

## Unit 41: Fluid Mechanics

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Unit code: T/601/1445

QCF Level: 4

Credit value: 15

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### OUTCOME 1 STATIC FLUID SYSTEMS

#### TUTORIAL 1 - HYDROSTATICS

#### 1. Be able to determine the behavioural characteristics and parameters of static fluid systems

*Immersed surfaces:* rectangular and circular surfaces e.g. retaining walls, tank sides, sluice gates, inspection covers, valve flanges

*Centre of pressure:* use of parallel axis theorem for immersed rectangular and circular immersed surfaces

*Devices:* hydraulic presses; hydraulic jacks; hydraulic accumulators; braking systems; determine outputs for given inputs

On completion of this tutorial you should be able to do the following.

- Define the main *fundamental properties* of liquids.
- Explain and calculate *hydrostatic pressure*.
- Calculate the hydrostatic forces and moments on *submerged surfaces*.
- Explain and solve problems involving simple *hydrostatic devices*.

Before you start you should make sure that you fully understand first and second moments of area. If you are not familiar with this, you should do the pre-requisite tutorial preceding this one. Let's start this tutorial by studying the fundamental properties of liquids.

## 1. SOME FUNDAMENTAL STUDIES

### 1.1 IDEAL LIQUIDS

An ideal liquid is defined as follows.

It is *INVISCID*. This means that molecules require no force to separate them. The topic is covered in detail later.

It is *INCOMPRESSIBLE*. This means that it would require an infinite force to reduce the volume of the liquid.

### 1.2 REAL LIQUIDS

#### VISCOSITY

Real liquids have *VISCOSITY*. This means that the molecules tend to stick to each other and to any surface with which they come into contact. This produces fluid friction and energy loss when the liquid flows over a surface. Viscosity defines how easily a liquid flows. The lower the viscosity, the easier it flows.

#### BULK MODULUS

Real liquids are compressible and this is governed by the *BULK MODULUS K*. This is defined as follows.

$$K = V\Delta p/\Delta V$$

$\Delta p$  is the increase in pressure,  $\Delta V$  is the reduction in volume and  $V$  is the original volume.

DENSITY Density  $\rho$  relates the mass and volume such that  $\rho = m/V$  kg/m<sup>3</sup>

#### PRESSURE

Pressure is the result of compacting the molecules of a fluid into a smaller space than it would otherwise occupy. Pressure is the force per unit area acting on a surface. The unit of pressure is the N/m<sup>2</sup> and this is called a *PASCAL*. The Pascal is a small unit of pressure so higher multiples are common.

$$1 \text{ kPa} = 10^3 \text{ N/m}^2$$

$$1 \text{ MPa} = 10^6 \text{ N/m}^2$$

Another common unit of pressure is the *bar* but this is not an SI unit.

$$1 \text{ bar} = 10^5 \text{ N/m}^2$$

$$1 \text{ mb} = 100 \text{ N/m}^2$$

#### GAUGE AND ABSOLUTE PRESSURE

Most pressure gauges are designed only to measure and indicate the pressure of a fluid above that of the surrounding atmosphere and indicate zero when connected to the atmosphere. These are called *gauge pressures* and are normally used. Sometimes it is necessary to add the atmospheric pressure onto the gauge reading in order to find the true or *absolute pressure*.

Absolute pressure = gauge pressure + atmospheric pressure.

Standard atmospheric pressure is 1.013 bar.

## 2. HYDROSTATIC FORCES

When you have completed this section, you should be able to do the following.

- Calculate the pressure due to the depth of a liquid.
- Calculate the total force on a vertical surface.
- Define and calculate the position of the centre of pressure for various shapes.
- Calculate the turning moments produced on vertically immersed surfaces.
- Explain the principles of simple hydraulic devices.
- Calculate the force and movement produced by simple hydraulic equipment.

### 2.1 HYDROSTATIC PRESSURE

#### 2.1.1 PRESSURE INSIDE PIPES AND VESSELS

Pressure results when a liquid is compacted into a volume. The pressure inside vessels and pipes produce stresses and strains as it tries to stretch the material. An example of this is a pipe with flanged joints. The pressure in the pipe tries to separate the flanges. The force is the product of the pressure and the bore area.

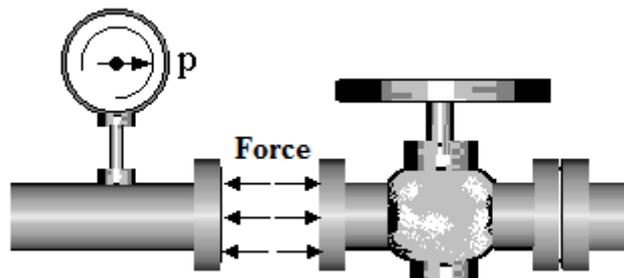


Fig.1

#### WORKED EXAMPLE No. 1

Calculate the force trying to separate the flanges of a valve (Fig.1) when the pressure is 2 MPa and the pipe bore is 50 mm.

#### SOLUTION

Force = pressure x bore area

Bore area =  $\pi D^2/4 = \pi \times 0.05^2/4 = 1.963 \times 10^{-3} \text{ m}^2$

Pressure =  $2 \times 10^6 \text{ Pa}$

Force =  $2 \times 10^6 \times 1.963 \times 10^{-3} = 3.927 \times 10^3 \text{ N}$  or **3.927 kN**

## 2.1.2 PRESSURE DUE TO THE WEIGHT OF A LIQUID

Consider a tank full of liquid as shown. The liquid has a total weight  $W$  and this bears down on the bottom and produces a pressure  $p$ . Pascal showed that the pressure in a liquid always acts normal (at  $90^\circ$ ) to the surface of contact so the pressure pushes down onto the bottom of the tank. He also showed that the pressure at a given point acts equally in all directions so the pressure also pushes up on the liquid above it and sideways against the walls.

