SOLID MECHANICS

TUTORIAL – MECHANISMS

KINEMATICS - VELOCITY AND ACCELERATION DIAGRAMS

This work covers elements of the syllabus for the Engineering Council exams C105 Mechanical and Structural Engineering and D225 Dynamics of Mechanical Systems.

On completion of this short tutorial you should be able to do the following.

- Describe a mechanism.
- Define relative and absolute velocity.
- Define relative and absolute acceleration.
- Define radial and tangential velocity.
- Define radial and tangential acceleration.
- Describe a four bar chain.
- Solve the velocity and acceleration of points within a mechanism.
- Use mathematical and graphical methods.
- Construct velocity and acceleration diagrams.
- Define the Coriolis Acceleration.
- Solve problems involving sliding links.

It is assumed that the student is already familiar with the following concepts.

- Vector diagrams.
- Simple harmonic motion.
- Angular and linear motion.
- Inertia force.
- Appropriate level of mathematics.

All these above may be found in the pre-requisite tutorials.

1. INTRODUCTION

A mechanism is used to produce mechanical transformations in a machine. This transformation could be any of the following.

- It may convert one speed to another speed.
- It may convert one force to another force.
- It may convert one torque to another torque.
- It may convert force into torque.
- It may convert one angular motion to another angular motion.
- It may convert angular motion into linear motion.
- It may convert linear motion into angular motion.

A good example is a crank, connecting rod and piston mechanism.

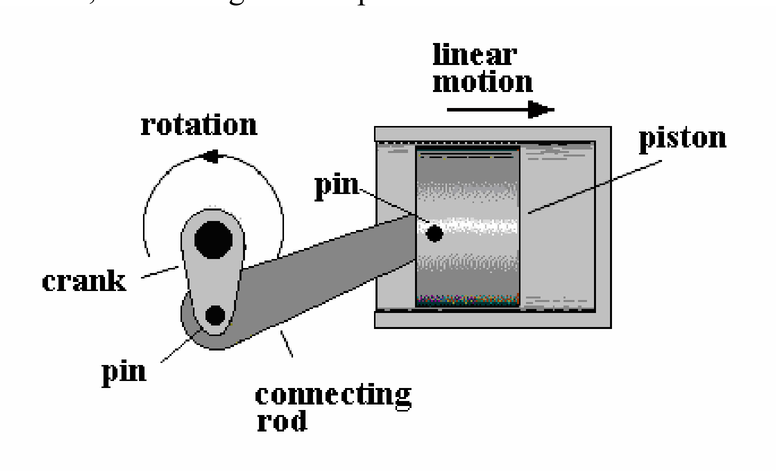


Figure 1

If the crank is turned, angular motion is converted into linear motion of the piston and input torque is transformed into force on the piston. If the piston is forced to move, the linear motion is converted into rotary motion and the force into torque. The piston is a sliding joint and this is called ***PRISMATIC*** in some fields of engineering such as robotics. The pin joints allow rotation of one part relative to another. These are also called ***REVOLUTE*** joints in other areas of engineering.

Consider the next mechanism used in shaping machines and also known as the Whitworth quick-return mechanism.

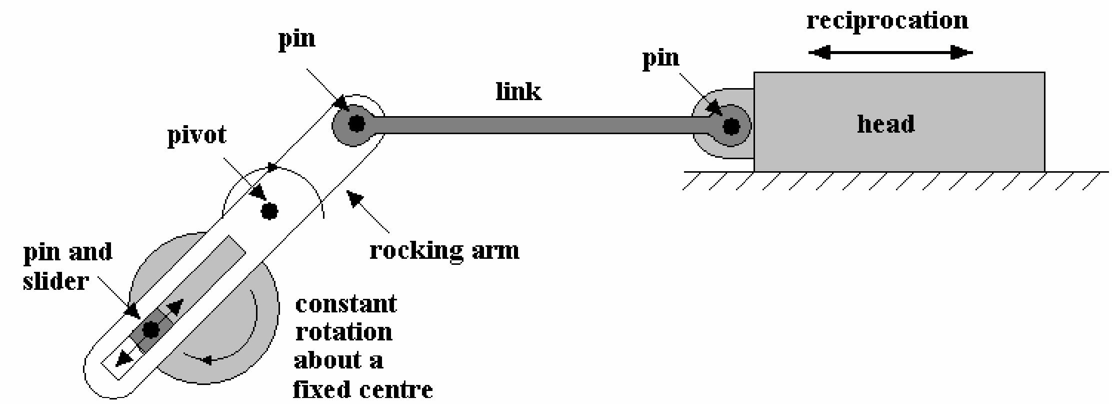


Figure 2

The input is connected to a motor turning at constant speed. This makes the rocking arm move back and forth and the head (that carries the cutting tool) reciprocates back and forth. Depending on the lengths of the various parts, the motion of the head can be made to move forwards at a fairly constant cutting speed but the return stroke is quick. Note that the pin and slider must be able to slide in the slot or the mechanism would jam. This causes problems in the solution because of the sliding link and this is covered later under Coriolis acceleration.

The main point is that the motion produced is anything but simple harmonic motion and at any time the various parts of the mechanism have a displacement, velocity and acceleration. The acceleration gives rise to inertia forces and this puts stress on the parts in addition to the stress produced by the transmission of power. For example the acceleration of a piston in an internal combustion engine can be enormous and the connecting rod is subjected to high stresses as a result of the inertia as well as due to the power transmission.

You will find in these studies that the various parts are referred to as links and it can be shown that all mechanisms are made up of a series of four links. The basic four bar link is shown below. When the input link rotates the output link may for example swing back and forth. Note that the fourth link is the frame of the machine and it is rigid and unable to move. With experience you should be able to identify the four bar chains in a mechanism. All the links shown are rigid links which means they may push or pull. It is possible to have links made of chain or rope which can only pull.

