EDEXCEL NATIONAL CERTIFICATE/DIPLOMA

UNIT 5 - ELECTRICAL AND ELECTRONIC PRINCIPLES
NQF LEVEL 3

OUTCOME 2 - CAPACITANCE

2 Understand the concepts of capacitance and determine capacitance values in DC circuits

Capacitors**:** types (electrolytic, mica, plastic, paper, ceramic, fixed and variable capacitors); typical capacitance values and construction (plates, dielectric materials and strength, flux density, permittivity); function e.g. energy stored, circuits (series, parallel, combination); working voltage

Charging and discharging of a capacitor**:** measurement of voltage, current and time; tabulation of data and graphical representation of results; time constants

DC network that includes a capacitor**:** e.g. DC power source with two/three capacitors connected in series, DC power source with two/three capacitors connected in parallel

INTRODUCTION

Capacitors are two terminal devices which are used in d.c. and a.c. circuits. In this outcome we shall be restricted to the theory and construction of capacitors and d.c. applications. In outcome 4 we will examine the affect of capacitors on alternating current.

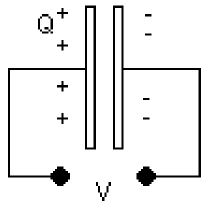
Capacitors can be used to do the following:

- Store static electricity in the form of electrons with a charge Q
- Store electrical energy
- Block the flow of direct current.
- Apparently allow the flow of A.C.

1. BASIC THEORY AND FLUID ANALOGY

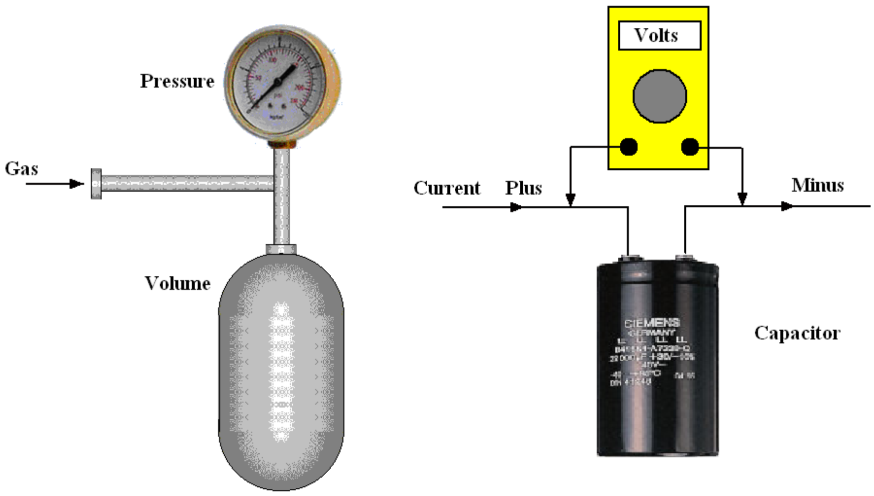
We learned in Outcome 1 that Electric charge 'Q' is a quantity of electricity measured in Coulombs. The energy possessed by the charge depends upon the voltage. Relative to zero potential, the energy that a charge possesses is Q V Joules. When a charge is transferred as electrons in a conductor, the charge per second is the current 'I' Amperes. 1 Amp =1 Coulomb/s.

Capacitors are basically two parallel plates separated by an insulating material called a dielectric. There are many types for a variety of purposes but the basic theory is the same for all. If a voltage V is connected across the plates, electrons are forced onto one plate and a corresponding number is removed from the other plate. If a plate has a surplus of electrons it becomes negatively charged and the plate with a deficiency of electrons becomes positively charged. In other words the capacitor has been made to store charge and hence energy.



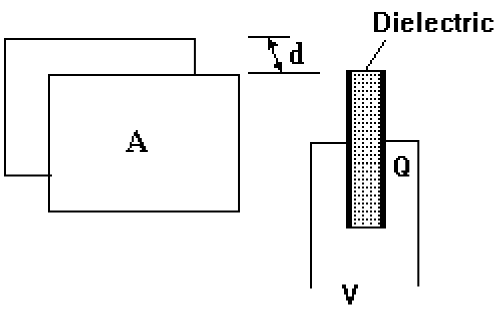
We may make an analogy with a vessel used to store gas under pressure. These are known as accumulators. If a gas is pumped into a container, the pressure 'p' rises as the quantity of gas increases (e.g. pumping up a bicycle tire). Work is done during the pumping and the energy used is stored in the container. The energy stored is directly proportional to the pressure and the volume of the vessel 'V' and the energy may be calculated from the formula E = pV.