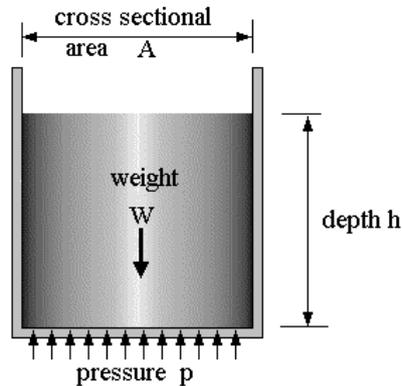


Fig. 2

The volume of the liquid is  $V = A h \text{ m}^3$

The mass of liquid is hence  $m = \rho V = \rho A h \text{ kg}$

The weight is obtained by multiplying by the gravitational constant  $g$ .

$$W = mg = \rho A h g \text{ Newton}$$

The pressure on the bottom is the weight per unit area  $p = W/A \text{ N/m}^2$

It follows that the pressure at a depth  $h$  in a liquid is given by the following equation.

$$p = \rho g h$$

The unit of pressure is the  $\text{N/m}^2$  and this is called a **PASCAL**. The Pascal is a small unit of pressure so higher multiples are common.

### WORKED EXAMPLE 2

Calculate the pressure and force on an inspection hatch 0.75 m diameter located on the bottom of a tank when it is filled with oil of density  $875 \text{ kg/m}^3$  to a depth of 7 m.

### SOLUTION

The pressure on the bottom of the tank is found as follows.  $p = \rho g h$

$$\rho = 875 \text{ kg/m}^3$$

$$g = 9.81 \text{ m/s}^2$$

$$h = 7 \text{ m}$$

$$p = 875 \times 9.81 \times 7 = 60086 \text{ N/m}^2 \text{ or } \mathbf{60.086 \text{ kPa}}$$

The force is the product of pressure and area.

$$A = \pi D^2/4 = \pi \times 0.75^2/4 = 0.442 \text{ m}^2$$

$$F = p A = 60.086 \times 10^3 \times 0.442 = 26.55 \times 10^3 \text{ N or } \mathbf{26.55 \text{ Kn}}$$

### 2.1.3 PRESSURE HEAD

When  $h$  is made the subject of the formula, it is called the pressure head.  $h = p/\rho g$

Pressure is often measured by using a column of liquid. Consider a pipe carrying liquid at pressure  $p$ . If a small vertical pipe is attached to it, the liquid will rise to a height  $h$  and at this height, the pressure at the foot of the column is equal to the pressure in the pipe.

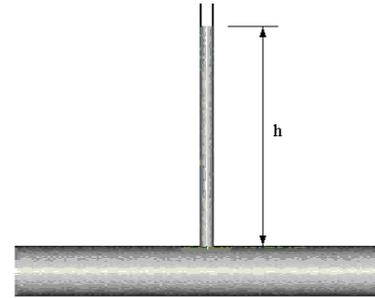


Fig.3

This principle is used in barometers to measure atmospheric pressure and manometers to measure gas pressure.

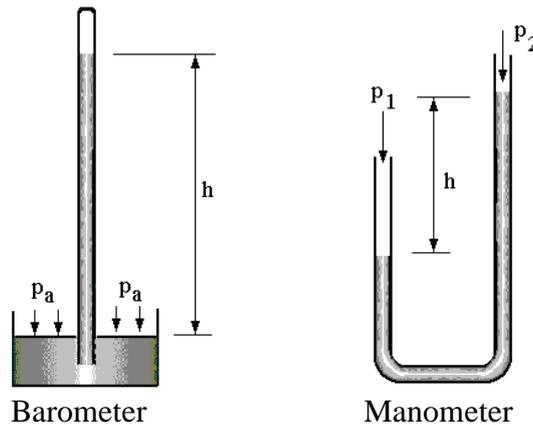


Fig.4

In the manometer, the weight of the gas is negligible so the height  $h$  represents the difference in the pressures  $p_1$  and  $p_2$ .

$$p_1 - p_2 = \rho g h$$

In the case of the barometer, the column is closed at the top so that  $p_2 = 0$  and  $p_1 = p_a$ . The height  $h$  represents the atmospheric pressure. Mercury is used as the liquid because it does not evaporate easily at the near total vacuum on the top of the column.

$$P_a = \rho g h$$

#### WORKED EXAMPLE No.3

A manometer (fig.4) is used to measure the pressure of gas in a container. One side is connected to the container and the other side is open to the atmosphere. The manometer contains oil of density  $750 \text{ kg/m}^3$  and the head is 50 mm. Calculate the gauge pressure of the gas in the container.

#### SOLUTION

$$p_1 - p_2 = \rho g h = 750 \times 9.81 \times 0.05 = 367.9 \text{ Pa}$$

Since  $p_2$  is atmospheric pressure, this is the gauge pressure.  $p_2 = 367.9 \text{ Pa (gauge)}$

### **SELF ASSESSMENT EXERCISE No.1**

1. A mercury barometer gives a pressure head of 758 mm. The density is 13 600 kg/m<sup>3</sup>. Calculate the atmospheric pressure in bar. (1.0113 bar)
2. A manometer (fig.4) is used to measure the pressure of gas in a container. One side is connected to the container and the other side is open to the atmosphere. The manometer contains water of density 1000 kg/m<sup>3</sup> and the head is 250 mm. Calculate the gauge pressure of the gas in the container. (2.452.5 kPa)
3. Calculate the pressure and force on a horizontal submarine hatch 1.2 m diameter when it is at a depth of 800 m in seawater of density 1030 kg/m<sup>3</sup>. (8.083 MPa and 9.142 MN)

### 3. FORCES ON SUBMERGED SURFACES

#### 3.1 TOTAL FORCE

Consider a vertical area submerged below the surface of liquid as shown.

