# ENGINEERING SCIENCE H1

# OUTCOME 1 - TUTORIAL 4

# COLUMNS

## EDEXCEL HNC/D ENGINEERING SCIENCE LEVEL 4 – H1
## FORMERLY UNIT 21718P

This material is duplicated in the Mechanical Principles module H2 and those studying the Mechanical Engineering course will find this a good introduction to that module.

You should judge your progress by completing the self assessment exercises.

These may be sent for marking or you may request copies of the solutions at a cost (see home page).

On completion of this tutorial you should be able to do the following.

- Explain the difference between a column and a strut.
- Define and calculate slenderness ratio.
- Calculate the stress in a column.
- Calculate the stress in a column with offset loads.
- Determine the maximum offset for a safe load.

***It is assumed that students doing this tutorial already understand direct stress and bending stress.***

## INTRODUCTION

Compression members are loaded in the direction of their length and not transversely (beams). They may be long relative to their cross section in which case they are STRUTS or short in which case they are COLUMNS. There is obviously an 'in between' case called intermediate members. In this module you are required to study columns but you do need to appreciate the difference.

STRUTS fail by bending and buckling so they very limited as a structural element.

COLUMNS fail in compression. In civil engineering they are often made of brittle material which is strong in compression such as cast iron, stone and concrete. These materials are weak in tension so it is important to ensure that bending does not produce tensile stresses in them. If the compressive stress is too big, they fail by crumbling and cracking. Structural steel is also used as columns and the cross section properties of standard rolled steel columns (RSC) are found in British Standard **BS4 part 1**. A sample of this table is attached at the end.

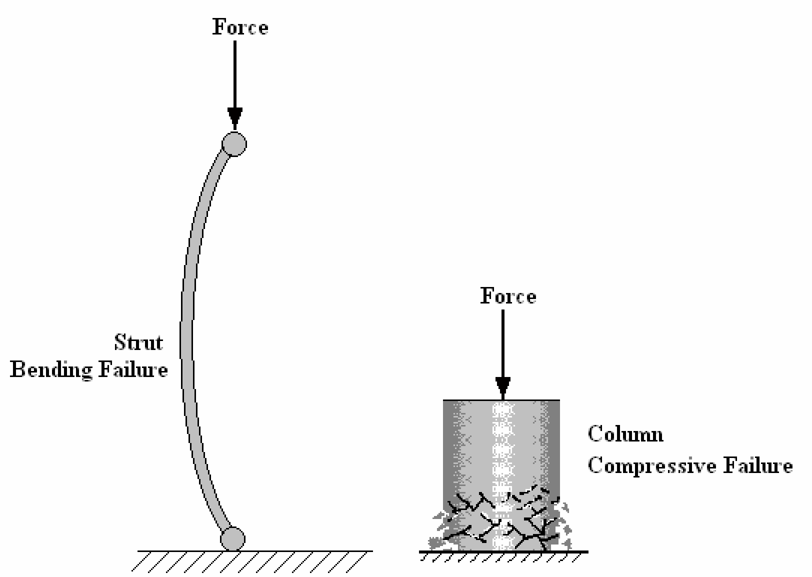


Figure 1

## 2. SLENDERNESS RATIO

One way of deciding whether a compression member is long relative to its cross section is the use of slenderness ratio. This is defined as:

$$\text{S.R.} = \frac{L}{k}$$

L is the effective length and k is the radius of gyration for the cross sectional area. A strut is defined as having a slenderness ratio is greater than 120 when made of steel and 80 when made of aluminium.

## 3. RADIUS OF GYRATION k

The radius of gyration is defined as $k = \sqrt{\frac{I}{A}}$

I is the 2nd moment of area and A is the cross sectional area.

These properties may be looked up in tables for standard RSC but must be calculated for other sections.

## WORKED EXAMPLE No.1

Derive formulae for the radius of gyration of a circle diameter D and a rectangle width B and depth D.

**Circle** $$I = \frac{\pi D^4}{64} \qquad A = \frac{\pi D^2}{4} \qquad k = \sqrt{\frac{4\pi D^4}{64\pi D^2}} = \frac{D}{4}$$

**Rectangle** $$I = \frac{BD^3}{12} \qquad A = BD \qquad k = \sqrt{\frac{BD^3}{12BD}} = \frac{D}{\sqrt{12}}$$

## WORKED EXAMPLE No.2

Calculate the slenderness ratio of a strut made from a hollow tube 20 mm outside diameter and 16 mm inside diameter and 1.2 metres long.