A capacitor is the electrical equivalent of this. Electrons (charge 'Q') are pumped into the capacitor and the voltage 'V' rises as the charge increases. The energy stored is given by the formula E = VQ



Consider two plates each with an area A and separated by a distance d. The material between them is the dielectric. The charge Q stored on a pair of plates depends upon the voltage and the physical dimensions and properties of the plates and dielectric. For a given capacitor it is found that charge is directly proportional to voltage such that

$$Q = CV$$



The constant C is the **CAPACITANCE**. Capacitance is measured in Farads (F) but this is a large unit so we use the following multiples.

$mF = F \times 10^{-3}$ $\mu F = F \times 10^{-6}$ $nF = F \times 10^{-9}$ $pF = F \times 10^{-12}$

WORKED EXAMPLE No. 1

A capacitor stores 0.05 Coulombs when the terminal voltage is 160V. What is the capacitance?

SOLUTION

C = Q/V = 0.05/160 = 0.0003125 F or 312.5 μF

2. ENERGY STORED

A charge of 1 Coulomb carries energy of 1 Joule for each volt. The energy in a charge is hence QV. When a capacitor is charged, the voltage has to build up from 0 to V so the average voltage is V/2. The energy stored is hence QV/2. Substituting Q = CV the energy stored in a capacitor is hence

$$E = \frac{CV^2}{2}$$

WORKED EXAMPLE No. 2

A capacitor has a value of 800 μF and it is charged until the terminal voltage is 15V. Calculate the energy stored.

SOLUTION

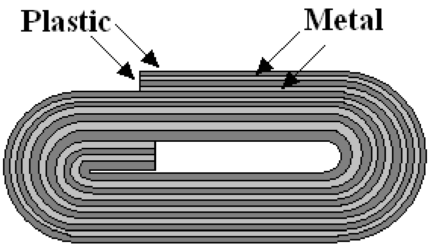
$$E = \frac{CV^2}{2} = \frac{800x10^{-6} x 15^2}{2} = 0.09 \text{ J}$$

3. TYPES AND CONSTRUCTION

Some typical types of capacitors are shown below. **Each capacitor has a maximum working voltage** which if exceeded might cause the dielectric to break down and conduct.

3.1 Paper and Plastic

The dielectric is made of paper or plastic, plastic being the more modern type. In the case of paper, two sheets of foil and two sheets of paper are interleaved and rolled into a cylinder. In the case of plastic, the plastic is coated in a metal film. The finished product is encapsulated in a plastic or metal tube with terminals connecting to the metal sections.

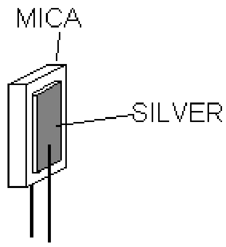


The capacitor may be made in box shapes in a similar way. The end connections are designed for mounting vertically or horizontally on circuit boards or between terminals.

The types of plastics commonly used are polycarbonate, polystyrene, polypropylene and polyester. The type used depends upon the intended voltage and accuracy.

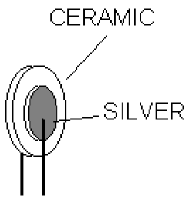
3.2 Silver Mica

A piece of mica is the dielectric with silver deposited on each side to form the plates. They are encapsulated in wax impregnated with ceramic. They may be disc shaped or flat. Their advantage is high working voltage (typically 350V) and accuracy.



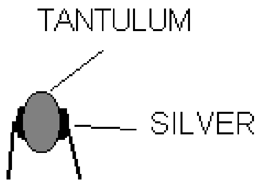
3.3 Ceramic

These may be disc shaped, tubular or flat. The dielectric is ceramic and metal films are deposited to form the plates. They are encapsulated in resin or epoxy.



3.4. Tantalum Bead

Tantalum is used as the dielectric and produces a large capacitance in small volume but with low working voltages. These small capacitors are encapsulated in resin.

