SOLID MECHANICS
DYNAMICS

WORK, POWER AND ENERGY TRANSFER IN DYNAMIC ENGINEERING SYSTEMS

This is set at the British Edexcel National level NQF 3.

On completion of this tutorial you should be able explain and solve problems involving:

- Work, power and energy transfer in dynamic systems.
- The principle of conservation of momentum and the principle of conservation of energy in dynamic systems.
- The theory and consequences of dry friction
- Explain and apply D'Alembert's theorem in dynamic force systems.

Contents

1. Introduction

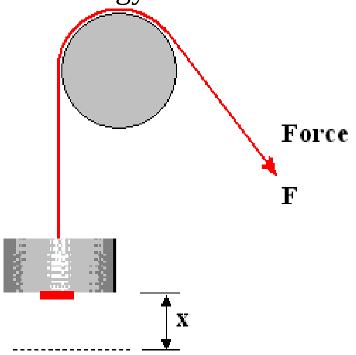
In the study of dynamic systems there are certain rules of a similar form that apply. These are the conservation of energy, the conservation of momentum and D'Alembert's principle governing the balance of forces. Before we study these we need to study work energy and power.

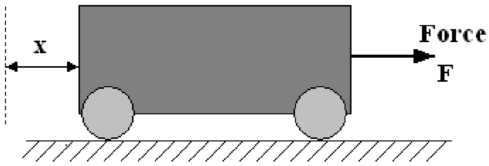
2. Work

When a force 'F' moves a distance 'x', the work done 'W' is **W = F x**

In order to do work, you must use an equal amount of energy and since energy cannot be destroyed, it must have been transferred somewhere else (Law of conservation of energy). For example if a mass is raised on a pulley, work is done and the energy of the mass increases as it is lifted. The energy used cannot have been destroyed so it must be stored in the mass as an increase in its potential (gravitational) energy.

Another example is a truck being accelerated along a floor. A force is needed to accelerate the truck and as it moves more and more work is done. The energy used to accelerate the mass becomes stored in it as kinetic energy.

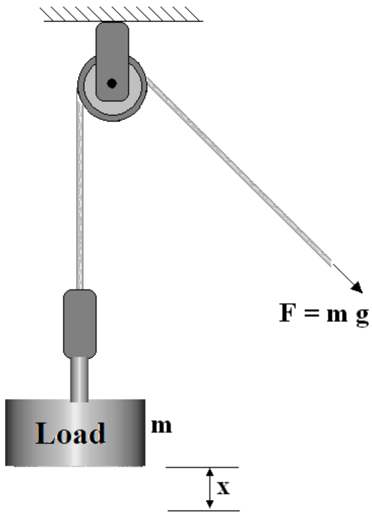




Both the examples show that energy may be transferred to a mass by doing work. It follows that ***Energy is Stored Work***

3. Energy Forms

Potential Energy



This is the energy stored in a body by virtue of its altitude z and is also called gravitational energy.
Consider a mass M kg raised a height z metres against the force of gravity. The weight is Mg.

The work done = weight × distance moved

$$W = Mgz$$

Since this energy cannot be destroyed it is stored in the mass and may be recovered. The Potential energy is:

$$P.E. = mgz$$

Note that 'z' is the S. I. symbol for altitude but 'h' for height and 'x, for distance is also commonly used.

WORKED EXAMPLE No. 1

If the mass being lifted is 200 kg and it is raised 0.6 m, determine the work done and the change in P.E. of the mass.

SOLUTION

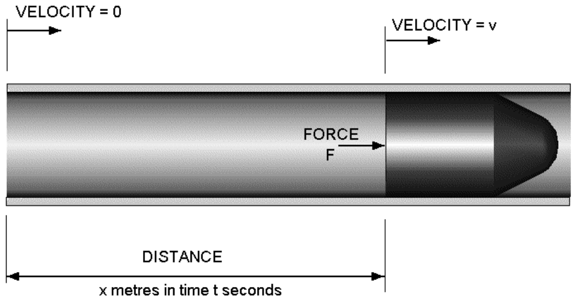
The weight is mg so the force to be overcome is F = 200 $\times$ 9.81 = 1 962 N

W = 1 962 $\times$ 0.6 = 1177.2 J

The change in P. E. is the same assuming no energy was wasted.