For a hollow tube the second moment of area is

$$I = \frac{\pi\left(D^4 - d^4\right)}{64} = \frac{\pi\left(20^4 - 16^4\right)}{64} = 4637\text{mm}^4$$

$$A = \frac{\pi\left(D^2 - d^2\right)}{4} = \frac{\pi\left(20^2 - 16^2\right)}{4} = 113.1\text{ mm}^2$$

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{4637}{113.1}} = 6.4\text{ mm}$$

$$\text{S.R.} = \frac{L}{k} = \frac{1200\text{ mm}}{6.4\text{ mm}} = 187.5$$

## SELF ASSESSMENT EXERCISE No.1

1. Find the radius of gyration and the slenderness ratio of a strut made from 5 m length of hollow tube 50 mm outer diameter and 40 mm inner diameter.
   **(Ans 16 mm and 312.3)**

## 4. COMPRESSION STRESS

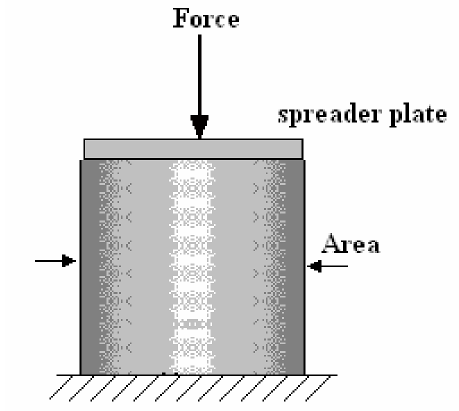


Figure 2

It is bad practise to apply a load at a point on brittle columns because high local stress results in that region. A steel plate should be used to spread the load over the section. Ideally the load is applied at the centre of area and it is assumed that the compressive stress spreads out evenly over the section. If the load is F and the cross sectional area is A then the direct (compressive) stress is $\sigma_D = -F/A$ (compression is negative)

## 5. OFFSET LOADS

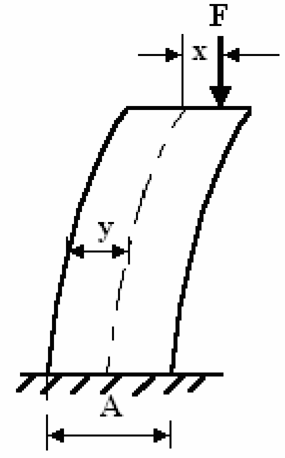


Figure 3

If the load is not applied at the centre of area, bending is induced in the column and it is more likely to fail. Brittle columns in particular must not be allowed to go into tension or they will crack. This is illustrated in figure 3.

When the load is applied a distance 'x' from the centroid, a bending moment is induced in the column as shown. The bending moment is $M = Fx$ where x is the offset distance.
From the well known formula for bending stress we have $\sigma_B = My/I$
y is the distance from the centroid to the edge of the column.
The stress produced will be +ve (tensile) on one edge and -ve (compressive) on the other.

On the compressive edge this will add to the direct compressive stress making it larger so that

$$\sigma = \sigma_B + \sigma_D = -My/I - F/A$$

On the tensile edge the resulting stress is $\sigma = \sigma_B + \sigma_D = My/I - F/A$

Substitute M = F x $\sigma = \frac{Fxy}{I} - \frac{F}{A}$

Note that offset loads induce bending and makes buckling easier if the column is long enough to be affected by it.

### WORKED EXAMPLE No.2

A column is 0.5 m diameter and carries a load of 500 kN offset from the centroid by 0.1m. Calculate the extremes of stresses.