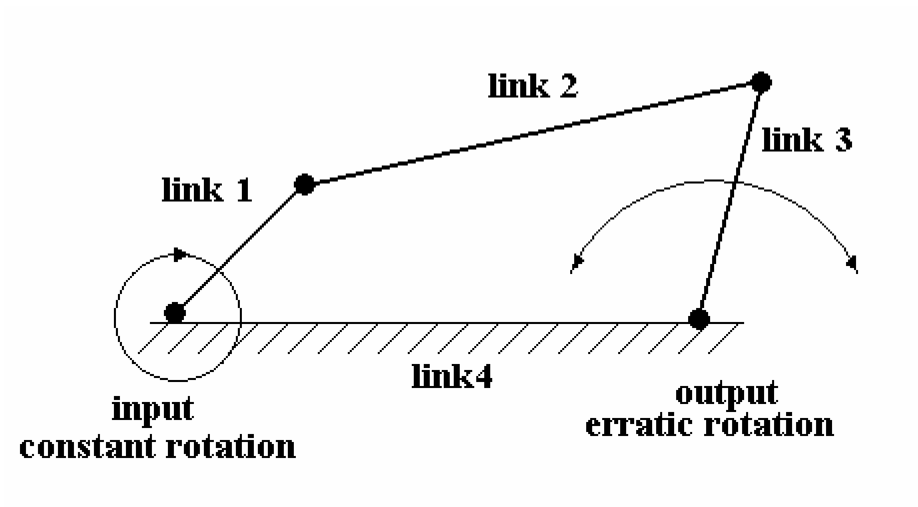


Figure 3

2. DISPLACEMENT, VELOCITY AND ACCELERATION

All parts of a mechanism have displacement, velocity and acceleration. In the tutorial on free vibration, a mechanism called the Scotch Yoke was examined in order to explain sinusoidal or harmonic motion. The wheel turns at a constant speed and the yoke moves up and down.

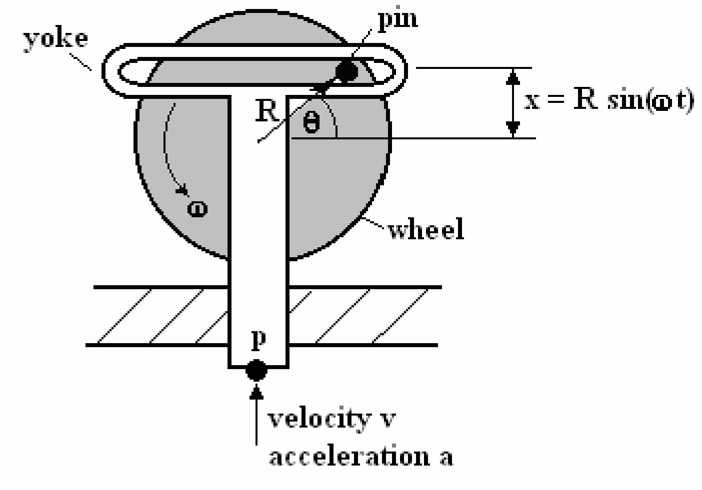


Figure 4

It was shown that the displacement 'x', velocity 'v' and acceleration 'a' of point p was given as follows.

Angle $\theta = \omega t$

Displacement $x = R \sin(\omega t)$.

Velocity $v = dx/dt = \omega R \cos(\omega t)$

Acceleration $a = dv/dt = -\omega^2 R \sin(\omega t)$

The values can be calculated for any angle or moment of time. The acceleration could then be used to calculate the inertia force needed to accelerate and decelerate the link. Clearly it is the maximum values that are needed. Other mechanisms can be analysed mathematically in the same way but it is more difficult. The starting point is to derive the equation for displacement with respect to angle or time and then differentiate twice to get the acceleration. Without the aid of a computer to do this, the mathematics is normally much too difficult and a graphical method should be used as shown later.

WORKED EXAMPLE No.1

A crank, con rod and piston mechanism is shown below. Determine the maximum acceleration of the piston when the crank speed is 30 rev/min clockwise.

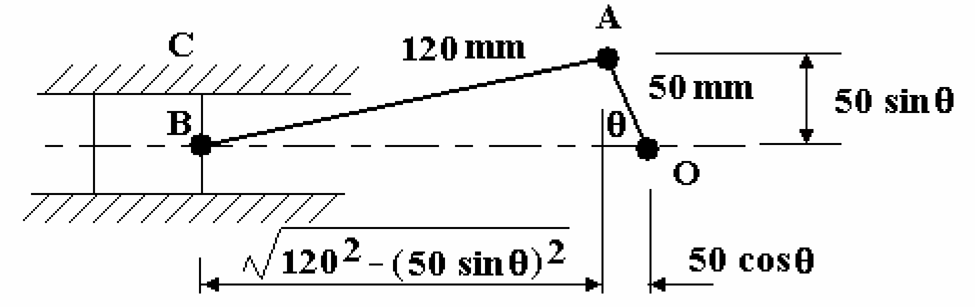


Figure 5

SOLUTION

When θ = 0 the piston will be furthest left at a distance of 170 mm from point O. Take this as the reference point and measure displacement x from there. Remember that θ = ωt and ω = 2π x 30/60 = 3.142 rad/s. The displacement is then

$$\mathbf{x} = 170 - (50\cdot\cos(\omega\cdot t)) - \left[\sqrt{120^2 - (50\cdot\sin(\omega\cdot t))^2}\right]$$

Differentiate to get the velocity

$$\mathbf{v} = 50\cdot\sin(\omega\cdot t)\cdot\omega + \frac{2500}{\sqrt{14400 - 2500\cdot\sin(\omega\cdot t)^2}}\cdot\sin(\omega\cdot t)\cdot\cos(\omega\cdot t)\cdot\omega$$

Differentiate again to get the acceleration.

$$\mathbf{a} = 50\cdot\cos(\omega\cdot t)\cdot\omega^2 + \frac{6250000}{\left(14400 - 2500\cdot\sin(\omega\cdot t)^2\right)^{\left(\frac{3}{2}\right)}}\cdot\sin(\omega\cdot t)^2\cdot\cos(\omega\cdot t)^2\cdot\omega^2$$
$$+ \frac{2500}{\sqrt{14400 - 2500\cdot\sin(\omega\cdot t)^2}}\cdot\cos(\omega\cdot t)^2\cdot\omega^2 - \frac{2500}{\sqrt{14400 - 2500\cdot\sin(\omega\cdot t)^2}}\cdot\sin(\omega\cdot t)^2\cdot\omega^2$$

The diagram shows a plot of displacement, velocity and acceleration against angle. It should be noted that none of them are sinusoidal and not harmonic (in particular, the acceleration).