Kinetic Energy

This is the energy stored in a body by virtue of its velocity. Consider a mass, which is at rest. A force is applied to it and it accelerates at a m/s^2 and after t seconds it achieves a velocity of v m/s and has travelled x metres.



From the laws of motion we know that

F = M a

a = v/t   x = vt/2

The work done is

$$W = Fx = M\frac{v}{t} \times \frac{vt}{2} = \frac{Mv^2}{2}$$

Since the energy used up cannot be destroyed, it is stored in the mass and may be recovered. The Kinetic Energy is

$$\mathbf{K.E.} = \frac{\mathbf{Mv^2}}{\mathbf{2}}$$

WORKED EXAMPLE No. 2

A force of 80 N is used to pull a truck 200 m along a horizontal floor. Determine the work done and the increase in K. E.

SOLUTION

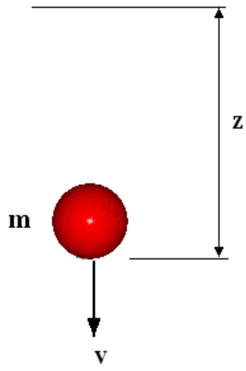
W = F x = 80 $\times$ 200 = 16 000 J If there was no energy lost due to friction then this must end up as kinetic energy in the truck.

3. *Energy Conservation*

The law of conservation of energy states that energy cannot be created nor destroyed. It may however change from form to another. When friction is involved, energy is changed into heat and often this is regarded as an energy loss because it is excluded from the problem.

Many engineering problems involve the conversion of energy from one form to another. The law of energy conservation tells us that the total energy before the conversion must be the same after the conversion. This may be used to solve problems. In later problems we shall deal with conversions with friction but first let's study simple case with no losses in the conversion process.

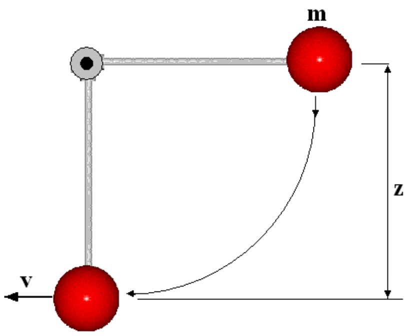
Falling Bodies



A body, which is z metres above a point, has potential energy of mgz. As it falls, the potential energy is converted in kinetic energy $mv^2/2$. If the energy conversion is perfect, then we may equate the two

$mgz = mv^2/2$ hence

$$v = \sqrt{(2gz)}$$



The formula may also be applied to swinging hammers or pendulums. The hammer starts at height z and swings down so that at the bottom of the swing the kinetic energy is equal to the potential energy lost and again

$$v = \sqrt{(2gz)}.$$

WORKED EXAMPLE No. 3

A ball of mass 0.4 kg swings on the end of a thin rod with negligible mass with length 60 mm. The ball is held horizontal and released. Calculate the following:

i. The velocity of the head as it passes through the lowest position.
ii. The loss of potential energy.
iii. The gain in kinetic energy.

SOLUTION

The ball will swing through a vertical height of 60 mm 06 0.06 m so z = 0.06

$$v = \sqrt{(2gz)} = \sqrt{(2 \times 9.81 \times 0.06)} = 1.085 \text{ m/s}.$$

Change in P. E. = $mgz = 0.4 \times 9.81 \times 0.06 = 0.2354$ J

The gain in Kinetic Energy = $mv^2/2 = 0.4 \times 1.0852/2 = 0.2354$ J
Note the energy is conserved.

SELF ASSESSMENT EXERCISE No. 1

1. An object of mass 20 kg is dropped onto a surface from a height of 50 m. Calculate the energy and velocity just before it hits the surface.

   (9 810 J and 31.3 m/s)
2. A swinging hammer must have 50 Joules of energy and a velocity of 2 m/s at the bottom of the swing. Calculate the mass and height of the hammer before it is released.

   (25 kg and 0.204 m)
3. A swinging hammer has a mass of 2 kg and is raised 0.2 m. Calculate the energy and velocity at the bottom of the swing.

   (1.98 m/s)

5. *Mechanical Power*

Power is the rate of using energy or doing work. Energy has units of Joules so power is Joules per second or Watts.

$$\mathbf{P = \frac{Energy\ used}{time\ taken}}$$

Mechanical power is the power developed by a force as opposed to say the rate of using heat which is also a form of power.

$$\text{Power} = \text{work done per second} = \frac{Fx}{t}$$

Since x/t is the velocity v of the force then we have a definition for mechanical power.

