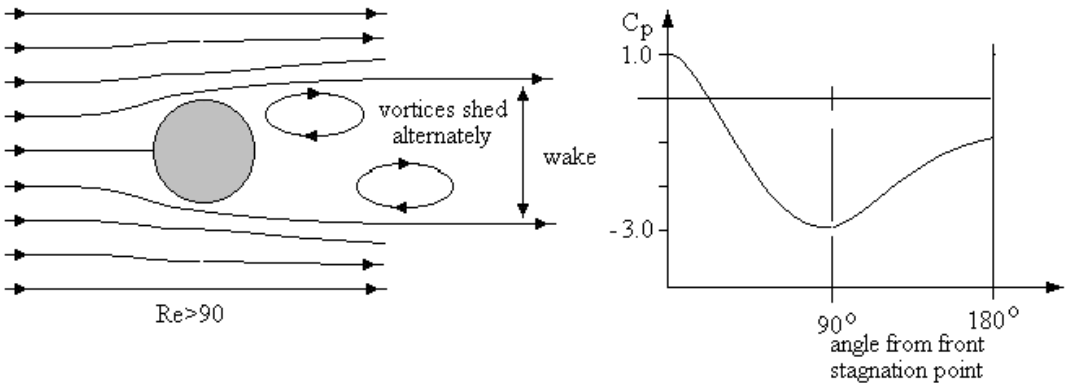

Fig.10

Up to a Reynolds number of about 2 x 10^5, the drag coefficient is constant with a value of approximately 1. The drag is now almost entirely due to pressure. Up to this velocity, the boundary layer has remained laminar but at higher velocities, flow within the boundary layer becomes turbulent. The point of separation moves back producing a narrow wake and a pronounced drop in the drag coefficient.

When the wake contains vortices shed alternately from the top and bottom, they produce alternating forces on the structure. If the structure resonates with the frequency of the vortex shedding, it may oscillate and produce catastrophic damage. This is a problem with tall chimneys and suspension bridges. The vortex shedding may produce audible sound.

Fig. 12 shows an approximate relationship between C_D and R_e for a cylinder and a sphere.

SELF ASSESSMENT EXERCISE No.2

1. Calculate the drag force for a cylindrical chimney 0.9 m diameter and 50 m tall in a wind blowing at 30 m/s given that the drag coefficient is 0.8. The density of the air is 1.2 kg/m^3. (19.44 N)

2. Using the graph (fig.12) to find the drag coefficient, determine the drag force per metre length acting on an overhead power line 30 mm diameter when the wind blows at 8 m/s. The density of air may be taken as 1.25 kg/m^3 and the kinematic viscosity as 1.5 x 10^{-5} m^2/s. (1.8 N).

1.5 APPLICATION TO SPHERES

The relationship between drag and Reynolds number is roughly the same as for a cylinder but it is more predictable. The Reynolds number is $Re = \frac{\rho u_o d}{\mu}$ where d is the diameter of the sphere. The projected area of a sphere of diameter d is ¼ πd^2. In this case, the expression for the drag coefficient is as follows. $C_D = \frac{8R}{\rho u^2 \times \pi d^2}$.

At very small Reynolds numbers (less than 0.2) the flow stays attached to the sphere all the way around and this is called Stokes flow. The drag is at its highest in this region.

As the velocity increases, the boundary layer separates at the rear stagnation point and moves forward. A toroidal vortex is formed. For 0.2<Re<500 the flow is called Allen flow.

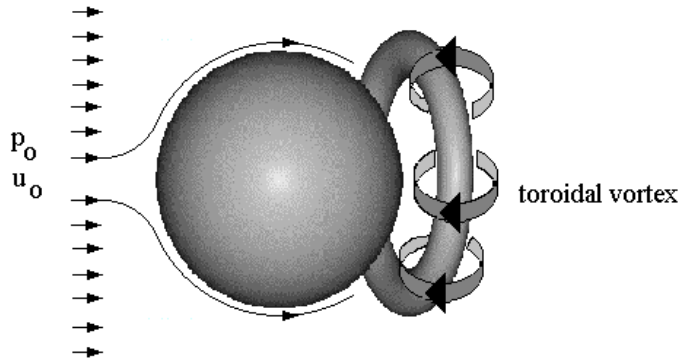

Fig.11

The breakaway or separation point reaches a stable position approximately 80° from the front stagnation point and this happens when R_e is about 1000. For 500<R_e the flow is called Newton flow. The drag coefficient remains constant at about 0.4. Depending on various factors, when R_e reaches 10^5 or larger, the boundary layer becomes totally turbulent and the separation point moves back again forming a smaller wake and a sudden drop in the drag coefficient to about 0.3. An empirical formula that covers the range $0.2 < R_e < 10^5$ is as follows.

$$C_D = \frac{24}{R_e} + \frac{6}{1+\sqrt{R_e}} + 0.4$$

Fig. 12 shows this approximate relationship between C_D and Re.