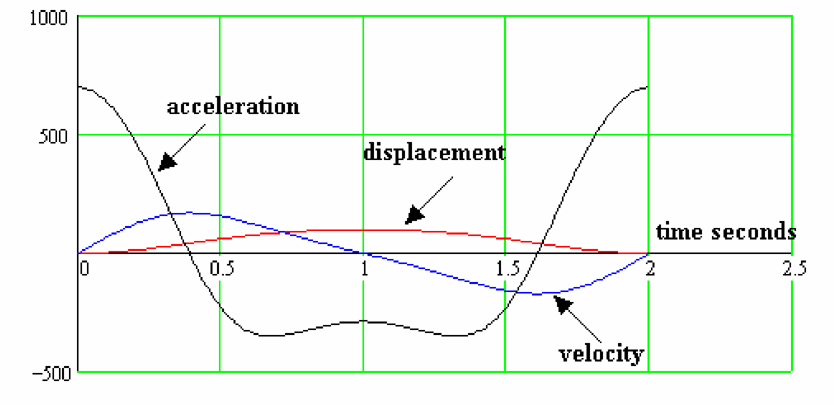


Figure 6

The units are all in mm and seconds. The above was done with a computer package. Plotting the above functions over a complete rotation shows that the maximum acceleration occurs at t = 0 (θ = 0) and evaluating gives an answer of 700 mm/s^2.

If the radius of the crank is small in comparison to the length of the connecting rod, the motion becomes close to sinusoidal. To illustrate this, here is the plot with the crank radius reduced to 10 mm. The acceleration is now almost a cosine curve.

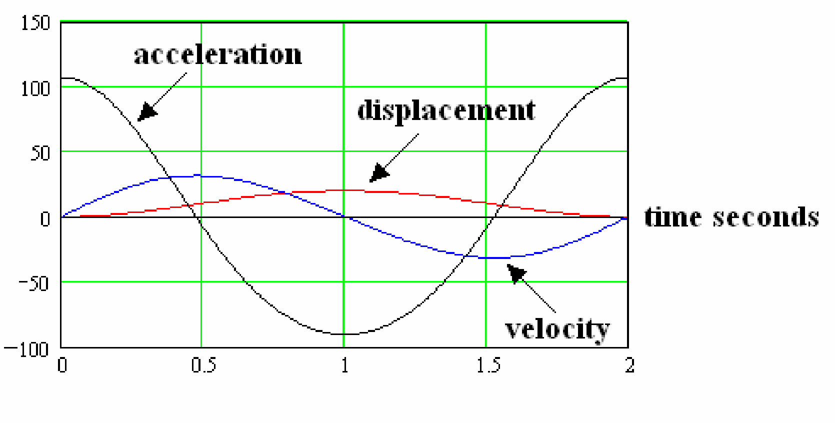


Figure 7

Solving these problems mathematically is difficult so we will now look at a graphical method.

3. VELOCITY DIAGRAMS

This section involves the construction of diagrams which needs to be done accurately and to a suitable scale. Students should use a drawing board, ruler, compass, protractor and triangles and possess the necessary drawing skills.

ABSOLUTE AND RELATIVE VELOCITY

An absolute velocity is the velocity of a point measured from a fixed point (normally the ground or anything rigidly attached to the ground and not moving). Relative velocity is the velocity of a point measured relative to another that may itself be moving.

TANGENTIAL VELOCITY

Consider a link A B pinned at A and revolving about A at angular velocity ω. Point B moves in a circle relative to point A but its velocity is always tangential and hence at 90^o to the link. A convenient method of denoting this tangential velocity is $(v_B)_A$ meaning the velocity of B relative to A. This method is not always suitable.

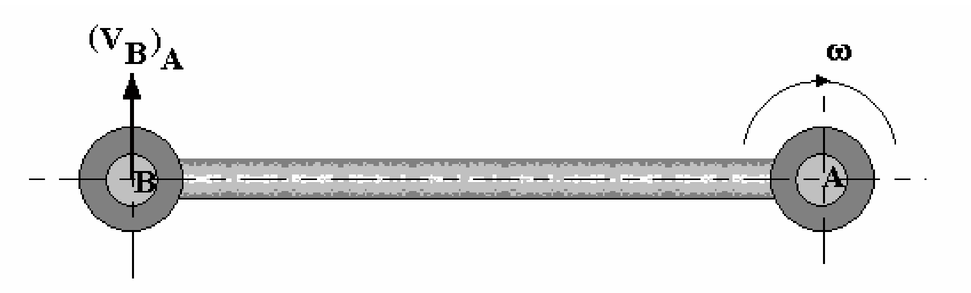


Figure 8

RADIAL VELOCITY

Consider a sliding link C that can slide on link AB. The direction can only be radial relative to point A as shown. If the link AB rotates about A at the same time then link C will have radial and tangential velocities.

