ENGINEERING SCIENCE H1

OUTCOME 2 - TUTORIAL 1

BASIC DYNAMICS

EDEXCEL HNC/D ENGINEERING SCIENCE LEVEL 4 – H1
FORMERLY UNIT 21718P

Students following the mechanical course will find this material is basically revision of work on basic dynamics from national level. Students who have not done basic mechanical dynamics might find it useful to study the modules on basic dynamics on the web site.

You should judge your progress by completing the self assessment exercises.

These may be sent for marking or you may request copies of the solutions at a cost (see home page).

On completion of this tutorial you should be able to do the following.

- **Examine Newton's Laws of motion.**
- **Explain the affect of friction on motion.**
- **Explain the force of gravity.**
- **Examine the laws of motion for angular motion.**
- **Explain the relationship between torque and angular acceleration.**

It is assumed that students doing this tutorial already understands BASIC DYNAMICS including velocity, acceleration, inertia, momentum and angular motion.Most of this tutorial is revision of the material you should already have studied with some extra studies added to it.

1. NEWTON'S LAWS OF MOTION

Newton studied the way that forces make bodies change their motion and came up with his three laws.

1. A body at rest or with constant motion will remain so until acted on by an external force.

2. The rate of change in momentum is directly proportional to the force and takes place in the direction of the force.

3. Every force has an equal and opposite reaction.

The essence of these laws is that in order to change the motion of a body you must accelerate it or decelerate it by applying a force to it. When you apply a force to a body, it also applies an equal and opposite force to you.

The INERTIA of a body is the property that enables it to resist changes in ts motion.

The work youneed to study is mainly concerned with the second law.

NEWTON'S SECOND LAW.

The rate of change of momentum is directly proportional to the applied force.

Momentum = Mass x Velocity   Units kg m/s

Rate of change of momentum = $\Delta mv/t$ in the case of solids, m is fixed so $\Delta mv = m\Delta v$

$$\mathbf{F = M \,\Delta\, v/t = M\, a}$$

This is the best known version of the law which in words states **Force = Mass x Acceleration**

Force is that which is required to produce a change in motion of the body such that $F = M\,\Delta v/t$.

The unit of force is hence the kg m/s^2. This has a derived name of the Newton so **1 N = 1 kg m/s^2**.

SELF ASSESSMENT EXERCISE No.1

1. A vehicle of mass 1100 kg moves at 3 m/s. The brakes are applied and the vehicle reduces speed to 0.5 m/s in 45 s. Calculate the force needed.

2. A rocket of mass 200 kg in outer space moves at 360 m/s. It accelerates in a straight line by firing its motors with a force of 50 N. Calculate how long it takes to reach a velocity of 700 m/s.

2. THE EFFECT OF FRICTION

Friction is an external force that always acts to oppose motion.

When a body is accelerated, the force causing acceleration is the ***NETT FORCE.***

Nett Force = Applied Force - Friction Force.

SELF ASSESSMENT EXERCISE No.2

1. The applied force on a vehicle is 6000 N but the wind and road resistance is 2000 N. Calculate the acceleration of the vehicle. The mass is 2000 kg

2. Calculate the force needed to accelerate a piston of mass 0.8 kg in a cylinder at 3 m/s^2 if the resisting force is 3 N.

3. GRAVITY

All bodies exert a force of attraction on each other but this force diminishes with distance. Unless one of the bodies is very large and/or very close, the effect is very small. In the case of the Earth, it is large and we are close so it exerts a force of gravity on everything close to it such that any unrestrained body would accelerate downwards towards the centre of the Earth at 9.81 m/s^2. This is the gravitational constant "g". Gallileo showed that large and small bodies fall at the same rate with his famous experiment when he dropped two cannon balls of different sizes from the leaning tower of Pisa. They hit the ground together. This experiment works for compact masses but if you dropped a spanner and a feather, the feather would float around on the air currents. Astronauts on the moon showed that in the absence of air, a spanner and a feather fall together.

Since all bodies are accelerated downwards at the same rate then the force acting on it is F = M g. In order to stop a body falling, an equal and opposite force upwards must be applied. This is usually exerted by the ground on all stationary bodies and gives rise to the idea of weight. The weight of a body is simply the force of gravity acting on it so **Weight = M g.**

If a body moves in a vertical direction, the force required must include the force of gravity.
F = Ma + Mg = M(a + g) and acceleration 'a' is positive when upwards.

For a hovering aircraft a = 0 and the upwards force is equal to its weight. A body accelerating upwards must exert a force greater than its weight. A body accelerating downwards must exert a force less than the weight.

SELF ASSESSMENT EXERCISE No.3

1. Calculate the force required from a rocket engine if it must accelerate the rocket upwards at 3 m/s^2. The mass is 5 000 kg.

2. A lift of mass 500 kg is accelerated upwards at 2 m/s^2. Calculate the force in the rope.

3. The same lift is accelerated downwards at 2 m/s^2. What is the force in the rope then?

It is important to note that if a body falls at a constant speed, the acceleration is zero so the nett force is zero and the force resisting motion is equal to the weight. For example a parachutist has a large air resistance and falls at approximately 13 m/s. If the air resistance is smaller, say a free fall diver, the terminal speed is about 55 m/s. Similarly if something sinks in a liquid it will have a terminal velocity because of the fluid friction. For very small particles in a viscous liquid, this may be a fraction of a mm per year.