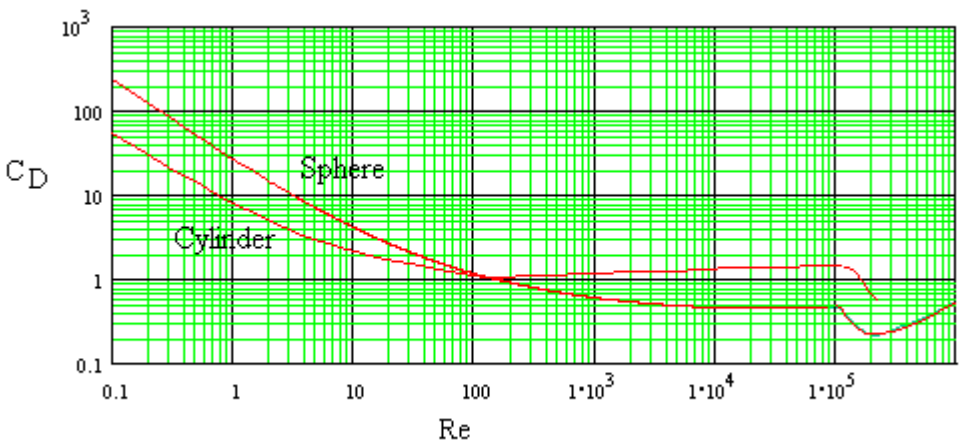

Fig.12

WORKED EXAMPLE No.3

A sphere diameter 40 mm moves through a fluid of density 750 kg/m^3 and dynamic viscosity 50 cP with a velocity of 0.6 m/s. Note 1 cP = 0.001 Ns/m^2.

Calculate the drag on the sphere.

Use the graph to obtain the drag coefficient.

SOLUTION

$$Re = \frac{\rho u d}{\mu} = \frac{750 \text{ x } 0.6 \text{ x } 0.04}{0.05} = 360$$

from the graph $C_D = 0.8$

$$C_D = \frac{2R}{\rho u^2 \text{ x projected Area}} \qquad \text{Projected area} = \pi\frac{d^2}{4} = \pi\frac{0.04^2}{4} = 1.2566 \text{ x } 10^{-3}\,\text{m}^2$$

$$R = \frac{C_D \rho u^2 A}{2} = \frac{0.8 \text{ x } 750 \text{ x } 0.6^2 \text{ x } 1.2566 \text{ x } 10^{-3}}{2} = 0.136 \text{ N}$$

1.6 TERMINAL VELOCITY

When a body falls under the action of gravity, a point is reached, where the drag force is equal and opposite to the force of gravity. When this condition is reached, the body stops accelerating and the terminal velocity reached. Small particles settling in a liquid are usually modelled as small spheres and the preceding work may be used to calculate the terminal velocity of small bodies settling in a liquid. A good application of this is the falling sphere viscometer described in chapter one.

For a body immersed in a liquid, the buoyant weight is W and this is equal to the viscous resistance R when the terminal velocity is reached.

R = W = volume x gravity x density difference $= \dfrac{\pi d^3 g(\rho_s - \rho_f)}{6}$

ρ_s = density of the sphere material
ρ_f = density of fluid
d = sphere diameter

STOKES' FLOW

For R_e<0.2 the flow is called Stokes flow and Stokes showed that $R = 3\pi d\mu u_t$
For a falling sphere viscometer, Stokes flow applies. Equating the drag force and the buoyant weight we get

$$3\pi\pi d\mu_t = \frac{\pi d^3 g(\rho_s - \rho_f)}{6}$$

$$\mu = \frac{d^2 g(\rho_s - \rho_f)}{18u_t}$$

The terminal velocity for Stokes flow is $u_t = \dfrac{d^2 g(\rho_s - \rho_f)}{18\mu}$

This formula assumes a fluid of infinite width but in a falling sphere viscometer, the liquid is squeezed between the sphere and the tube walls and additional viscous resistance is produced. The Faxen correction factor F is used to correct the result.