The area of the elementary strip is  
 $dA = B dy$

You should already know that the pressure at depth  $h$  in a liquid is given by the equation  $p = \rho gh$  where  $\rho$  is the density and  $h$  the depth.

In this case, we are using  $y$  to denote depth so  $p = \rho gy$

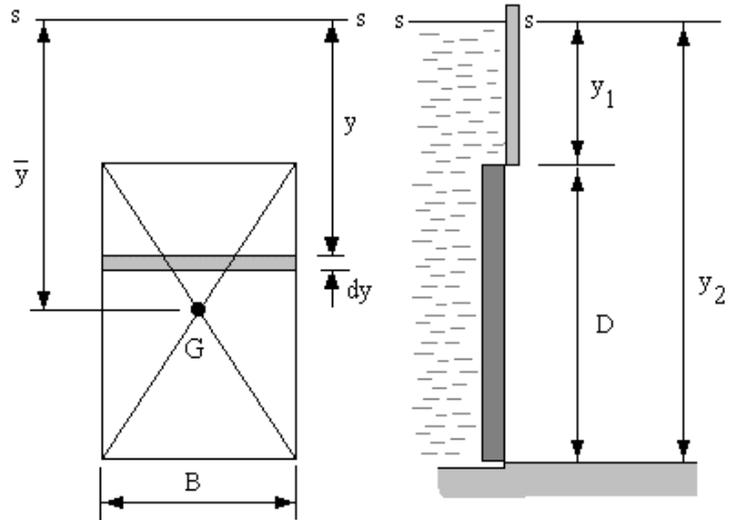


Fig.5

The force on the strip due to this pressure is  $dF = p dA = \rho B gy dy$

The total force on the surface due to pressure is denoted  $R$  and it is obtained by integrating this expression between the limits of  $y_1$  and  $y_2$ .

It follows that 
$$R = \rho g B \left( \frac{y_2^2 - y_1^2}{2} \right)$$

This may be factorised. 
$$R = \rho g B \frac{(y_2 - y_1)(y_2 + y_1)}{2}$$

$(y_2 - y_1) = D$  so  $B(y_2 - y_1) = BD = \text{Area of the surface } A$

$(y_2 + y_1)/2$  is the distance from the free surface to the centroid  $y$ .

It follows that the total force is given by the expression

$$R = \rho g A y$$

The term  $Ay$  is the first moment of area and in general, the total force on a submerged surface is

$$R = \rho g \times \text{1st moment of area about the free surface.}$$

### 3.2

### CENTRE OF PRESSURE

The centre of pressure is the point at which the total force may be assumed to act on a submerged surface. Consider the diagram again. The force on the strip is  $dF$  as before. This force produces a turning moment with respect to the free surface  $s - s$ . The turning moment due to  $dF$  is as follows.

$$dM = y dF = \rho g B y^2 dy$$

The total turning moment about the surface due to pressure is obtained by integrating this expression between the limits of  $y_1$  and  $y_2$ .

$$M = \int_{y_1}^{y_2} \rho g B y^2 dy = \rho g B \int_{y_1}^{y_2} y^2 dy$$

$$\text{By definition } I_{ss} = B \int_{y_1}^{y_2} y^2 dy$$

Hence

$$M = \rho g I_{ss}$$

This moment must also be given by the total force  $R$  multiplied by some distance  $h$ . The position at depth  $h$  is called the **CENTRE OF PRESSURE**.  $h$  is found by equating the moments.

$$M = h R = h \rho g A \bar{y} = \rho g I_{ss}$$

$$\bar{h} = \frac{\rho g I_{ss}}{\rho g A \bar{y}} = \frac{I_{ss}}{A \bar{y}}$$

$$\bar{h} = \frac{2^{\text{nd}} \text{ moment of area}}{1^{\text{st}} \text{ moment of area}} \text{ about } s - s$$

In order to be competent in this work, you should be familiar with the parallel axis theorem (covered in part 1) because you will need it to solve 2<sup>nd</sup> moments of area about the free surface. The rule is as follows.

$$I_{ss} = I_{gg} + A y^2$$

$I_{ss}$  is the 2<sup>nd</sup> moment about the free surface and  $I_{gg}$  is the 2<sup>nd</sup> moment about the centroid.

You should be familiar with the following standard formulae for 2<sup>nd</sup> moments about the centroid.

$$\text{Rectangle } I_{gg} = BD^3/12$$

$$\text{Rectangle about its edge } I = BD^3/3$$

$$\text{Circle } I_{gg} = \pi D^4/64$$

### WORKED EXAMPLE No.4

Show that the centre of pressure on a vertical retaining wall is at  $2/3$  of the depth. Go on to show that the turning moment produced about the bottom of the wall is given by the expression  $\rho gh^3/6$  for a unit width.

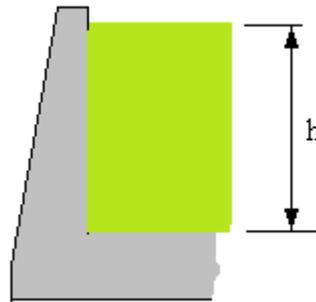


Fig.6

### SOLUTION

For a given width B, the area is a rectangle with the free surface at the top edge.

$$\bar{y} = \frac{h}{2} \quad A = bh$$

$$1^{\text{st}} \text{ moment of area about the top edge is } A\bar{y} = B \frac{h^2}{2}$$

$$2^{\text{nd}} \text{ moment of area about the top edge is } B \frac{h^3}{3}$$

$$\bar{h} = \frac{2^{\text{nd}} \text{ moment}}{1^{\text{st}} \text{ moment}} = \frac{B \frac{h^3}{3}}{B \frac{h^2}{2}}$$

$$\bar{h} = \frac{2h}{3}$$

It follows that the centre of pressure is  $h/3$  from the bottom.