### SOLUTION

$F = 500$ kN $x = 0.1$ m $y = D/2 = 0.25$ m
$I = \pi D^4/64 = \pi \times 0.5^4/64 = 0.00307\ \text{m}^4$ $A = \pi D^2/4 = \pi \times 0.5^2/4 = 0.196\ \text{m}^2$

Tensile Edge $\sigma = \frac{Fxy}{I} - \frac{F}{A} = \frac{500000 \times 0.1 \times 0.25}{0.00307} - \frac{500000}{0.196} = 1.52\ \text{MPa}$

Compressive Edge $\sigma = -\frac{Fxy}{I} - \frac{F}{A} = -\frac{500000 \times 0.1 \times 0.25}{0.00307} - \frac{500000}{0.196} = -6.62\ \text{MPa}$

## 6. NEUTRAL AXIS

The neutral axis is the axis of zero stress. In the above example, the stress varied from 1.528 MPa on one edge to -6.621 MPa on the other edge. Somewhere in between there must a value of y which makes the stress zero. This does not occur on the centroid but is by definition the position of the neutral axis. Ideally this axis should not be on the section at all so that no tensile stress occurs in the column. The position of the neutral axis can easily be found by drawing a stress distribution diagram and then either scaling off the position or calculate it from similar triangles.

### WORKED EXAMPLE No.3

Determine the position of the neutral axis for the column in example 2.

### SOLUTION

Drawing a graph of stress against position (y) along a diameter we get the figure shown (not drawn to scale). If it is drawn to scale the position of the neutral axis may be scaled off.

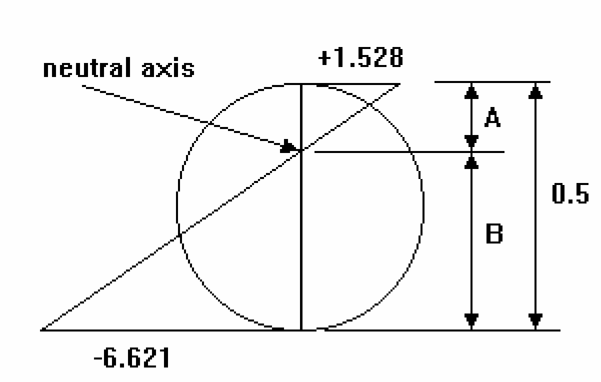


Figure 5

Using similar triangles we arrive at the solution as follows.

A + B = 0.5
A = 0.5 - B
A/1.528 = B/6.621
(0.5 - B)/1.528 = B/6.621
3.3105 - 6.621B = 1.528 B
B = 0.406 m

## 7. MAXIMUM OFFSET

If a column must not go into tension, then the maximum offset may be calculated. Consider a circular section first. The combined stress due to compression and bending is:

$$\sigma = \frac{Fxy}{I} - \frac{F}{A}$$

If the edge must not go into tension then the maximum stress will be zero so:

$$\frac{Fxy}{I} - \frac{F}{A} = 0 \qquad x(\max) = \frac{I}{Ay} = \frac{Z}{y}$$

For a round section $A = \pi D^2/4$ $I = \pi D^4/64$ and $y = D/2$ If we substitute we get

$$x(\max) = \frac{4\pi D^4}{64\ \pi D^2\ D/2} = \frac{D}{8}$$

The load must be no more than D/8 from the centroid.

If the column is a rectangular section $I = BD^3/12$ $A = BD$ and the critical value of y is D/2

$$x(\max) = \frac{I}{Ay} = \frac{2\ BD^3}{12\ BD\ D} = \frac{D}{6}$$ when the offset is on the short axis.

When the offset is on the long axis x(max) is B/6 . This means the offset must be within the middle 1/3 of the column and this is called the **middle third rule.** The shaded area on the diagram shows the safe region for applying the load.

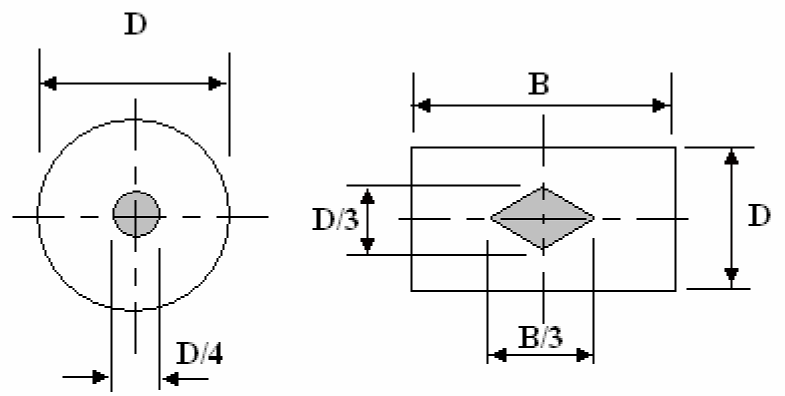


Figure 6

For any standard section such as those in BS4, the maximum offset is easily found from x = Z/y although for steel sections some tension is allowed.