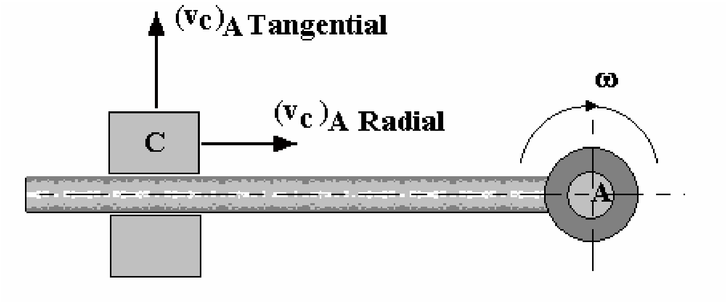


Figure 9

Note that both the tangential and radial velocities are denoted the same so the tags radial and tangential are added.

The sliding link has two relative velocities, the radial and the tangential. They are normal to each other and the true velocity relative to A is the vector sum of both added as shown. ***Note that lower case letters are used on the vector diagrams.*** The two vectors are denoted by c_1 and c_2. The velocity of link C relative to point A is the vector a c_2.

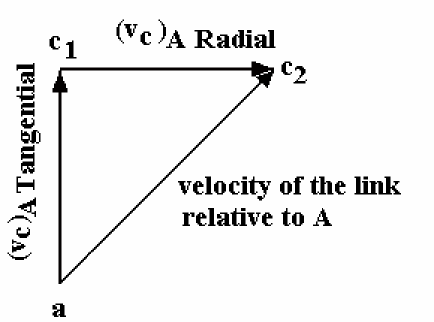


Figure 10

CRANK, CONNECTING ROD AND PISTON

Consider this mechanism again. Let's freeze the motion (snap shot) at the position shown. The diagram is called a space diagram.

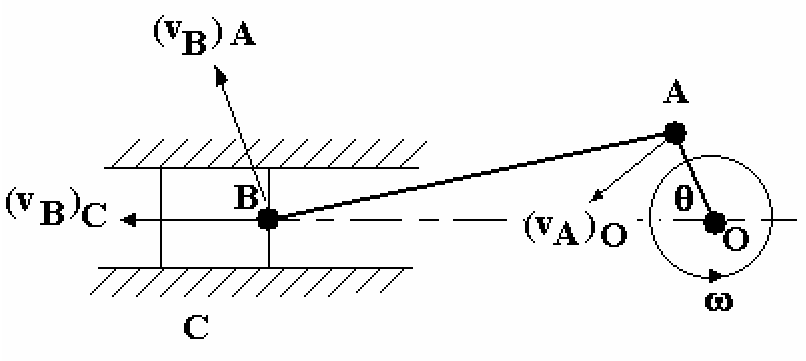


Figure 11

Every point on every link has a velocity through space. First we label the centre of rotation, often this is the letter O. Point A can only move in a tangential direction so the velocity of A relative to O is also its absolute velocity and the vector is normal to the crank and it is designated $(v_A)_O$. (Note the rotation is anticlockwise).

Now suppose that you are sat at point A and everything else moves relative to you. Looking towards B, it would appear the B is rotating relative to you (in reality it is you that is rotating) so it has a tangential velocity denoted $(V_B)_A$. The direction is not always obvious except that it is normal to the link.

Consider the fixed link OC. Since both points are fixed there is no velocity between them so so $(v_C)_O = 0$

Next consider that you at point C looking at point B. Point B is a sliding link and will move in a straight line in the direction fixed by the slider guides and this is velocity $(v_B)_C$. It follows that the velocity of B seen from O is the same as that seen from C so $(v_B)_C = (v_B)_O$

The absolute velocity of B is $(v_B)_C = (v_B)_O$ and this must be the vector sum of $(V_A)_O$ and $(v_B)_A$ and the three vectors must form a closed triangle as shown. The velocity of the piston must be in the direction in which it slides (conveniently horizontal here). This is a velocity diagram.

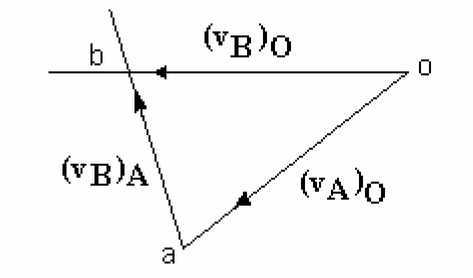


Figure 12

METHODOLOGY

First calculate the tangential velocity $(v_A)_O$ from $v = \omega \times \text{radius} = \omega \times OA$

Draw the vector o - a in the correct direction (note lower case letters).

We know that the velocity of B relative to A is to be added so the next vector ab starts at point a. At point a draw a line in the direction normal to the connecting rod but of unknown length.

We know that the velocity of B relative and absolute to O is horizontal so the vector ob must start at a. Draw a horizontal line (in this case) through o to intersect with the other line. This is point b. The vectors ab and ob may be measured or calculated. Usually it is the velocity of the slider that is required.

In a design problem, this velocity would be evaluated for many different positions of the crank shaft and the velocity of the piston determined for each position.

Remember that the slider direction is not always horizontal and the direction of o - b must be the direction of sliding.

WORKED EXAMPLE No.2

The mechanism shown has a crank 50 mm radius which rotates at 2000 rev/min. Determine the velocity of the piston for the position shown. Also determine the angular velocity of link AB about A.

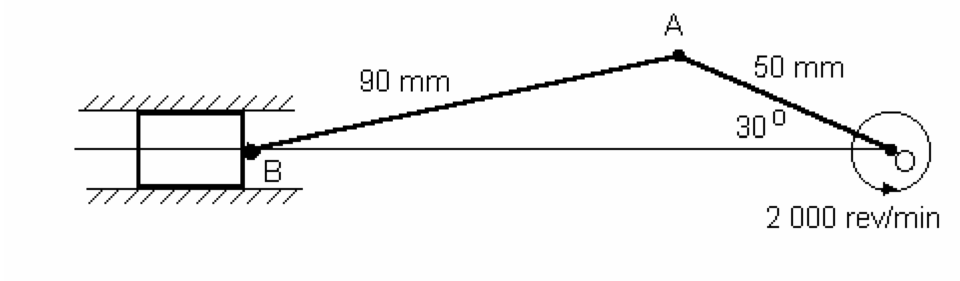


Figure 13

SOLUTION

Note the diagrams are not drawn to scale. The student should do this using a suitable scale for example 1 cm = 1 m/s. This is important so that the direction at 90^o to the link AB can be transferred to the velocity diagram.