The total force is  $R = \rho gAy = \rho gBh^2/2$  and for a unit width this is  $\rho gh^2/2$

The moment about the bottom is  $R \times h/3 = (\rho gh^2/2) \times h/3 = \rho gh^3/6$

### SELF ASSESSMENT EXERCISE No.2

1. A vertical retaining wall contains water to a depth of 20 metres. Calculate the turning moment about the bottom for a unit width. Take the density as  $1000 \text{ kg/m}^3$ .  
(13.08 MNm)
2. A vertical wall separates seawater on one side from fresh water on the other side. The seawater is 3.5 m deep and has a density of  $1030 \text{ kg/m}^3$ . The fresh water is 2 m deep and has a density of  $1000 \text{ kg/m}^3$ . Calculate the turning moment produced about the bottom for a unit width.  
(59.12 kNm)

### WORKED EXAMPLE No.5

A concrete wall retains water and has a hatch blocking off an outflow tunnel as shown. Find the total force on the hatch and the position of the centre of pressure. Calculate the total moment about the bottom edge of the hatch. The water density is  $1000 \text{ kg/m}^3$ .

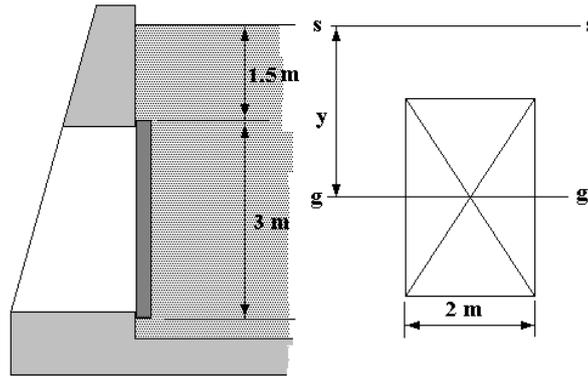


Fig.7

### SOLUTION

Total force =  $R = \rho g A y$

For the rectangle shown  $y = (1.5 + 3/2) = 3 \text{ m}$ .  $A = 2 \times 3 = 6 \text{ m}^2$ .

$R = 1000 \times 9.81 \times 6 \times 3 = 176580 \text{ N}$  or  $176.58 \text{ kN}$

$h = \text{2nd mom. of Area} / \text{1st mom. of Area}$

$1^{\text{st}}$  moment of Area =  $Ay = 6 \times 3 = 18 \text{ m}^3$ .

$2^{\text{nd}}$  mom of area =  $I_{SS} = (BD^3/12) + Ay^2 = (2 \times 3^3/12) + (6 \times 3^2)$

$I_{SS} = 4.5 + 54 = 58.5 \text{ m}^4$ .

$h = 58.5/18 = 3.25 \text{ m}$

The distance from the bottom edge is  $x = 4.5 - 3.25 = 1.25 \text{ m}$

Moment about the bottom edge is  $= Rx = 176.58 \times 1.25 = 220.725 \text{ kNm}$ .

### WORKED EXAMPLE No.6

Find the force required at the top of the circular hatch shown in order to keep it closed against the water pressure outside. The density of the water is  $1030 \text{ kg/m}^3$ .

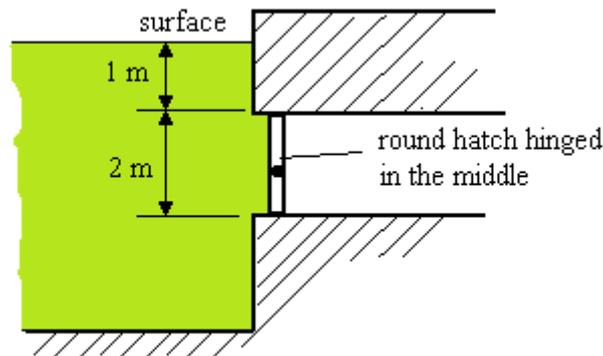


Fig. 8

$y = 2 \text{ m}$  from surface to middle of hatch.

$$\text{Total Force} = R = \rho g A y = 1030 \times 9.81 \times (\pi \times 2^2/4) \times 2 = 63487 \text{ N or } 63.487 \text{ kN}$$

Centre of Pressure  $h = 2^{\text{nd}} \text{ moment} / 1^{\text{st}} \text{ moment}$

$2^{\text{nd}}$  moment of area.

$$I_{SS} = I_{gg} + Ay^2 = (\pi \times 2^4/64) + (\pi \times 2^2/4) \times 2^2$$

$$I_{SS} = 13.3518 \text{ m}^4.$$

$1^{\text{st}}$  moment of area

$$Ay = (\pi \times 2^2/4) \times 2 = 6.283 \text{ m}^3.$$

Centre of pressure.

$$h = 13.3518 / 6.283 = 2.125 \text{ m}$$

This is the depth at which, the total force may be assumed to act. Take moments about the hinge.

$F =$  force at top.

$R =$  force at centre of pressure which is  $0.125 \text{ m}$  below the hinge.