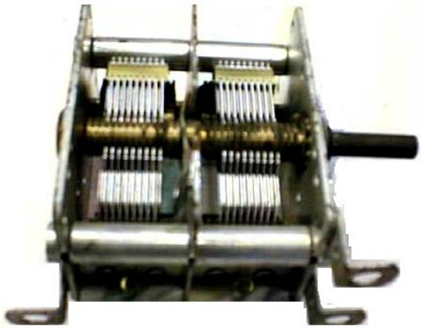


3.5. Electrolytic

The construction consists of metal foil and impregnated cotton gauze layered and rolled up into a cylinder. The dielectric is a chemical film formed by electrolytic action between the gauze and the foil. Very large capacitances are possible. The dielectric is broken down by reversed current so they cannot be used with A.C. and polarity of the terminals must be observed. Sizes range from less than 1 micro-Farad to many thousands of micro-Farads. The physical size also depends upon the working voltage.

3.6. Variable

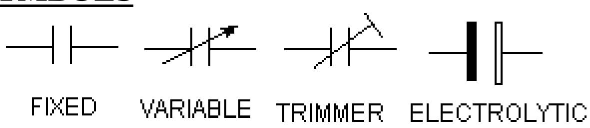
The picture shows a multiplate air spaced variable capacitor. There are two sets of metal plates or vanes, fixed and moving. The fixed vanes are all attached to the frame. The moving vanes are attached to the spindle. The plates are made to interleave and the area common to both sets of plates is varied by rotating the spindle. The dielectric is air or plastic. The dielectric is air. If plastic is used between the plates, they may be made more compact. These mechanical designs are expensive and for many applications they have been replaced by a device called a varicap diode. A typical use is in radio tuners.



3.7. Preset or Trimmers.

These are small variable capacitors designed to be adjusted only occasionally. When a capacitance value is critical, a trimmer may be used instead of a fixed type so that the exact capacitance may be obtained. One design uses interleaving plates that rotate. Another design uses two metal plates that are squeezed onto a mica sheet by turning a screw.

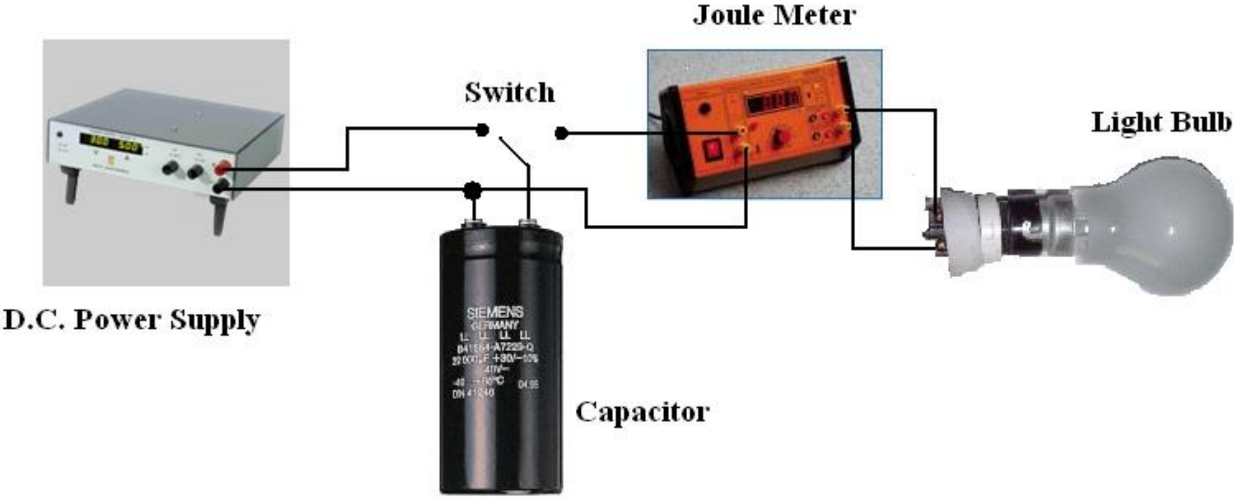
4. <u>CAPACITOR SYMBOLS</u>



SELF ASSESSMENT EXERCISE No.1

You could do this as an experiment.