**<u>WORKED EXAMPLE No.3</u>**

A column is made from an universal 'I' section 305 x 305 x 97. A load of 2 MN is applied on the x axis 200 mm from the centroid. Calculate the stress at the outer edges of the x axis.

If the column is 5 m tall, what is the slenderness ratio?

**<u>SOLUTION</u>**

The offset is x = 0.2 m and the load F = 2 MN

From the table $I = 22249 \times 10^{-8}\ m^{-4}$ $A = 123 \times 10^{-4}\ m^2$ $y = h/2 = 0.154$ m

$$\sigma_c = -\frac{Fxy}{I} - \frac{F}{A} = -\frac{(2 \times 10^6)(0.2)(0.154)}{I} - \frac{2 \times 10^6}{123 \times 10^{-4}} = -439\ \text{MPa}$$

$$\sigma_T = \frac{Fxy}{I} - \frac{F}{A} = \frac{(2 \times 10^6)(0.2)(0.154)}{I} - \frac{2 \times 10^6}{123 \times 10^{-4}} = 114\ \text{MPa}$$

There are two radii of gyration. $k_x = 0.134$ $k_y = 0.0769$ m

Slenderness Ratio about the x axis is = $l/k_x = 37.3$

Slenderness Ratio about the y axis is = $l/k_y = 65$

These are well below the limit of 120 for steel but the bending might cause collapse and would be worth checking.

## SELF ASSESSMENT EXERCISE No.2

1. A column is 0.4 m diameter. It has a vertical load of 300 kN acting 0.05m from the centroid. Calculate the stresses on the extreme edges.
   **(Answers 0 MPa and -4.77 MPa).**

2. A column is 0.3 m diameter. Calculate the offset position of the load which just prevents the one edge from going into tension. **(Answer 0.038 m).**

3. A column is made from a rectangular block of concrete with a section 600 mm x 300 mm. What is the maximum offset of a point load that just prevents the edge going into tension.
   **(Answer 50 mm).**

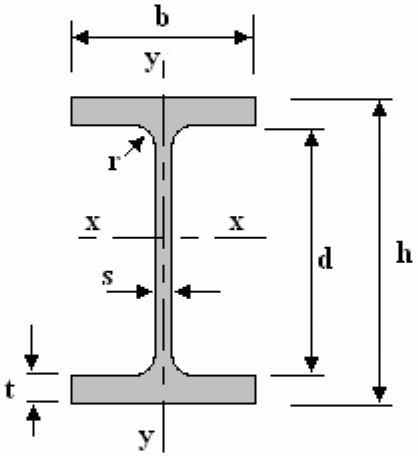
4. A column is made from cast iron tube 0.4 m outside diameter with a wall 40 mm thick. The top is covered with a flat plate and a vertical load of 70 kN is applied to it. Calculate the maximum allowable offset position of the load if the material must always remain in compression.
   **(Answer 0.082 m)**

5. A hollow cast iron pillar, 38 cm outside diameter and wall thickness 7.5 cm, carries a load of 75 kN along a line parallel to, but displaced 3 cm from, the axis of the pillar. Determine the maximum and minimum stresses in the pillar.

   What is the maximum allowable eccentricity of the load relative to the axis of the pillar if the stresses are to be compressive at all points of the cross section?