Angular speed of the crank $\omega = 2\pi N/60 = 2\pi \times 2000/60 = 209.4$ rad/s
$(v_A)_O = \omega \times \text{radius} = 209.4 \times 0.05 = 10.47$ m/s.

First draw vector oa. (diagram a)
Next add a line in the direction ab (diagram b)
Finally add the line in the direction of ob to find point b and measure ob to get the velocity. (diagram C).

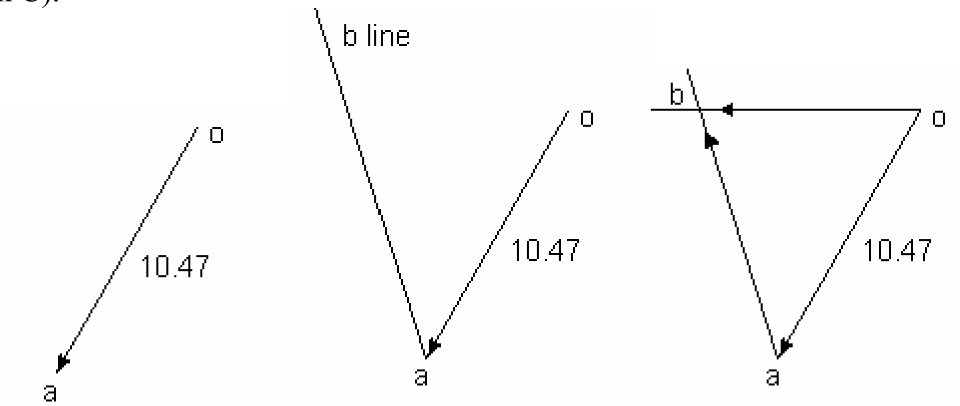


Figure 14a Figure 14b Figure 14c

The velocity of B relative to O is 7 m/s.
The tangential velocity of B relative to A is the vector ab and this gives 9.2 m/s.
The angular velocity of B about A is found by dividing by the radius (length of AB).
ω for AB is then 9.2/0.09 = 102.2 rad/s. (note this is relative to A and not an absolute angular velocity)

SELF ASSESSMENT EXERCISE No.1

Find the velocity of the piston for each case below and the angular velocity of AB about point A.

1. The crank OA rotates anti-clockwise at 3000 rev/min.

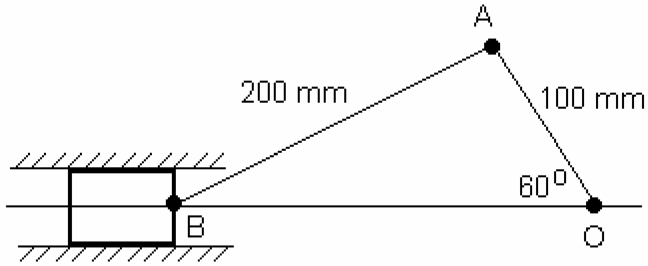


Figure 15

Answer 34 m/s and 21.25 rad/s

2. The crank revolves clockwise at 300 rev/min. Note that the vector ob is still horizontal because the piston can only slide horizontally relative to O. Also the rotation of the crank is opposite to the previous cases so the direction of oa is down to the right.

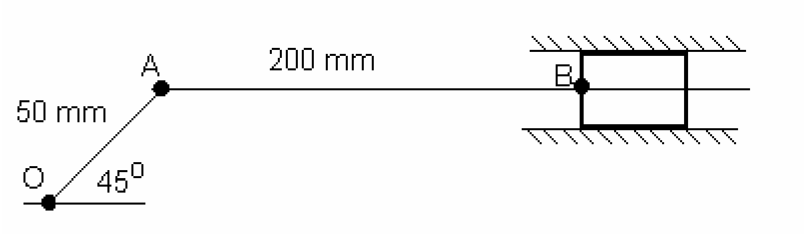


Figure 16

Answer 1.11 m/s to the right and 5.55 rad/s

3. The crank O-A rotates at 200 rev/min clockwise. Note the vector ob is at 45^o to the horizontal as the piston must slide in this direction.

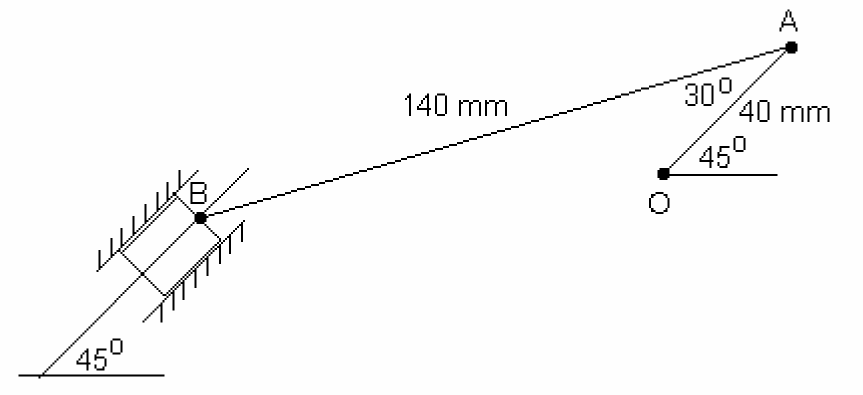


Figure 17

Answer 0.49 m/s and 6.92 rad/s.