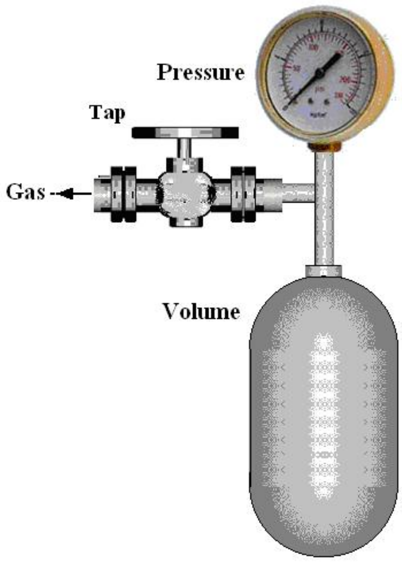
A circuit is set up as illustrated to measure the energy stored in the capacitor. The D.C. power supply is set to 20V. When the switch is connected to the power supply the capacitor charges up. When the switch is connected to the Joule meter, the capacitor discharges through the meter and lights up the bulb which quickly dims and as the charge is used up. The process is repeated 10 times and the total energy discharged is 50 Joules. Calculate the capacitance.

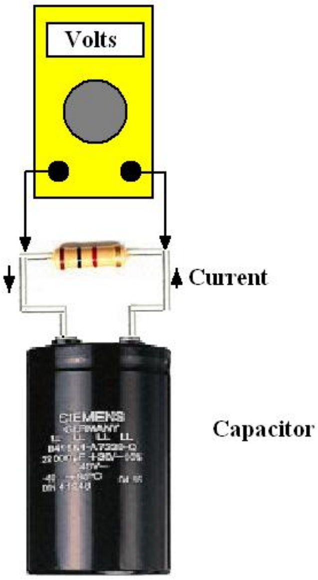


(Answer 25 000 μF)

5. CHARGING AND DISCHARGING THROUGH A RESISTANCE

It might be useful here to return to our analogy. Consider a gas bottle with a tap connected to it. If we crack open the tap and let the gas out, the pressure will fall with time until the bottle is empty. This is like taking a fully charged capacitor and connecting a resistor across the terminals. The charge will flow through the resistor and the terminal voltage will fall with time.



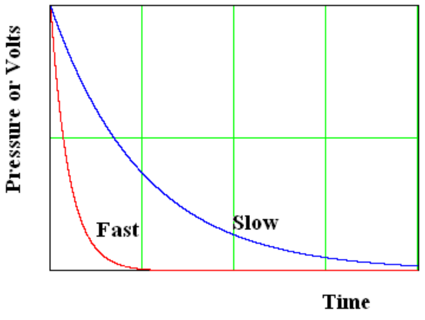


The pressure and voltage will fall in a manner called an EXPONENTIAL DECAY. The speed at which the pressure and voltage fall depends on the capacity and the resistance. If we open the valve more, the bottle will discharge more quickly. If we put a lower value resistor across the terminals, the voltage will fall more quickly. The graph shows a slow and fast discharge.

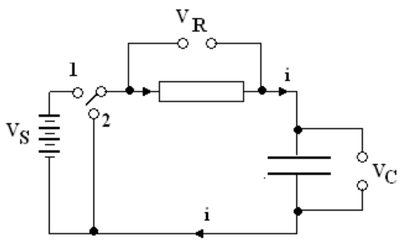
In the case of the electrical system, the equation that gives the discharge curve is: $V = V_s e^{\frac{-t}{T}}$

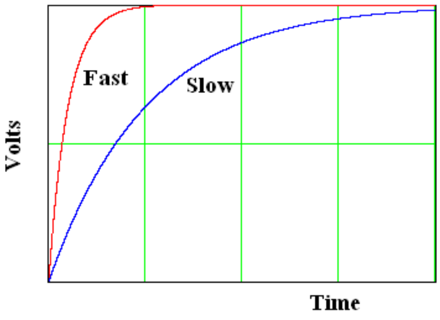
V_s is the starting voltage, t is the time and T is a time constant given by T = RC

We see that if R and C are big, T is big and the discharge is slow.



The opposite of discharging is charging. The circuit shown below can be used to charge and discharge a capacitor. When the switch is connected to (1) the capacitor charges through the resistor and the voltage across the capacitor rises exponentially to the source voltage. When the switch is connected to (2), the capacitor discharges through the resistor.