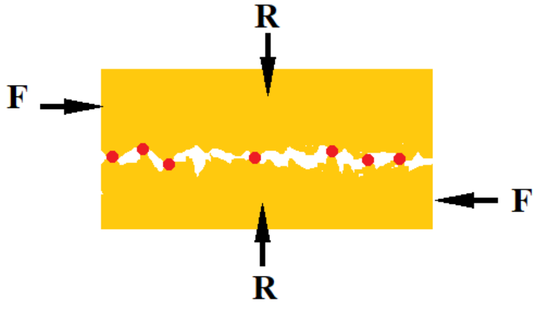
$$\mathbf{P = F\,v}$$

6. *Friction*

Friction occurs between two surfaces and resists the sliding motion of one over the other. This means that a force F is needed to overcome friction. If the force applied to the body is smaller than the friction force, it will not slide. If the force is bigger, then some force is left over to produce acceleration of the body. Friction can be reduced by lubrication to separate the surfaces but if the surfaces are dry we have ***Dry Friction.***

Coulomb's Theory

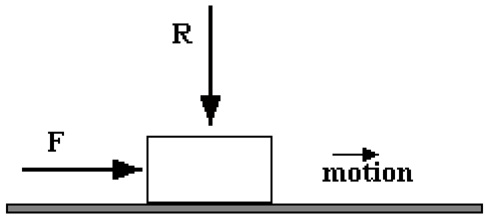
No matter how hard we try, we cannot produce a surface which is perfectly smooth and flat. If we look at a polished surface under a microscope we see it is covered in microscopic canyons and mountains. Consequently, when two surfaces come into contact, only the high points touch. If a load R is applied to squeeze the surfaces together, the points will yield and spread at the points of contact (in red) until there is enough area to take the load.



The consequence of this is that no matter how much surface area there is, the area of contact is the same. This is the first law of friction - ***the area of the surfaces do not affect the friction force.***

If the surfaces are perfectly clean, the points of contact weld together. In order to make the surfaces slide, the points must be sheared. It follows that the friction force depends upon the shear strength of the materials and that materials with low shear strength such as Teflon, PTFE, graphite and Indium, have low friction.

The area of contact will increase directly with the normal load R. It follows that the friction force F must be directly proportional to the load R. The constant of proportionality is the coefficient of friction μ



It follows that $\mathbf{F = \mu R}$

This is the second law of friction or ***Coulomb's Law***. It applies to anything sliding be it horizontally, on a slope or on a curved surface. The affect of lubrication of any kind is dramatic but this definition of the coefficient of friction is used even when it is not constant under all circumstances. For this tutorial it is important to understand the consequences of friction.

The Consequences of Friction

When a force moves the work done is Work = Force $\times$ distance moved

Doing work uses up energy. It follows that when a body moves and friction is overcome, energy is used up doing it. This energy is lost as heat. The heating is caused by the rubbing and shearing of the high points.

The shearing of the high points will cause the surface to wear away. If one surface is different to the other, the material on one may be removed and ends up on the other. You can see this for yourself if you rub a steel block on a copper plate.

Energy loss and wear of surfaces is costly and should be avoided. The science of reducing friction and wear is called ***Tribology.*** The main ways of reducing them are to use lubrication but this is not to be studied at this stage. Friction always causes energy to be wasted in the form of heat. It is ever present in mechanical mechanisms. When solving problems involving energy conversion, the conversion is less than 100%, the difference being the energy wasted by friction.

SELF ASSESSMENT EXERCISE No. 2

1. When a car skids with its wheels locked, the friction force is 3 000 N. The car weighs 2 Tonnes (1 Tonne = 1 000 kg). Determine the coefficient of friction between the car tyres and the road.
(Answer 0.153)

2. Assuming the same coefficient of friction as in Q1, determine the braking force for a lorry when it skids with its wheels locked. The lorry has a mass of 40 Tonnes.
(Answer 60 kN)

3. The coefficient of friction between the brake blocks and the wheel rim on a bicycle is 0.25. Calculate the force required to produce a friction force of 10 N. Why are bicycle brakes less effective in the rain?
(40 N)

7. *Efficiency*

Efficiency (symbol η eta) is the ratio of the energy conversion process so

$$\eta\% = \frac{\textbf{Energy Out}}{\textbf{Energy In}} \times \mathbf{100}$$

The energy wasted is

Energy Out – Energy In

In devices like friction brakes, all the energy is converted into heat by friction so the efficiency does not mean anything.