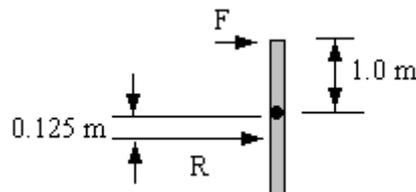


Fig. 9

$$\text{For equilibrium } F \times 1 = 63.487 \times 0.125$$

$$F = 7.936 \text{ kN}$$

### WORKED EXAMPLE No.7

The diagram shows a hinged circular vertical hatch diameter  $D$  that flips open when the water level outside reaches a critical depth  $h$ . Show that for this to happen the hinge must be located at a position  $x$  from the bottom given by the formula  $x = \frac{D}{2} \left\{ \frac{8h - 5D}{8h - 4D} \right\}$

Given that the hatch is 0.6 m diameter, calculate the position of the hinge such that the hatch flips open when the depth reaches 4 metres.

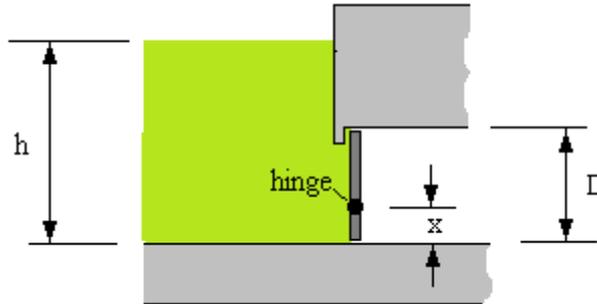


Fig.10

### SOLUTION

The hatch will flip open as soon as the centre of pressure rises above the hinge creating a clockwise turning moment. When the centre of pressure is below the hinge, the turning moment is anticlockwise and the hatch is prevented from turning in that direction. We must make the centre of pressure at position  $x$ .

$$\bar{y} = h - \frac{D}{2}$$

$$\bar{h} = h - x$$

$$\bar{h} = \frac{\text{second moment of area}}{\text{first moment of area}} \text{ about the surface}$$

$$\bar{h} = \frac{I_{gg} + A\bar{y}^2}{A\bar{y}} = \frac{\frac{\pi D^4}{64} + \frac{\pi D^2}{4} \bar{y}^2}{\frac{\pi D^2}{4} \bar{y}} = \frac{D^2}{16\bar{y}} + \bar{y}$$

Equate for  $\bar{h}$

$$\frac{D^2}{16\bar{y}} + \bar{y} = h - x$$

$$x = h - \frac{D^2}{16\bar{y}} - \bar{y} = h - \frac{D^2}{16\left(h - \frac{D}{2}\right)} - \left(h - \frac{D}{2}\right)$$

$$x = -\frac{D^2}{(16h - 8D)} + \frac{D}{2} = \frac{D}{2} - \frac{D^2}{(16h - 8D)}$$

$$x = \frac{D}{2} \left\{ 1 - \frac{D}{8h - 4D} \right\} = \frac{D}{2} \left\{ \frac{8h - 4D - D}{8h - 4D} \right\} = \frac{D}{2} \left\{ \frac{8h - 5D}{8h - 4D} \right\}$$

Putting  $D = 0.6$  and  $h = 4$  we get  $x = 0.5 \text{ m}$

### SELF ASSESSMENT EXERCISE No.3

1. A circular hatch is vertical and hinged at the bottom. It is 2 m diameter and the top edge is 2 m below the free surface. Find the total force, the position of the centre of pressure and the force required at the top to keep it closed. The density of the water is  $1000 \text{ kg/m}^3$ .  
(92.469 kN, 3.08 m, 42.5 kN)
2. A large tank of sea water has a door in the side 1 m square. The top of the door is 5 m below the free surface. The door is hinged on the bottom edge. Calculate the force required at the top to keep it closed. The density of the sea water is  $1036 \text{ kg/m}^3$ .  
(27.11 N)
3. A culvert in the side of a reservoir is closed by a vertical rectangular gate 2 m wide and 1 m deep as shown in fig. 11. The gate is hinged about a horizontal axis which passes through the centre of the gate. The free surface of water in the reservoir is 2.5 m above the axis of the hinge. The density of water is  $1000 \text{ kg/m}^3$ .

Assuming that the hinges are frictionless and that the culvert is open to atmosphere, determine

- (i) the force acting on the gate when closed due to the pressure of water. (55.897 kN)
- (ii) the moment to be applied about the hinge axis to open the gate. (1635 Nm)

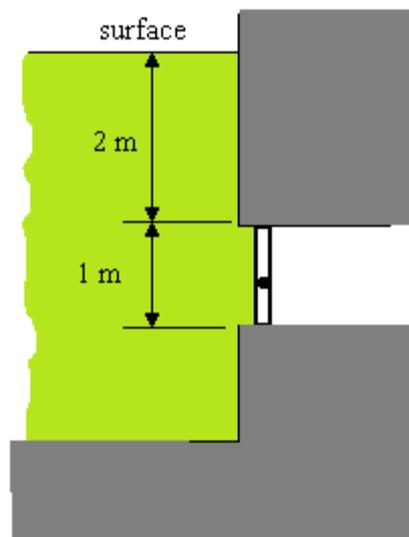


Fig.11

4. The diagram shows a rectangular vertical hatch breadth  $B$  and depth  $D$ . The hatch flips open when the water level outside reaches a critical depth  $h$ . Show that for this to happen the hinge must be located at a position  $x$  from the bottom given by the formula

$$x = \frac{D}{2} \left\{ \frac{6h - 4D}{6h - 3D} \right\}$$

Given that the hatch is 1 m deep, calculate the position of the hinge such that the hatch flips open when the depth reaches 3 metres. (0.466 m)

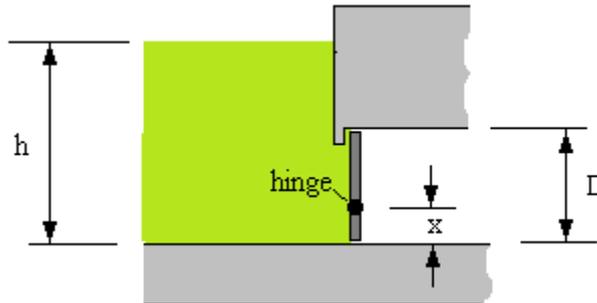


Fig.12

5. Fig.13 shows an L shaped spill gate that operates by pivoting about hinge O when the water level in the channel rises to a certain height  $H$  above O. A counterweight  $W$  attached to the gate provides closure of the gate at low water levels. With the channel empty the force at sill S is 1.635 kN. The distance  $l$  is 0.5m and the gate is 2 m wide.