The equation governing the charging curve is $V_c = V_s\left(1 - e^{\frac{-t}{T}}\right)$

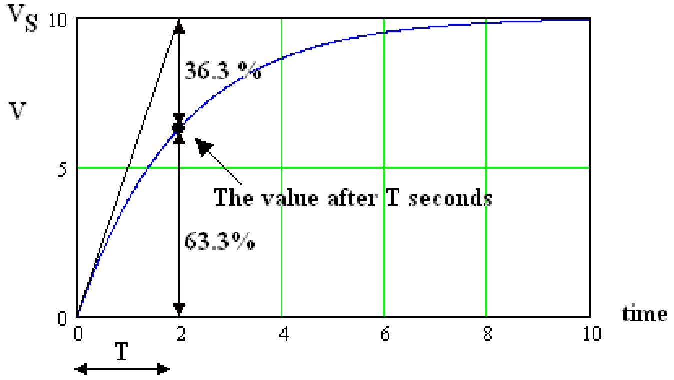
THE MEANING OF THE TIME CONSTANT

Starting with the equation $V_c = V_s\left(1 - e^{\frac{-t}{T}}\right)$

If we put t = T we have

$$V_C = V_S\left(1 - e^{-\frac{t}{T}}\right) = V_S\left(1 - e^{-1}\right) = 0.633\,V_S$$

So the time constant is the time taken for the voltage to change to 63.3% of the final value. This is useful when finding T from a graph.



Note in theory that the final voltage is never reached but if we calculate the voltage at t = 4T we find V = 98.2%V_S. We can say effectively that it takes 4T for a capacitor to almost completely charge or discharge.

WORKED EXAMPLE No. 3

A capacitor of value 50 μF is charged from zero to 100 V through a 5 MΩ resistor. Calculate the time constant and the time taken for the voltage to rise to 50 V.

SOLUTION

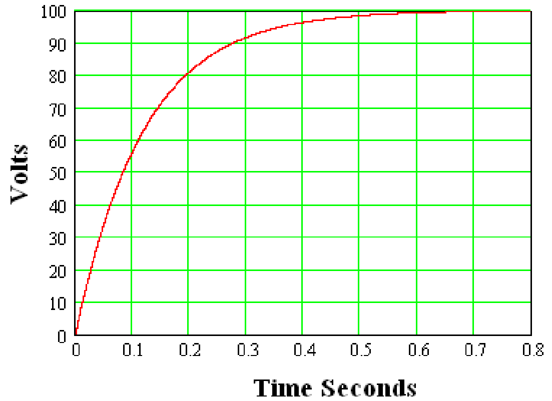
$T = RC = 50 \times 10^{-6} \times 5 \times 10^{6} = 250$

$$V_C = V_S\left(1 - e^{-\frac{t}{T}}\right) \qquad 50 = 100\left(1 - e^{\frac{-t}{250}}\right) \qquad 0.5 = \left(1 - e^{\frac{-t}{250}}\right) \qquad e^{\frac{-t}{250}} = 1 - 0.5 = 0.5$$

$$\frac{-t}{250} = \ln(0.5) = -0.6931 \qquad t = 250 \times 0.6931 = 173.2 \text{ seconds}$$

SELF ASSESSMENT EXERCISE No. 2

The graph shows a charging curve for a capacitor and resistance. Work out the time constant T and determine the capacitance if the resistor value is 6 kΩ.

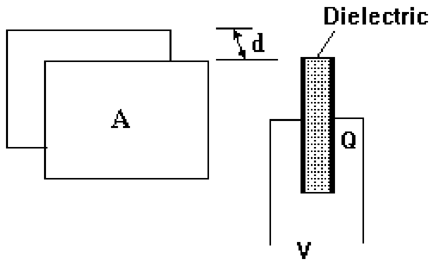


(Answers 0.012 s and 20 μF)

5. ELECTROSTATIC THEORY

Each electron carries a negative quantity of energy of 160 x 10^{-21} Joules per volt. If a plate has excess electrons it has a negative charge and if it loses electrons it becomes positively charged.

Plates which have opposite charge attract each other. Plates with a similar charge repel each other. These forces are electrostatic forces.



5.1 ELECTRIC FIELD E

The electric field strength is defined as:

E = V/d Volts/metre

5.2 ELECTRIC FLUX D

Electric flux is the charge that exists in common between two plates. It is expressed as the charge per square meter of common plate area. It is defined as follows.