WORKED EXAMPLE No. 4

A pulley is used to raise a mass of 85 kg a distance of 12 m. The efficiency is 60%. Calculate the work done.

SOLUTION

The change in P. E. = mg z = 85 × 9.81 × 12 = 10 kJ

The Work Done is the energy in so W = 10/60% = 10/0.6 = 16.67 kJ

SELF ASSESSMENT EXERCISE No. 3

1. 5 000 J of energy is used up in 20 seconds. What is the power? (250 W)
2. A vehicle is propelled 25 000 m by a force of 2 000 N in 12 seconds. Calculate the work done and the power used. (4.17 MW)
3. A block of mass 500 kg is raised at a constant rate by a hoist at a rate of 0.15 m/s. Calculate the force in the rope and hence the power used. (4905 N and 735.8 W)
4. A load is raised by a pulley. The force in the rope is 40 N and it moves 3 m in 11 seconds. The process is 70% efficient. Calculate the mechanical power. (15.6 W)
5. A rocket flies at 120 m/s under a propulsion force of 3 000 N. Calculate the power used. (360 kW)
6. A lifting jack must raise a force of 4 kN a distance of 0.3 m. Due to friction the efficiency is only 35%. Calculate the energy used to raise the load. (3429 Joules)
7. An electric hoist raises a mass of 60 kg at a rate of 0.2 m/s. The process is 30% efficient. Calculate the power input to the hoist. (392 W)

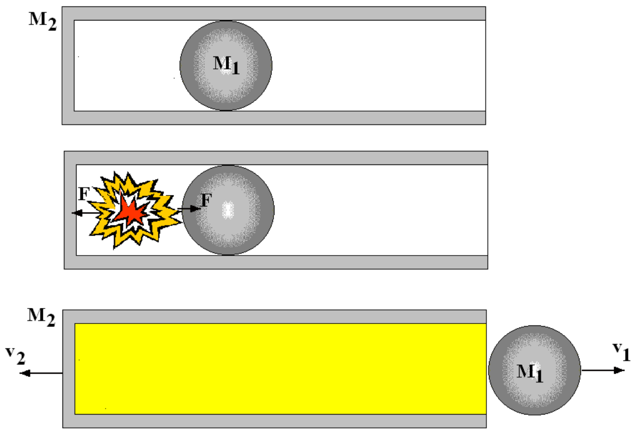
8. *Law of Conservation of Momentum*

This states that: ***The total momentum in a closed system is conserved.***

Put another way, if a body or bodies in a system undergoes changes to its motion with no external input from outside the system, then:

The total momentum in the system before the changes is equal to the total momentum after the changes.

Consider a cannon of mass M_2 that fires a cannon ball of mass M_1 as shown. Before the gunpowder is ignited the velocity of both is zero and so the total momentum of the system is zero.



When the powder is ignited the explosion exerts a force on the ball and an equal and opposite force on the cannon for the same period of time. It follows that the impulse is the same for both so the change in momentum (from zero) must be the same for both but equal and opposite.

Final momentum of the cannon = $M_2 v_2$
Final momentum of the ball = $M_1 v_1$
Total momentum = $M_1 v_1 + M_2 v_2$
Since $M_1 v_1 = - M_2 v_2$ the total momentum is zero and hence the total momentum before the change is the same as after the change and this is the law of conservation of momentum.

WORKED EXAMPLE No. 5

Suppose the cannon has a mass of 80 kg and the ball a mass of 3 kg. The explosion produces a mean force of 400 N that lasts for 0.2 seconds. Calculate the final velocity of the cannon and the ball and show that the final total momentum is zero.

SOLUTION

Impulse = F t = $400 \times 0.2 = 80$ N s

Change in momentum of the ball = $3 (v_1 - 0) = 3 v_1$
Impulse = change in momentum so $80 = 3 v_1$ $\qquad v_1 = 26.667$ m/s

Change in momentum of the cannon = $-80 (v_2 - 0) = -80 v_2$
Impulse = change in momentum so $80 = -80 v_2$
$v_2 = -1$ m/s Minus because it is acting to the left.