6. A column is 4 m tall and made from an universal ‘I’ section 152 x 152 x 23. A load of 60 kN is applied on the x axis 110 mm from the centroid. Calculate the stress at the outer edges of the x axis.
   (35.2 MPa tensile and 45 MPa compressive, SL = 61 about the x axis and 108 about y axis)

# SAMPLE OF TABLE FOR UNIVERSAL COLUMNS WITH ‘I’ SECTION



| Designation | Mass per m | Depth of Section | Width of Section | Thickness of | | Root Radius | Depth between Fillets | Area of Section | Second Moment Area | | Radius of Gyration | | Elastic Modulus | | Plastic Modulus | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Web | Flange | | | | Axis x-x | Axis y-y | Axis x-x | Axis y-y | Axis x-x | Axis y-y | Axis x-x | Axis y-y |
| | M | h | b | s | t | r | d | A | Ix | Iy | rx | ry | Zx | Zy | Sx | Sy |
| | kg/m | mm | mm | mm | mm | mm | mm | cm$^2$ | cm$^4$ | cm$^4$ | cm | cm | cm$^3$ | cm$^3$ | cm$^3$ | cm$^3$ |
| 356x406x634 | 633.9 | 474.6 | 424 | 47.6 | 77 | 15.2 | 290.2 | 808 | 274845 | 98125 | 18.4 | 11 | 11582 | 4629 | 14235 | 7108 |
| 356x406x551 | 551 | 455.6 | 418.5 | 42.1 | 67.5 | 15.2 | 290.2 | 702 | 226938 | 82671 | 18 | 10.9 | 9962 | 3951 | 12076 | 6058 |
| 356x406x467 | 467 | 436.6 | 412.2 | 35.8 | 58 | 15.2 | 290.2 | 595 | 183003 | 67834 | 17.5 | 10.7 | 8383 | 3291 | 10002 | 5034 |
| 356x406x393 | 393 | 419 | 407 | 30.6 | 49.2 | 15.2 | 290.2 | 501 | 146618 | 55367 | 17.1 | 10.5 | 6998 | 2721 | 8222 | 4154 |
| 356x406x340 | 339.9 | 406.4 | 403 | 26.6 | 42.9 | 15.2 | 290.2 | 433 | 122543 | 46853 | 16.8 | 10.4 | 6031 | 2325 | 6999 | 3544 |
| 356x406x287 | 287.1 | 393.6 | 399 | 22.6 | 36.5 | 15.2 | 290.2 | 366 | 99875 | 38677 | 16.5 | 10.3 | 5075 | 1939 | 5812 | 2949 |
| 356x406x235 | 235.1 | 381 | 394.8 | 18.4 | 30.2 | 15.2 | 290.2 | 299 | 79085 | 30993 | 16.3 | 10.2 | 4151 | 1570 | 4687 | 2383 |
| 356x368x202 | 201.9 | 374.6 | 374.7 | 16.5 | 27 | 15.2 | 290.2 | 257 | 66261 | 23688 | 16.1 | 9.6 | 3538 | 1264 | 3972 | 1920 |
| 356x368x177 | 177 | 368.2 | 372.6 | 14.4 | 23.8 | 15.2 | 290.2 | 226 | 57118 | 20529 | 15.9 | 9.54 | 3103 | 1102 | 3455 | 1671 |
| 356x368x153 | 152.9 | 362 | 370.5 | 12.3 | 20.7 | 15.2 | 290.2 | 195 | 48589 | 17553 | 15.8 | 9.49 | 2684 | 948 | 2965 | 1435 |
| 356x368x129 | 129 | 355.6 | 368.6 | 10.4 | 17.5 | 15.2 | 290.2 | 164 | 40246 | 14611 | 15.6 | 9.43 | 2264 | 793 | 2479 | 1199 |
| 305x305x283 | 282.9 | 365.3 | 322.2 | 26.8 | 44.1 | 15.2 | 246.7 | 360 | 78872 | 24635 | 14.8 | 8.27 | 4318 | 1529 | 5105 | 2342 |
| 305x305x240 | 240 | 352.5 | 318.4 | 23 | 37.7 | 15.2 | 246.7 | 306 | 64203 | 20315 | 14.5 | 8.15 | 3643 | 1276 | 4247 | 1951 |
| 305x305x198 | 198.1 | 339.9 | 314.5 | 19.1 | 31.4 | 15.2 | 246.7 | 252 | 50904 | 16299 | 14.2 | 8.04 | 2995 | 1037 | 3440 | 1581 |
| 305x305x158 | 158.1 | 327.1 | 311.2 | 15.8 | 25 | 15.2 | 246.7 | 201 | 38747 | 12569 | 13.9 | 7.9 | 2369 | 808 | 2680 | 1230 |
| 305x305x137 | 136.9 | 320.5 | 309.2 | 13.8 | 21.7 | 15.2 | 246.7 | 174 | 32814 | 10700 | 13.7 | 7.83 | 2048 | 692 | 2297 | 1053 |
| 305x305x118 | 117.9 | 314.5 | 307.4 | 12 | 18.7 | 15.2 | 246.7 | 150 | 27672 | 9059 | 13.6 | 7.77 | 1760 | 589 | 1958 | 895 |
| 305x305x97 | 96.9 | 307.9 | 305.3 | 9.9 | 15.4 | 15.2 | 246.7 | 123 | 22249 | 7308 | 13.4 | 7.69 | 1445 | 479 | 1592 | 726 |