Determine the magnitude of  $H$ .

(i) when the gate begins to open due to the hydrostatic load. (1 m)

(ii) when the force acting on the sill becomes a maximum. What is the magnitude of this force. (0.5 m)

Assume the effects of friction are negligible.

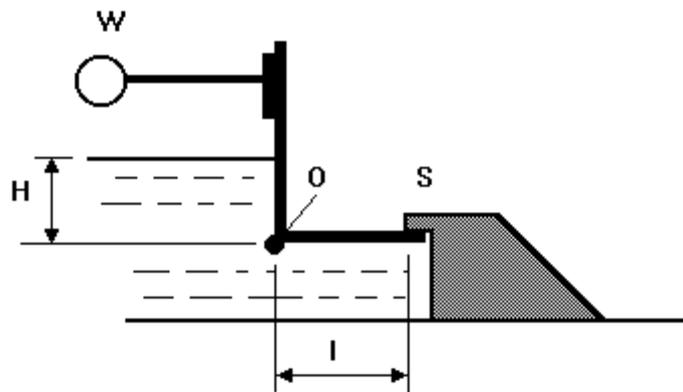


Fig.13

## 4 HYDROSTATIC DEVICES

In this section, you will study the following.

- Pascal's Laws.
- A simple hydraulic jack.
- Basic power hydraulic system.

### 4.1 PASCAL'S LAWS

- *Pressure always acts normal to the surface of contact.*

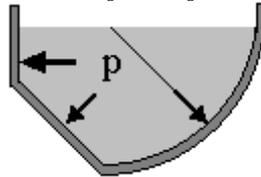


Fig.1.4

- *The force of the molecules pushing on neighbouring molecules is equal in all directions so long as the fluid is static (still).*
- *The force produced by a given pressure in a static fluid is the same on all equal areas.*

These statements are the basis of **PASCAL'S LAWS** and the unit of pressure is named after Pascal. These principles are used in simple devices giving force amplification.

### 4.2 CAR BRAKE

A simple hydraulic braking system is shown in fig.15

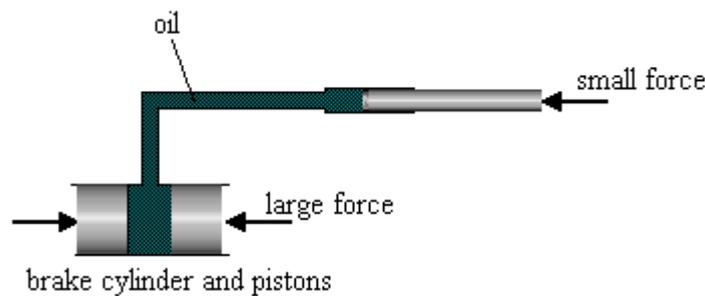


Fig.15

The small force produced by pushing the small piston produces pressure in the oil. The pressure acts on the larger pistons in the brake cylinder and produces a large force on the pistons that move the brake pads or shoes.

Fig. 16 shows the basis of a simple hydraulic jack.

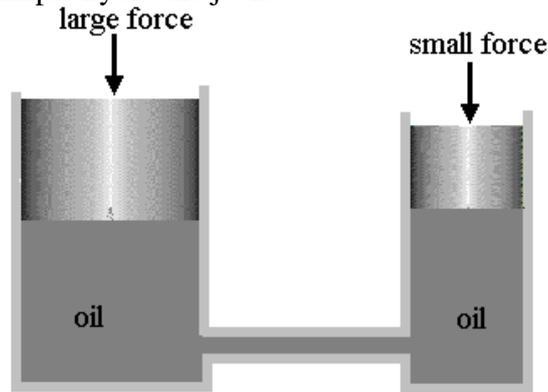


Fig.16

The small force pushing on the small piston produces a pressure in the oil. This pressure acts on the large piston and produces a larger force. This principle is used in most hydraulic systems but many modifications are needed to produce a really useful machine. In both the above examples, force is amplified because the same pressure acts on different piston areas. In order to calculate the force ratio we use the formula  $p = F/A$ .

### **FORCE RATIO**

Let the small piston have an area  $A_1$  and the large piston an area  $A_2$ . The force on the small piston is  $F_1$  and on the large piston is  $F_2$ .

The pressure is the same for both pistons so  $p = F_1/A_1 = F_2/A_2$

From this the force amplification ratio is  $F_2/F_1 = A_2/A_1$

Note the area ratio is not the same as the diameter ratio. If the diameters are  $D_1$  and  $D_2$  then the ratio becomes  $F_2/F_1 = A_2/A_1 = D_2^2/D_1^2$

### **MOVEMENT RATIO**

The simple hydraulic jack produces force amplification but it is not possible to produce an increase in the energy, power or work. It follows that if no energy is lost nor gained, the large piston must move a smaller distance than the small piston.