Total momentum after the firing is $M v_1 + M v_2 = 3 \times 26.667 - 1 \times 80 = 0$

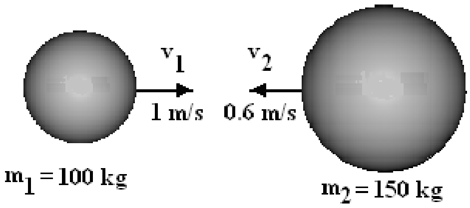
The principle of momentum conservation applies to bodies colliding. Consider two bodies of mass m_1 and m_2 moving at velocities u_1 and u_2 in the same direction. After collision the velocities change to v_1 and v_2 respectively.

The total momentum before the collision is equal to the total momentum after the collision.

We will apply this to a basic problem as follows.

WORKED EXAMPLE No. 6

A mass of 100 kg moves along a straight line at 1 m/s. It collides with a mass of 150 kg moving the opposite way along the same straight line at 0.6 m/s. The two masses join together on colliding to form one mass. Determine the velocity of the joint mass.



SOLUTION

The normal sign convention must be used namely that motion from left to right is positive and from right to left is negative.

$m_1 = 100$ kg $\quad\quad m_2 = 150$ kg

$u_1 = 1$ m/s $\quad\quad u_2 = -0.6$ m/s

Initial momentum = $(100 \times 1) + \{150 \times (-0.6)\} = 10$ kg m/s
Final momentum = 10 kg m/s (conserved)

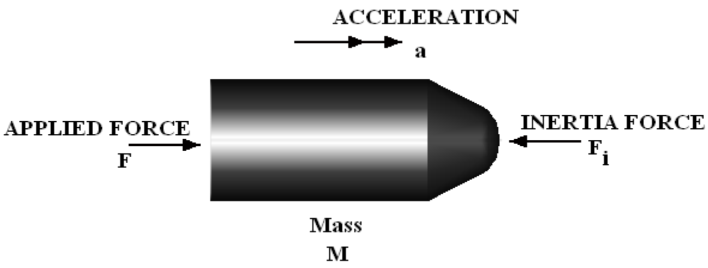
After collision the mass is 250 kg and the velocity is v.
Final momentum = 250 v = 10 $\quad$ v = 10/250 = 0.04 m/s
The combined mass ends up moving to the right at 0.04 m/s.

SELF ASSESSMENT EXERCISE No. 4

1. A railway wagon of mass 3 000 kg travelling at 0.42 m/s collides with a stationary wagon of mass 3 500 kg and becomes coupled. Determine the common velocity after collision.
(Ans. 0.194 m/s)
2. A bullet of mass 20 g is fired at 300 m/s into a bag of sand hanging on a rope and becomes embedded in the bag. The sand has a mass of 20 kg. Assuming that the rope plays no part in the problem, and the bag momentarily moves in a straight line with no rotation, calculate the velocity of the bag immediately after the impact.
(Ans. 0.2997 m/s)

9. *D'Alembert's Principle in Dynamics*

When a body accelerates, the applied force has to overcome the inertia. The inertia force resisting acceleration (or deceleration) is equal and opposite of the applied force. This means that total force acting on the body is zero. This is similar to the ideas of conservation of energy and momentum. Consider the free body diagram of an accelerating body.



The total force acting on the body is in equilibrium even though it is accelerating so we have

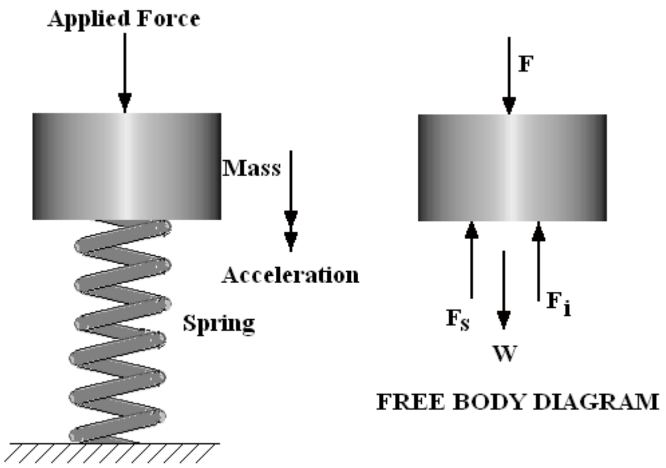
$$F + F_i = 0$$

We know F_i that will be negative in value (to the left) but when using symbols we always put plus as in this case. Evaluation of the numbers will yield the negative figure.