| Designation | Mass per m | Depth of Section | Width of Section | Thickness of | | Root Radius | Depth between Fillets | Area of Section | Second Moment Area | | Radius of Gyration | | Elastic Modulus | | Plastic Modulus | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Web | Flange | | | | Axis x-x | Axis y-y | Axis x-x | Axis y-y | Axis x-x | Axis y-y | Axis x-x | Axis y-y |
| 254x254x167 | 167.1 | 289.1 | 265.2 | 19.2 | 31.7 | 12.7 | 200.3 | 213 | 29998 | 9870 | 11.9 | 6.81 | 2075 | 744 | 2424 | 1137 |
| 254x254x132 | 132 | 276.3 | 261.3 | 15.3 | 25.3 | 12.7 | 200.3 | 168 | 22529 | 7531 | 11.6 | 6.69 | 1631 | 576 | 1869 | 878 |
| 254x254x107 | 107.1 | 266.7 | 258.8 | 12.8 | 20.5 | 12.7 | 200.3 | 136 | 17510 | 5928 | 11.3 | 6.59 | 1313 | 458 | 1484 | 697 |
| 254x254x89 | 88.9 | 260.3 | 256.3 | 10.3 | 17.3 | 12.7 | 200.3 | 113 | 14268 | 4857 | 11.2 | 6.55 | 1096 | 379 | 1224 | 575 |
| 254x254x73 | 73.1 | 254.1 | 254.6 | 8.6 | 14.2 | 12.7 | 200.3 | 93.1 | 11407 | 3908 | 11.1 | 6.48 | 898 | 307 | 992 | 465 |
| 203x203x86 | 86.1 | 222.2 | 209.1 | 12.7 | 20.5 | 10.2 | 160.8 | 110 | 9449 | 3127 | 9.28 | 5.34 | 850 | 299 | 977 | 456 |
| 203x203x71 | 71 | 215.8 | 206.4 | 10 | 17.3 | 10.2 | 160.8 | 90.4 | 7618 | 2537 | 9.18 | 5.3 | 706 | 246 | 799 | 374 |
| 203x203x60 | 60 | 209.6 | 205.8 | 9.4 | 14.2 | 10.2 | 160.8 | 76.4 | 6125 | 2065 | 8.96 | 5.2 | 584 | 201 | 656 | 305 |
| 203x203x52 | 52 | 206.2 | 204.3 | 7.9 | 12.5 | 10.2 | 160.8 | 66.3 | 5259 | 1778 | 8.91 | 5.18 | 510 | 174 | 567 | 264 |
| 203x203x46 | 46.1 | 203.2 | 203.6 | 7.2 | 11 | 10.2 | 160.8 | 58.7 | 4568 | 1548 | 8.82 | 5.13 | 450 | 152 | 497 | 231 |
| 152x152x37 | 37 | 161.8 | 154.4 | 8 | 11.5 | 7.6 | 123.6 | 47.1 | 2210 | 706 | 6.85 | 3.87 | 273 | 91.5 | 309 | 140 |
| 152x152x30 | 30 | 157.6 | 152.9 | 6.5 | 9.4 | 7.6 | 123.6 | 38.3 | 1748 | 560 | 6.76 | 3.83 | 222 | 73.3 | 248 | 112 |
| 152x152x23 | 23 | 152.4 | 152.2 | 5.8 | 6.8 | 7.6 | 123.6 | 29.2 | 1250 | 400 | 6.54 | 3.7 | 164 | 52.6 | 182 | 80.2 |