4 BAR CHAIN

The input link rotates at a constant angular velocity ω_1. The relative velocity of each point relative to the other end of the link is shown. Each velocity vector is at right angles to the link. The output angular velocity is ω_2 and this will not be constant. The points A and D are fixed so they will appear as the same point on the velocity diagram. The methodology is the same as before and best shown with another example.

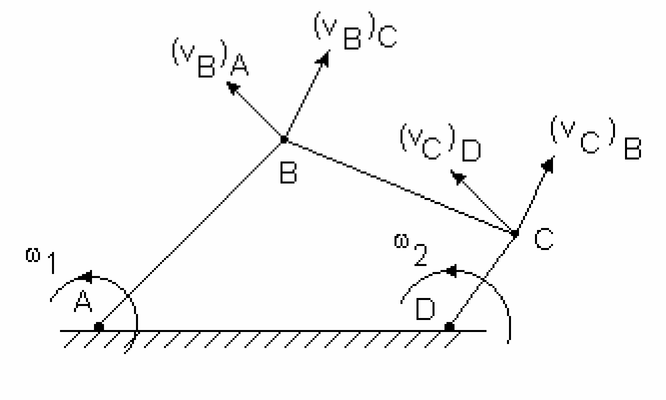


Figure 18

WORKED EXAMPLE No. 3

Find the angular velocity of the output link when the input rotates at a constant speed of 500 rev/min. The diagram is not to scale.

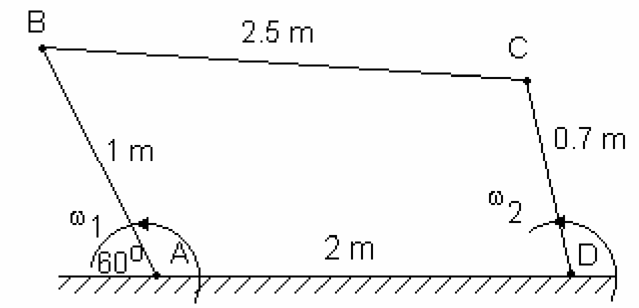


Figure 19

SOLUTION

First calculate ω_1.

$\omega_1 = 2\pi \times 500/60 = 52.36$ rad/s.

Next calculate the velocity of point B relative to A.

$(V_B)_A = \omega_1 \times AB = 52.36 \times 1 = 52.36$ m/s.

Draw this as a vector to an appropriate scale.

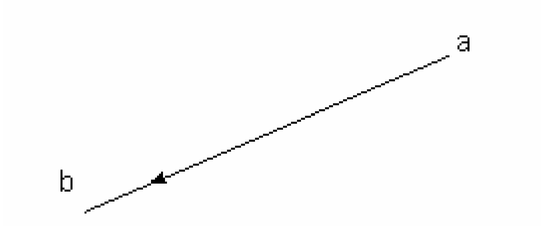


Figure 20a

Next draw the direction of velocity C relative to B at right angles to the link BC passing through point b on the velocity diagram.

Next draw the direction of the velocity of C relative to D at right angles to link DC passing through point a (which is the same as point d). Point c is where the two lines intersect,

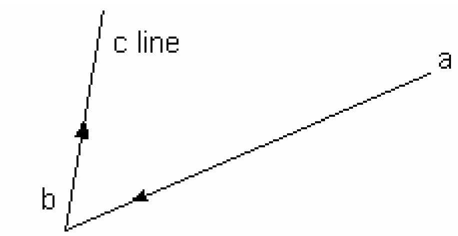


Figure 20b

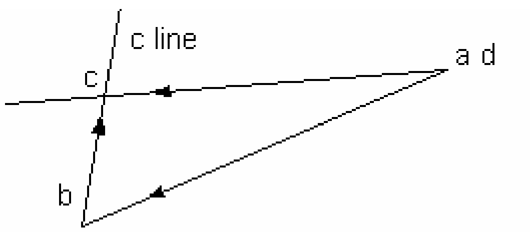


Figure 20c

Determine velocity cd by measurement or any other method. The velocity of point C relative to D and is 43.5 m/s.

Convert this into angular velocity by dividing the length of the link DC into it.

$\omega_2 = 43.5/0.7 = 62$ rad/s.

SELF ASSESSMENT EXERCISE No. 2

Determine the angular velocity of the link DC for each case shown and the direction of rotation. The diagrams are not to scale and should be constructed first. You are advised to use the best drawing instruments possible for accuracy.

1. The input rotates at 500 rev/min. Link BC is horizontal.

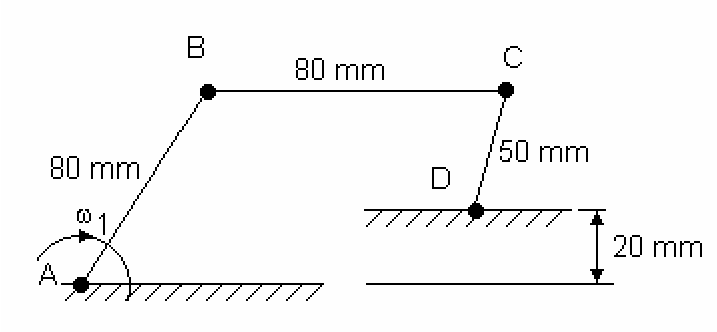


Figure 21

(Ans. 76 rad/s clockwise.)

2. The input link AB rotates at 60 rev/min in a clockwise direction.

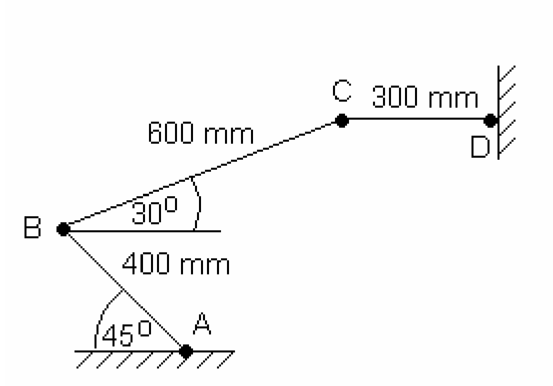


Figure 22

(Ans. 16 rad/s)

4. ACCELERATION DIAGRAMS

It is important to determine the acceleration of links because acceleration produces inertia forces in the link which stress the component parts of the mechanism. Accelerations may be relative or absolute in the same way as described for velocity.