Remember that work done is force x distance moved.  $W = F \times$

Let the small piston move a distance  $x_1$  and the large piston  $x_2$ . The work input at the small piston is equal to the work out at the large piston so

$$F_1 x_1 = F_2 x_2 \quad \text{Substituting that } F_1 = pA_1 \text{ and } F_2 = pA_2$$

$$pA_1 x_1 = pA_2 x_2 \quad \text{or } A_1 x_1 = A_2 x_2$$

The movement of the small piston as a ratio to the movement of the large piston is then  $x_1/x_2 = A_2/A_1 = \text{area ratio}$

#### 4.4 PRACTICAL LIFTING JACK

A practical hydraulic jack uses a small pumping piston as shown. When this moves forward, the non-return valve NRV1 opens and NRV2 closes. Oil is pushed under the load piston and moves it up. When the piston moves back, NRV1 closes and NRV2 opens and replenishes the pumping cylinder from the reservoir. The oil release valve, when open, allows the oil under the load cylinder to return to the reservoir and lowers the load.

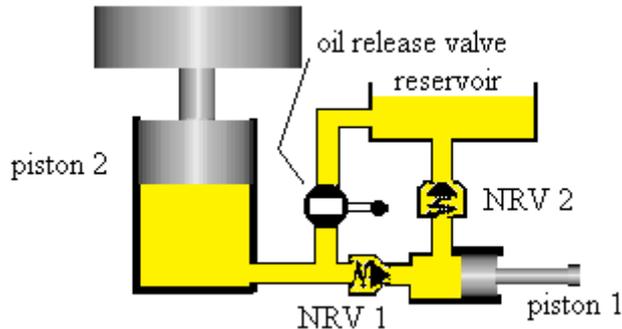


Fig.17

#### WORKED EXAMPLE No.8

A simple lifting jack has a pump piston 12 mm diameter and a load piston 60 mm diameter. Calculate the force needed on the pumping piston to raise a load of 8 kN. Calculate the pressure in the oil.

#### SOLUTION

$$\text{Force Ratio} = A_2/A_1 = D_2^2/D_1^2 = (60/12)^2 = 25$$

Force on the pumping piston is 1/25 of the load.

$$F_1 = 8 \times 10^3 / 25 = 320 \text{ N}$$

Pressure = Force/Area. Choosing the small piston

$$A_1 = \pi D_1^2/4 = \pi \times 0.012^2/4 = 113.1 \times 10^{-6} \text{ m}^2$$

$$p = F/A = 320 / 113.1 \times 10^{-6} = 2.829 \times 10^6 \text{ Pa or } 2.829 \text{ MPa}$$

Check using the large piston data.

$$F_2 = 8 \times 10^3 \text{ N}$$

$$A_2 = \pi D_2^2/4 = \pi \times 0.06^2/4 = 2.827 \times 10^{-3} \text{ m}^2$$

$$p = F/A = 8 \times 10^3 / 2.827 \times 10^{-3} = 2.829 \times 10^6 \text{ Pa or } 2.829 \text{ MPa}$$

### SELF ASSESSMENT EXERCISE No.4

1. Calculate  $F_1$  and  $x_2$  for the case shown below. (83.3 N, 555 mm)

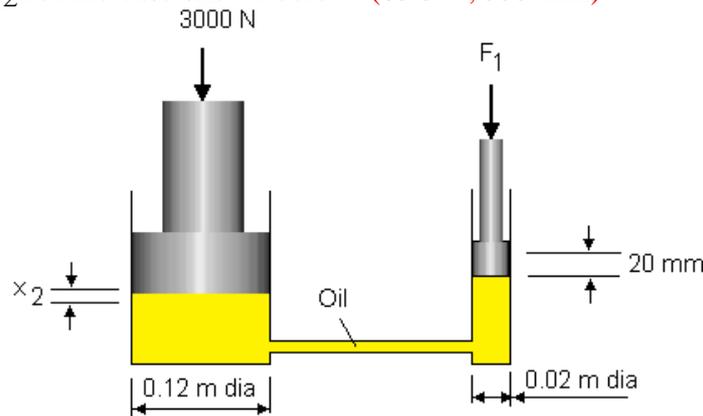


Fig.18

2. Calculate  $F_1$  and  $x_1$  for the case shown below. (312.5 kN, 6.25 mm)

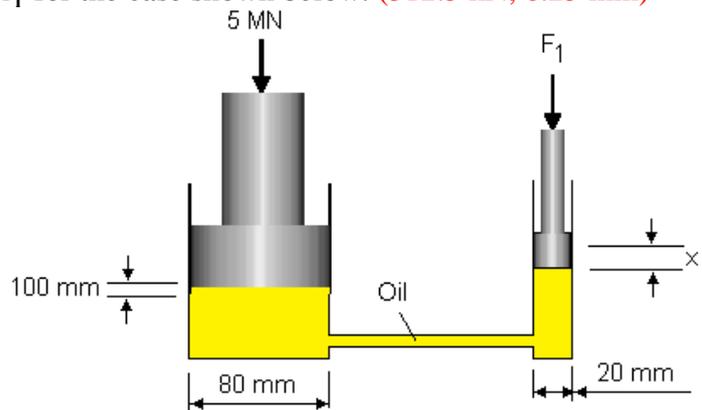


Fig.19

### 4.5 CYLINDERS

Cylinders are linear actuators that convert fluid power into mechanical power. They are also known as JACKS or RAMS. Hydraulic cylinders are used at high pressures. They produce large forces with precise movement. They are constructed of strong materials such as steel and designed to withstand large forces.