D = Q/A Coulombs/m^2

5.3 RELATIONSHIP BETWEEN ELECTRIC FIELD AND FLUX

It is found that for a given capacitor, the electric flux is directly proportional to the electric field.
D = constant x E and D = Q/A E = V/d so substituting we get:-

D/E = Constant = (Q/A)/(V/d) **Q d/A V = constant**

The ratio or constant is changed if a different dielectric is used. If a vacuum separates the plates it is found that D/E = Qd/AV = 8.85 x 10^{-12}

This constant is called the ABSOLUTE PERMITTIVITY of free space and is denoted as ε_o.

$$\varepsilon_o = 8.85 \times 10^{-12}$$

For other materials the constant is different. In order to take this into account the following formula is used.

$$D/E = Qd/AV = \text{constant} = \varepsilon_o \varepsilon_r$$

ε_r is a multiplier which states how many more times the constant is than when it is a vacuum. ε_r is called the RELATIVE PERMITTIVITY. Here are some values of ε_r.

MATERIAL	ε_r
Free Space	1.000
Air	1.006
Paper	2 approx.
Glass	7 approx.
Mica	4 approx.
Ceramic	6 approx.
Plastics	various

We said that D/E = Qd/AV = $\varepsilon_o\varepsilon_r$ so it follows that the capacitance C is given by:

$$C = Q/V = (A/d)\varepsilon_o \varepsilon_r$$

We put this in more simple terms by saying that capacitance is directly proportional to area but inversely proportional to the gap d. It also depends upon the relative permittivity of the dielectric.

WORKED EXAMPLE No. 4

A variable capacitor when fully closed has a total plate area of 50 000 mm^2. The plates are spaced 0.5 mm apart and the space is filled with air. Calculate the capacitance when fully closed.

SOLUTION

A = 50 000 x 10^{-6} m2
d = 0.5 x 10^{-3} m
ε_o = 8.85 x 10^{-12}
ε_r = 1.006

$$C = \frac{A}{d}\varepsilon_o\varepsilon_r = \frac{50\,000 \times 10^{-6}}{0.5 \times 10^{-3}} \times 8.85 \times 10^{-12} \times 1.006 = 890 \times 10^{-12}\,\text{F}$$

C = 890 pF

SELF ASSESSMENT EXERCISE No. 3

1. Calculate the charge stored on a capacitor of value 200 μF when 50 V are applied to the plates.
(0.01 Coulomb)

2. Calculate the value of a capacitor which stores 5 Coulombs at 20 V.
(0.25 F)

3. Calculate the Voltage required to store 20 Coulombs in a capacitor of value 50 μF.
(400 kV)

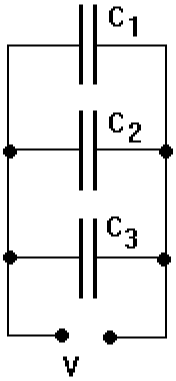
4. An overhead cable is 30 m from the ground at a potential difference of 20 kV to the ground. Calculate the electric field strength and the theoretical voltage at a height of 2 m from the ground.
(666.7V/m and 1333 V)

5. Calculate the capacitance of two parallel square plates 50 mm x 50 mm when separated by 2 mm of glass.
(77 pF)

   The plates are charged to 100 V and left isolated. The glass is removed and air replaces it. What happens to the voltage on the plates?

7. NETWORKS

7.1 CAPACITORS IN PARALLEL



Consider three capacitors C1, C2 and C3 connected in parallel as shown. The voltage is the same on each one. The charge stored on each is

$Q_1 = C_1 V \quad Q_2 = C_2 V \quad Q_3 = C_3 V$

The total charge stored $Q_T = Q_1 + Q_2 + Q_3$

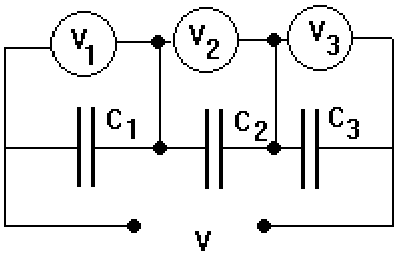
$Q_T = C_1 V + C_2 V + C_3 V = V(C_1 + C_2 + C_3)$

We define Q_T as $C_T V$ where C_T is the total capacitance so it follows that

$$C_T = (C_1 + C_2 + C_3)$$

It follows that capacitors in parallel are added together.

7.2 CAPACITORS IN SERIES



Consider three capacitors in series with a voltage V across the total but the voltage on each one is V_1, V_2 and V_3.

Since the movement of electrons must be the same onto and off all the plates, the charge stored on each capacitor is the same and equal to the total charge.