We shall consider two forms of acceleration, tangential and radial. Centripetal acceleration is an example of radial.

CENTRIPETAL ACCELERATION

A point rotating about a centre at radius R has a tangential velocity v and angular velocity ω and it is continually accelerating towards the centre even though it never moves any closer. This is centripetal acceleration and it is caused by the constant change in direction. It follows that the end of any rotating link will have a centripetal acceleration towards the opposite end.

The relevant equations are: $v = \omega R$ $a = \omega^2 R$ or $a = v^2/R$.

The construction of the vector for radial acceleration causes confusion so the rules must be strictly followed. Consider the link AB. The velocity of B relative to A is tangential $(v_B)_A$.

The centripetal acceleration of B relative to A is in a radial direction so a suitable notation might be a_R. It is calculated using $a_R = \omega \text{ x } AB$ or $a_R = v^2/AB$.

Note the direction is towards the centre of rotation but the vector starts at a and ends at b_1. It is very important to get this the right way round otherwise the complete diagram will be wrong.

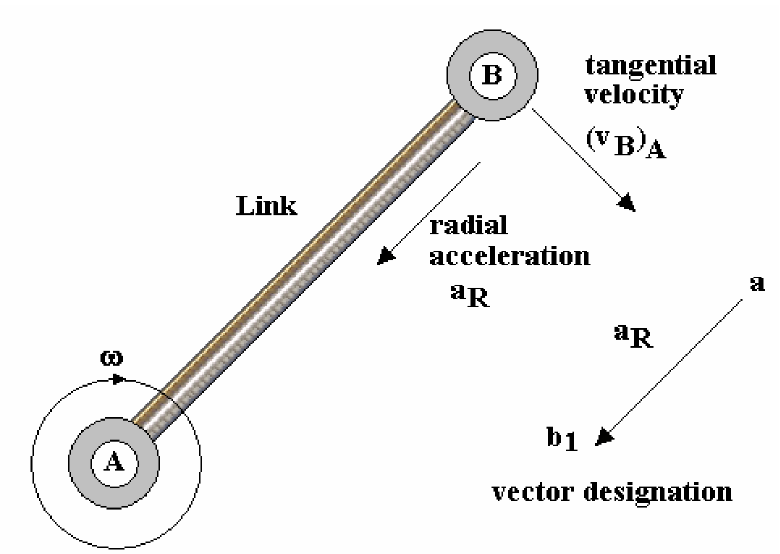


Figure 23

TANGENTIAL ACCELERATION

Tangential acceleration only occurs if the link has an angular acceleration α rad/s^2. Consider a link AB with an angular acceleration about A.

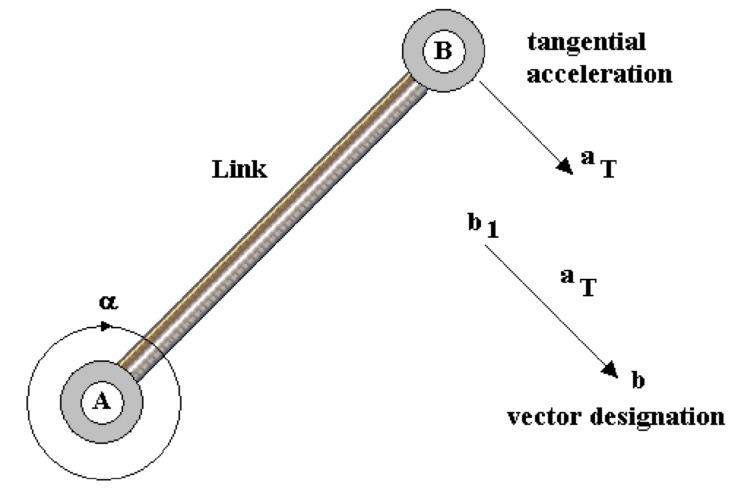


Figure 24

Point B will have both radial and tangential acceleration relative to point A. The true acceleration of point B relative to A is the vector sum of them. This will require an extra point. We will use b_1 and b on the vector diagram as shown.

Point B is accelerating around a circular path and its direction is tangential (at right angles to the link). It is designated a_T and calculated using $a_T = \alpha \times AB$. The vector starts at b_1 and ends at b. The choice of letters and notation are arbitrary but must be logical to aid and relate to the construction of the diagram.

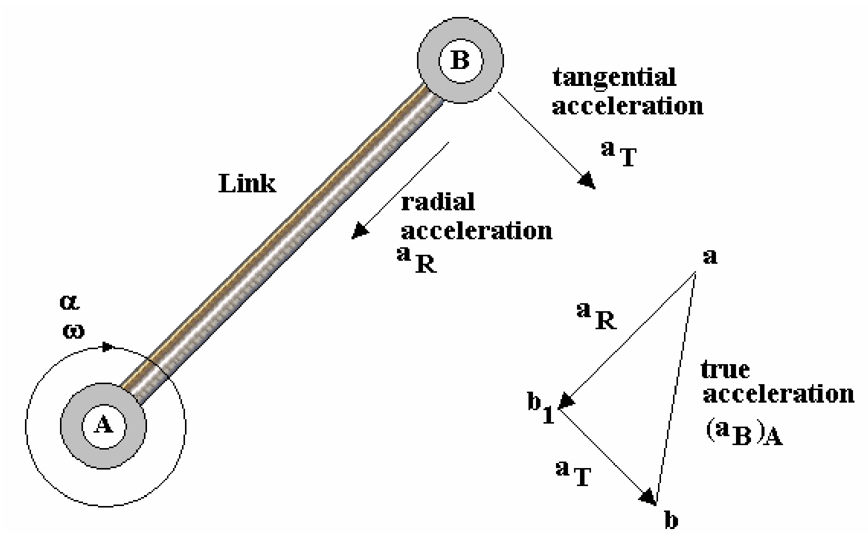


Figure 25

WORKED EXAMPLE No.4

A piston, connecting rod and crank mechanism is shown in the diagram. The crank rotates at a constant velocity of 300 rad/s. Find the acceleration of the piston and the angular acceleration of the link BC. The diagram is not drawn to scale.