WORKED EXAMPLE No.4

The terminal velocity of a steel sphere falling in a liquid is 0.03 m/s. The sphere is 1 mm diameter and the density of the steel is 7830 kg/m^3. The density of the liquid is 800 kg/m^3. Calculate the dynamic and kinematic viscosity of the liquid.

SOLUTION

Assuming Stokes' flow the viscosity is found from the following equation.

$$\mu = \frac{d^2 g(\rho_s - \rho_f)}{18u_t} = \frac{0.001^2 \text{ x } 9.81 \text{ x } (7830 - 800)}{18 \text{ x } 0.03} = 0.1277 \text{ Ns/m}^2 = 127.7 \text{ cP}$$

$$\nu = \frac{\mu}{\rho_s} = \frac{0.1277}{800} = 0.0001596 \text{ m}^2/\text{s} = 159.6 \text{ cSt}$$

Check the Reynolds number. $R_e = \dfrac{\rho_f ud}{\mu} = \dfrac{800 \text{ x } 0.03 \text{ x } 0.001}{0.0547} = 0.188$

As this is smaller than 0.2 the assumption of Stokes' flow is correct.

ALLEN FLOW

For $0.2 < R_e < 500$ the flow is called Allen flow and the following expression gives the empirical relationship between drag and Reynolds number. $\mathbf{C_D=18.5R_e^{-0.6}}$

Equating for C_D gives the following result. $C_D = \dfrac{8R}{\rho_f u_t^2 \ \pi^2} = 18.5R_e^{-0.6}$

Substitute $R = \dfrac{\pi d^3 g(\rho_s - \rho_f)}{6}$

$$C_D = \frac{8dg(\rho_s - \rho_f)}{6\rho_f u_t^2} = 18.5R_e^{-0.6} = 18.5\left(\frac{\rho_f u_t d}{\mu}\right)^{-0.6}$$

$$\frac{8dg(\rho_s - \rho_f)}{6\rho_f u_t^2} = 18.5\left(\frac{\rho_f u_t d}{\mu}\right)^{-0.6}$$

From this equation the velocity u_t may be found.

NEWTON FLOW

For $500 < R_e < 10^5$ C_D takes on a constant value of 0.44.

Equating for C_D gives the following. $C_D = \dfrac{8R}{\rho_f u_t^2 \ \pi^2} = 0.44$

Substitute $R = \dfrac{\pi d^3 g(\rho_s - \rho_f)}{6}$

$$\frac{8dg(\rho_s - \rho_f)}{6\rho_f u_t^2} = 0.44 \qquad u_t = \sqrt{\frac{29.73dg(\rho_s - \rho_f)}{\rho_f}}$$

When solving the terminal velocity, you should always check the value of the Reynolds number to see if the criterion used is valid.

WORKED EXAMPLE No.5

Small glass spheres are suspended in an up wards flow of water moving with a mean velocity of 1 m/s. Calculate the diameter of the spheres. The density of glass is 2630 kg/m^3. The density of water is 1000 kg/m^3 and the dynamic viscosity is 1 cP.

SOLUTION

First, try the Newton flow equation. This is the easiest.

$$u_t = \sqrt{\frac{29.73d\,g(\rho_s - \rho_f)}{\rho_f}}$$

$$d = \frac{u_t^2 \rho_f}{29.73\,g(\rho_s - \rho_f)} = \frac{1^2 \text{ x } 1000}{29.73 \text{ x } 9.81 \text{ x } (2630 - 1000)} = 0.0021\,\text{m} \text{ or } 2.1\,\text{mm}$$

Check the Reynolds number.

$$R_e = \frac{\rho_f u_t d}{\mu} = \frac{1000 \text{ x } 1 \text{ x } 0.0021}{0.001} = 2103$$

The assumption of Newton flow was correct so the answer is valid.

WORKED EXAMPLE No.6

Repeat the last question but this time with a velocity of 0.05 m/s. Determine the type of flow that exists.