$Q_T = Q_1 = Q_2 = Q_3 = Q$

$V = Q_T / C_T$ and $V_1 = Q_1 / C_1$ $\quad V_2 = Q_2 / C_2$ and $V_3 = Q_3 / C_3$

Since $V = V_1 + V_2 + V_3$ then $Q / C_T = Q/C_1 + Q / C_2 + Q/C_3$

Cancelling Q we have $1 / C_T = 1/C_1 + 1 / C_2 + 1/C_3$

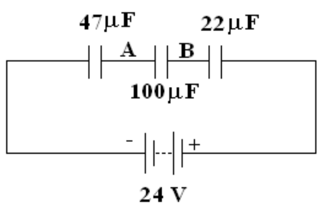
Hence $C_T = 1/(1/C_1 + 1 /C_2 + 1/C_3)$

Note that capacitors in series follow the same rule as resistors in parallel.

7.3 COMBINATION

Solving capacitor networks is similar to solving resistor networks. The following example shows this.

WORKED EXAMPLE No. 5



A 24 V battery is connected to three capacitors in series as shown. Calculate the total charge stored and the voltage at points A and B.

SOLUTION

$$C_T = \frac{1}{1/47 + 1/100 + 1/22} = 13.0325\,\mu F$$

$Q = C_T V = 13.025 \times 10^{-6} \times 24 = 312.78 \times 10^{-6}$ Coulomb

The charge is the same on each capacitor.

The voltage at point A is $312.78 \times 10^{-6}/47 \times 10^{-6} = 6.654$ V.

The voltage difference between A and B is $312.78 \times 10^{-6}/100 \times 10^{-6} = 3.128$ V.

The voltage at B is hence 6.654 + 3.128 = 9.782

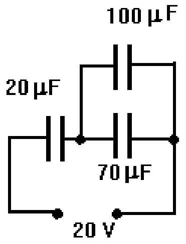
The voltage drop over the 22 µF capacitor is $312.78 \times 10^{-6}/22 \times 10^{-6} = 14.212$

Check the total, 9.782 + 14.212 = 24V.

The following problems are for those who want to something a bit more difficult.

WORKED EXAMPLE No. 6

Solve the total capacitance and the voltage across the 20 μF capacitor.

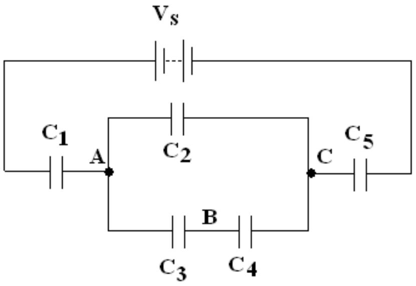


SOLUTION

First solve the parallel capacitors.
$C_p = 100 + 70 = 170$ μF
Next solve this in series with the 20.
$C_T = 1/(1/20 + 1/170) = 1/0.05588 = 17.89$ μF
Next solve the charge $Q = C_T V = 17.89 \times 10^{-6} \times 20 = 357.9 \times 10^{-6}$ Coulomb
The charge is the same on all series capacitors so the charge on the 20 μF is the same.
$Q = 357.9 \times 10^{-6} = 20 \times 10^{-6} \times V$ hence V = 17.89 V

WORKED EXAMPLE No. 7

A network of capacitors is connected to a battery as shown. Given the listed data below, calculate the total capacitance, and the voltage at points A B and C.

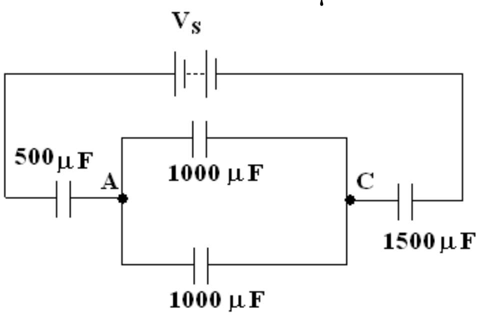


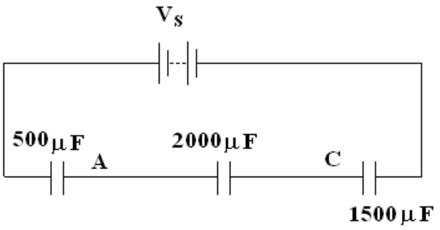
$V_s = 50$ V $C_1 = 500$ μF $C_2 = 1000$ μF $C_3 = C_4 = 2000$ μF $C_5 = 1500$ μF

SOLUTION

First evaluate the combined value of C_3 and C_4 $C_{34} = \dfrac{1}{\dfrac{1}{2000} + \dfrac{1}{2000}} = \dfrac{1}{0.001} = 1000$

The combined value is 1000 μF. The circuit may be redrawn for clarity.