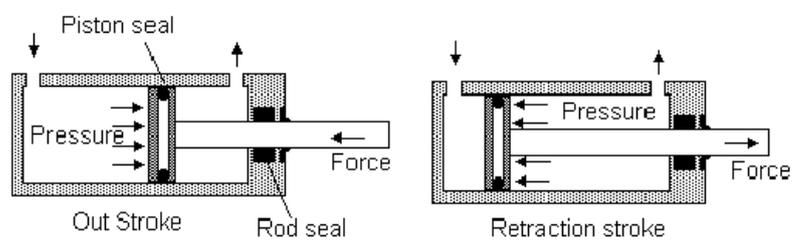


Fig.20

The diagram shows a double acting cylinder. Assume that the pressure on the other side of the piston is atmospheric. In this case, if we use gauge pressure, we need not worry about the atmospheric pressure.  $A$  is the full area of the piston. If the pressure is acting on the rod side, then the area is  $(A - a)$  where  $a$  is the area of the rod.

$$F = pA \quad \text{on the full area of piston.}$$

$$F = p(A-a) \quad \text{on the rod side.}$$

This force acting on the load is often less because of friction between the piston and piston rod and the seals.

### **WORKED EXAMPLE No.9**

A single rod hydraulic cylinder must pull with a force of 5 kN. The piston is 75 mm diameter and the rod is 30 mm diameter. Calculate the pressure required.

### **SOLUTION**

The pressure is required on the annular face of the piston in order to pull. The area acted on by the pressure is A - a

$$A = \pi \times 0.075^2 / 4 = 4.418 \times 10^{-3} \text{ m}^2$$

$$a = \pi \times 0.03^2 / 4 = 706.8 \times 10^{-6} \text{ m}^2$$

$$A - a = 3.711 \times 10^{-3} \text{ m}^2$$

$$p = F / (A - a) = 5 \times 10^3 / 3.711 \times 10^{-3} = 1.347 \times 10^6 \text{ Pa or } \mathbf{1.347 \text{ MPa}}$$

## **4.6 BASIC HYDRAULIC POWER SYSTEM**

The hand pump is replaced by a power driven pump. The load piston may be double acting so a directional valve is needed to direct the fluid from the pump to the top or bottom of the piston. The valve also allows the venting oil back to the reservoir.

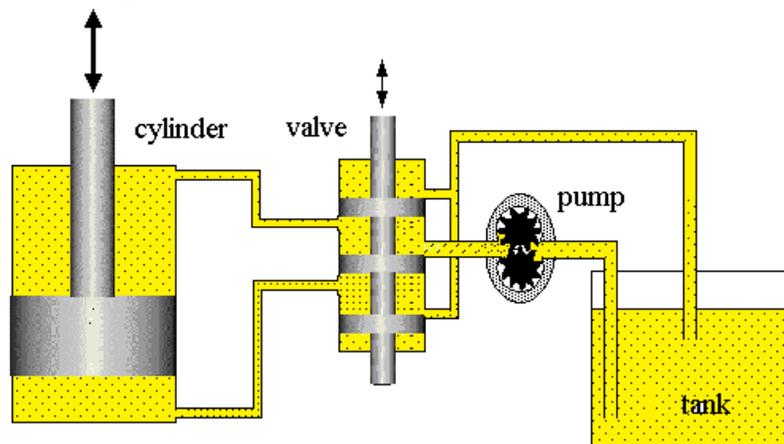


Fig.21

## **4.7 ACCUMULATORS**

An accumulator is a device for storing pressurised liquid. One reason for this might be to act as an emergency power source when the pump fails.

Originally, accumulators were made of long hydraulic cylinders mounted vertically with a load bearing down on them. If the hydraulic system failed, the load pushed the piston down and expelled the stored liquid

Modern accumulators use high pressure gas (Nitrogen) and when the pump fails the gas expels the liquid.

### WORKED EXAMPLE No.10

A simple accumulator is shown in fig.22. The piston is 200 mm diameter and the pressure of the liquid must be maintained at 30 MPa. Calculate the mass needed to produce this pressure.

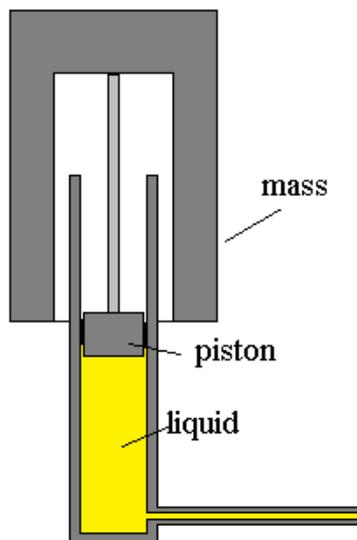


Fig.22

### SOLUTION

Weight = pressure x area

$$\text{Area} = \pi D^2/4 = \pi \times 0.2^2/4 = 0.0314 \text{ m}^2$$

$$\text{Weight} = 30 \times 10^6 \times 0.0314 = 942.5 \times 10^3 \text{ N or } 942.5 \text{ kN}$$

$$\text{Mass} = \text{Weight/gravity} = 942.5 \times 10^3 / 9.81 = 96.073 \times 10^3 \text{ kg or } 96.073 \text{ Tonne}$$

### SELF ASSESSMENT EXERCISE No.5

1. A double acting hydraulic cylinder with a single rod must produce a thrust of 800 kN. The operating pressure is 100 bar gauge. Calculate the bore diameter required. (101.8 mm)
2. The cylinder in question 1 has a rod diameter of 25 mm. If the pressure is the same on the retraction (negative) stroke, what would be the force available? (795 kN)
3. A single acting hydraulic cylinder has a piston 75 mm diameter and is supplied with oil at 100 bar gauge. Calculate the thrust. (44.18 kN)
4. A vertical hydraulic cylinder (fig.22) is used to support a weight of 50 kN. The piston is 100 mm diameter. Calculate the pressure required. (6.37 MPa)