Since no other forces are involved, $F = Ma = - F_i$ so $F_i = - Ma$

The inertia force always opposes acceleration so it is always negative when evaluated.

The principle can be extended to many other areas of work. Consider a mass supported by a spring. If a force F is applied downwards as shown, it will accelerate the mass downwards but it will also have to overcome the force exerted by stretching the spring. The mass also has a weigh W = Mg that acts downwards.

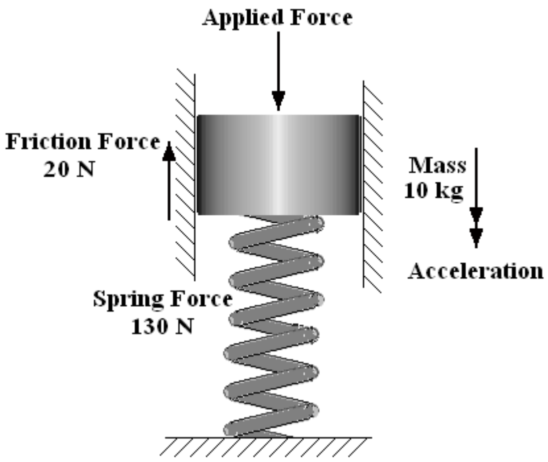


Applying D'Alembert's Principle, $F + F_s + F_i + W = 0$

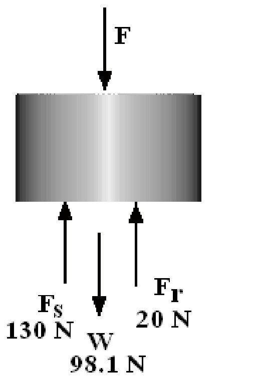
We know the spring force is up and the inertia force is up and will be opposite in sign to F and W when evaluated but when setting up the equation everything is assumed positive and we put in negative values when they the actual figures are known. Another force that might be considered is friction and this always opposes movement. The next example explains this.

WORKED EXAMPLE No. 7

A spring is fixed at the left end and has a sliding mass of 10 kg at the other as shown. A force of 100 N is applied to move the mass. At the given instant shown, the spring force is 130 N. A friction force of 20 N also exists. Draw the free body diagram and calculate the acceleration of the mass at that instant.



SOLUTION



FREE BODY DIAGRAM

Free Body Diagram

The spring force acts to the left. The friction force and the inertia force oppose movement and both act to the left. It follows that

$$F + F_i + F_s + F_r + W = 0$$

F = -100 N (Down)
W = -Mg = -10 × 9.81 = -98.1 N (Down)
F_s = 130 N (Up)
F_r = 20 N (Up)

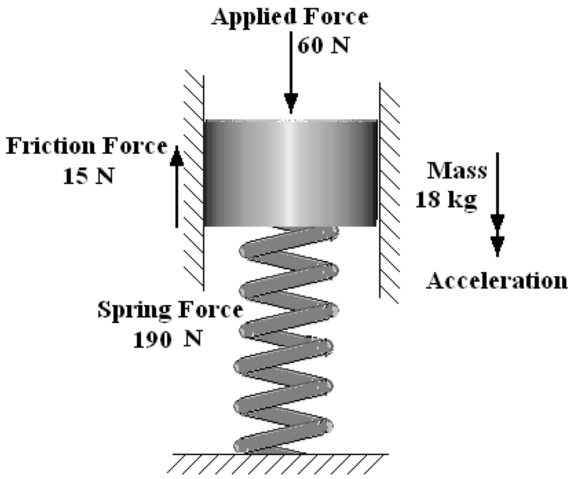
$$-100 + F_i + 130 + 20 - 98.1 = 0$$

F_i = 48.1 N (Up)
F_i = -M a   48.1 = -10 a   a = (48.1)/(-10) = -4.81m/s^2 negative so down as expected.

There is always confusion about the signs to use. Generally if the value is unknown and represented by a symbol, then make it plus even if you are sure the result is going to be minus and it will come out minus to confirm your assumption as in this case.

SELF ASSESSMENT EXERCISE No. 5

1. Calculate the acceleration of the mass at the instant shown for the system in the diagram.



(Ans. 1.75 m/s^2 down)