Next combine the parallel circuit $C_p = 1000 + 1000 = 2000$ μF. The circuit simplifies as shown.

Now find the final value of the three in series.

$$C_T = \frac{1}{\frac{1}{500}+\frac{1}{2000}+\frac{1}{1500}} = \frac{1}{0.00316666} = 315.79\ \mu F$$

Now find the total charge $Q = C_T V = 315.79 \times 10^{-6} \times 50 = 15.789 \times 10^{-3}$ Coulomb
The charge is the same on each series part of the circuit so this charge must exist on C_1 and C_5.
Identify that zero volts is on the left of the circuit and that we must now work in Farads.
$Q = C_1 V_1 = 15.789 \times 10^{-3}$ $V_1 = Q/C_1 = 15.789 \times 10^{-3}/ 500 \times 10^{-6} = 31.578$ V
The voltage at A is 31.578 V

$Q = C_5 V_5 = 15.789 \times 10^{-3}$ $V_5 = Q/C_5 = 15.789 \times 10^{-3}/ 1500 \times 10^{-6} = 10.526$ V
$V_C = 50 - 10.526 = 39.474$ V
The voltage difference between A and B is 39.474 - 31.578 = 7.895 V
Check from $Q = V_{AB}C_p = 15.789 \times 10^{-3}$ $V_{AB} = Q/C_p = 15.789 \times 10^{-3}/ 2000 \times 10^{-6} = 7.895$ V

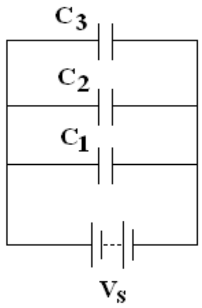
The charge divides up between C_2 and C_{34} But since these are both 1000 μF they get half each.

The charge stored on C_3 and C_4 is 7.8945×10^{-3}

The voltage difference between A and B is $V_{AB} = Q/C_3 = 7.8945 \times 10^{-3}/2000 \times 10^{-6} = 3.947$ V
The voltage at B is 31.578 + 3.947 = 35.526 V

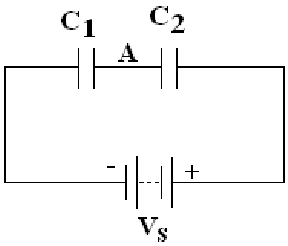
SELF ASSESSMENT EXERCISE No. 4

1. Given V_s = 6 V, C_1 = 22μF, C_2 = 47 μF, C_3 = 12 μF in the circuit below, calculate the total capacitance and the total charge.



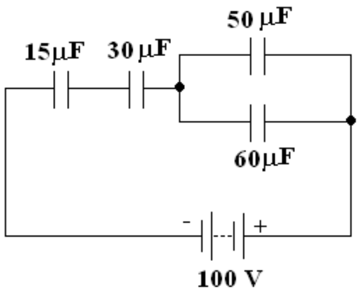
(Answers 81 μF and 486 x 10^{-6} Coulomb)

2. Given V_s = 12 V, C_1 = 220 pF and C_2 = 470 pF in the circuit below, calculate the total capacitance, the total charge and the voltage at A.



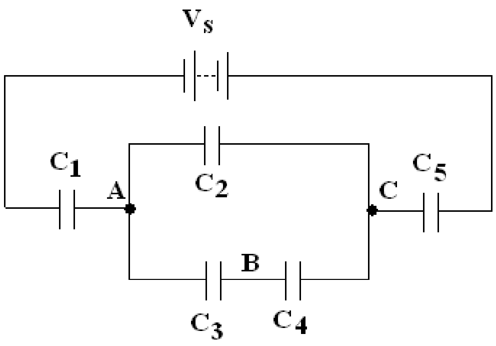
(Answers 149.855 pF, 1.798 x 10^{-9} Coulomb and 8.174 V)

3. Solve the total capacitance and the voltage at the junction between the 15 and 30 μF capacitor.



(Answers 9.167 μF and 6.11 V)

4. A network of capacitors is connected to a battery as shown. Given the listed data below, calculate the total capacitance, and the voltage at points A B and C.
V_s = 100 V C_1 = 250 μF C_2 = 470 μF C_3 = C_4 = 100 μF C_5 = 300 μF



(Answers 108 μF, 43.213 V, 53.601 V and 63.989)