SOLUTION

If no assumptions are made, we should use the general formula $C_D = \frac{24}{R_e} + \frac{6}{1+\sqrt{R_e}} + 0.4$

$$R_e = \frac{\rho_f u_t d}{\mu} = \frac{1000 \text{ x } 0.05 \text{ x } d}{0.001} = 50\,000d$$

$$C_D = \frac{24}{R_e} + \frac{6}{1+\sqrt{R_e}} + 0.4 = \frac{24}{50000d} + \frac{6}{1+\sqrt{50000d}} + 0.4$$

$$C_D = 0.00048d^{-1} + \frac{6}{1+223.6d^{0.5}} + 0.4$$

$$C_D = \frac{8dg(\rho_s - \rho_f)}{6\rho_f u^2} = \frac{8d \text{ x } 9.81 \text{ x } (2630 - 1000)}{6 \text{ x } 1000 \text{ x } 0.05^2} = 8528.16d$$

$$8528.16d = 0.00048d^{-1} + \frac{6}{1+223.6d^{0.5}} + 0.4$$

This should be solved by any method known to you such as plotting two functions and finding the point of interception.

$$f1(d) = 8528.16d$$

$$f2(d) = 0.00048d^{-1} + \frac{6}{1+223.6d^{0.5}} + 0.4$$

The graph below gives an answer of d = 0.35 mm.

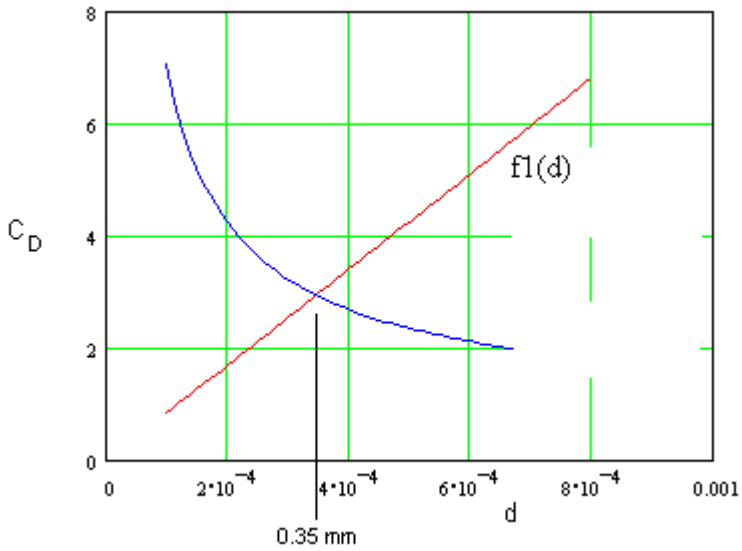

Fig. 13

Checking the Reynolds' number $R_e = \frac{\rho_f u_t d}{\mu} = \frac{1000 \text{ x } 0.05 \text{ x } 0.00035}{0.001} = 17.5$

This puts the flow in the Allen's flow section.

SELF ASSESSMENT EXERCISE No.3

1. a. Explain the term Stokes flow and terminal velocity.

b. Show that the terminal velocity of a spherical particle with Stokes flow is given by the formula
$u = d^2g(\rho_s - \rho_f)/18\mu$

Go on to show that $C_D=24/R_e$

2. Calculate the largest diameter sphere that can be lifted upwards by a vertical flow of water moving at 1 m/s. The sphere is made of glass with a density of 2630 kg/m^3. The water has a density of 998 kg/m^3 and a dynamic viscosity of 1 cP. (20.7 mm)

3. Using the same data for the sphere and water as in Q2, calculate the diameter of the largest sphere that can be lifted upwards by a vertical flow of water moving at 0.5 m/s. (5.95 mm).

4. Using the graph (fig. 12) to obtain the drag coefficient of a sphere, determine the drag on a totally immersed sphere 0.2 m diameter moving at 0.3 m/s in sea water. The density of the water is 1025 kg/m^3 and the dynamic viscosity is 1.05 x 10^{-3} Ns/m^2. (0.639 N).

5. A glass sphere of diameter 1.5 mm and density 2 500 kg/m^3 is allowed to fall through water under the action of gravity. The density of the water is 1000 kg/m^3 and the dynamic viscosity is 1 cP.

Calculate the terminal velocity assuming the drag coefficient is
$C_D = 24\ R_e^{-1}\ (1+ 0.15R_e^{0.687})$ (Ans. 0.215 m/s