4. MORE ON ANGULAR MOTION

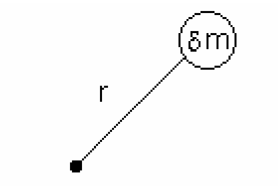
When a particle moves in a circle, it travels one circumference for every revolution about the centre. The distance travelled is along a circular line. The motion along the line is basically the same as along a straight line, it has a velocity v and acceleration a and travels a distance s. It also revolves about the centre point and has angular velocity ω, acceleration α and revolves through an angle θ. The common ink between them is the radius R.

The length of an arc is given by **$s = R\theta$**
$v = ds/dt = R\,d\theta/dt = R\,\omega$ **$v = R\omega$**
$a = dv/dt = R\,d\omega/dt = R\alpha$ **$a = R\alpha$**

These are important formulae that link linear and angular quantities.

4.1 MOMENT OF INERTIA

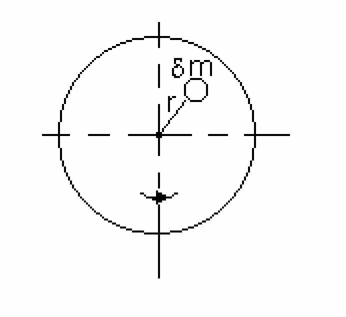
Consider a small mass δm rotating on a radius of r metres.



$r\,\delta m$ = first moment of mass
$r^2\,\delta m$ = 2nd moment of mass.
The symbol is I.

This is usually called the moment of inertia and it is an important of a rotating body. The moment of inertia of a body governs how easy or difficult it is to make it speed up or slow down as it rotates. Unfortunately most rotating bodies do not have the mass concentrated at one radius and the moment of inertia is not calculated as easily as indicated.

Consider a plane disc rotating about its centre.



The disc may be considered as made up of lots of small masses at various radii. The moment of inertia for the whole disc may be found by summing up the individual parts such that

$$I = \sum r^2\,\delta m$$

Another approach is to use an effective radius k. This is called the radius of gyration. If this is used then

$I = M\,k^2$ where m is the mass of the disc. This may be applied to any wheel or rotor but the problem is finding K.

For a simple plane disc, it may be shown that **$I = MR^2/2$** It follows that $k = 0.707\,R$.

For anything other than a plain disc **$I = Mk^2$**

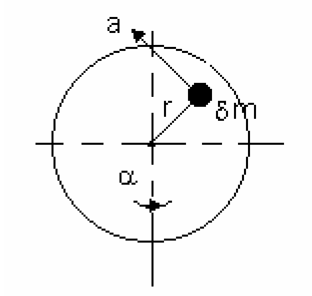
4.2 TORQUE AND ACCELERATION

Newton's 2nd Law of motion for a body accelerating in a straight line is F = ma

It will be shown that the equivalent law for a rotating body is $T = I\alpha$

Where T is the torque required to accelerate the wheel at α rad/s^2.

DERIVATION



Consider a wheel made up of many small masses δm.
The mass has an acceleration a. The force needed to accelerate it along a circular path is $\delta F = \delta m\, a$

Converting a into α produces $\delta F = \delta m\, \alpha r$

Multiply both sides by r $r\delta F = \delta m\, \alpha r^2$

$r\delta F = \delta T$ and this is the torque required to accelerate the small mass.

$$\delta T = \delta m\, \alpha r^2$$

The torque required to accelerate the whole wheel is found by integration. $T = \alpha\int r^2 \delta m$

By definition $\int \delta m\, r^2 = I$ so **$T = I\alpha$**

WORKED EXAMPLE No.1

A turbine has a mass of 1200 kg and a radius of gyration of 0.8 m. Calculate the braking torque required to slow it down from 50 rev/s to rest in 1 minute.

SOLUTION

$I = Mk^2 = 12200 \times 0.8^2 = 768$ kg m^2.
Initial angular speed $\omega = 2\pi N = 2\pi \times 50 = 314.2$ rad/s.
Change in speed = 0 - 314.2 = -314.2 rad/s
Time taken = 60 s
α = change in speed/time taken = -314.2/60 = -5.237 rad/s^2.

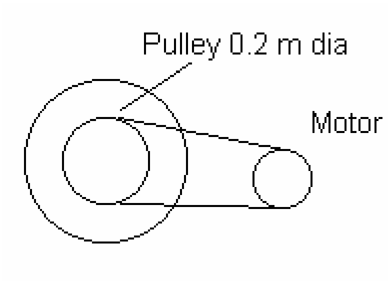
$T = I\alpha = -768 \times 5.237 = -4021.7$ N m. (Minus indicates braking torque)

SELF ASSESSMENT EXERCISE No.4

1. Calculate the torque required to accelerate a flywheel from rest to 10 rad/s in 3 seconds given the moment of inertia is 15 kg m^2.

2. A flywheel has a mass of 5 kg and radius of gyration of 0.12 m. Calculate the acceleration produced when a torque of 20 N m is applied.

3. A flywheel is driven by a pulley system as shown. Friction in the system exerts a braking torque of 2 Nm. Calculate the acceleration of the wheel.



The moment of inertia is 400 kg m^2.
The forces in the pulley belt are 120 N on the tight side and 80 N on the slack side.

4. A steam turbine and generator rotate at 3000 rev/min. The combined system has a mass of 520 kg and a radius of gyration of 2.2 m. The emergency stop is triggered and the system decelerates to 100 rev/min in 20 s. Calculate the braking torque acting on it.