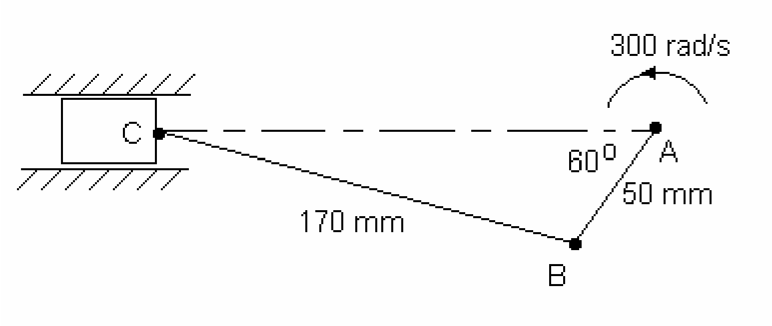


Figure 26

SOLUTION

First calculate the tangential velocity of B relative to A.
$(v_B)_A = \omega$ x radius = 300 x 0.05 = 15 m/s.
Next draw the velocity diagram and determine the velocity of C relative to B.

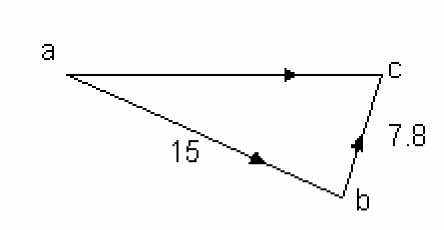


Figure 27

From the velocity diagram $(v_C)_B$ = 7.8 m/s

Next calculate all accelerations possible and construct the acceleration diagram to find the acceleration of the piston.

The tangential acceleration of B relative to A is zero in this case since the link has no angular acceleration ($\alpha = 0$).

The centripetal acceleration of B relative to A
$a_R = \omega^2 \text{x AB} = 300^2$ x 0.05 = 4500 m/s^2.

The tangential acceleration of C relative to B is unknown.

The centripetal acceleration of C to B
$a_R = v^2/BC = 7.8^2 /0.17 = 357.9$ m/s^2.

The stage by stage construction of the acceleration diagram is as follows.

First draw the centripetal acceleration of link AB (Fig.a). There is no tangential acceleration so designate it ab. Note the direction is the same as the direction of the link towards the centre of rotation but is starts at a and ends at b.

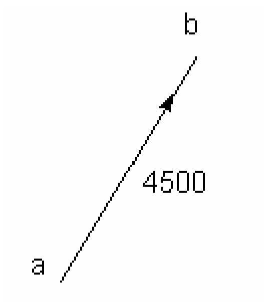


Figure 28a

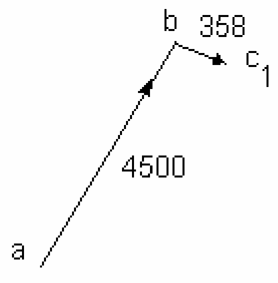


Figure 28b

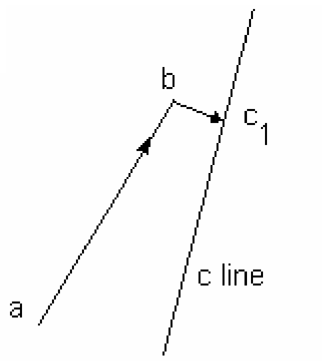


Figure 28c

Next add the centripetal acceleration of link BC (Figure b). Since there are two accelerations for point C designate the point c_1. Note the direction is the same as the direction of the link towards the centre of rotation.

Next add the tangential acceleration of point C relative to B (Figure c). Designate it c_1 c. Note the direction is at right angles to the previous vector and the length is unknown. Call the line a c line.

Next draw the acceleration of the piston (figure d) which is constrained to be in the horizontal direction. This vector starts at a and must intersect the c line. Designate this point c.

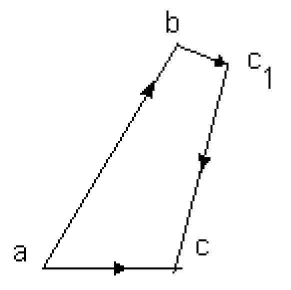


Figure 28d

The acceleration of the piston is vector ac so $(a_C)_B = 1505\ \text{m/s}^2$.

The tangential acceleration of C relative to B is c_1 c = 4000 m/s^2.

At the position shown the connecting rod has an angular velocity and acceleration about its end even though the crank moves at constant speed.

The angular acceleration of BC is the tangential acceleration divided by the length BC.

$\alpha_{(BC)} = 4000 / 0.17 = 23529\ \text{rad/s}^2$.

WORKED EXAMPLE No.5

The diagrams shows a "rocking lever" mechanism in which steady rotation of the wheel produces an oscillating motion of the lever OA. Both the wheel and the lever are mounted in fixed centres. The wheel rotates clockwise at a uniform angular velocity (ω) of 100 rad/s. For the configuration shown, determine the following.

(i) The angular velocity of the link AB and the absolute velocity of point A.
(ii) The centrifugal accelerations of BC, AB and OA.
(iii) The magnitude and direction of the acceleration of point A.

The lengths of the links are as follows.
BC = 25 mm AB = 100 mm OA = 50 mm OC = 90 mm

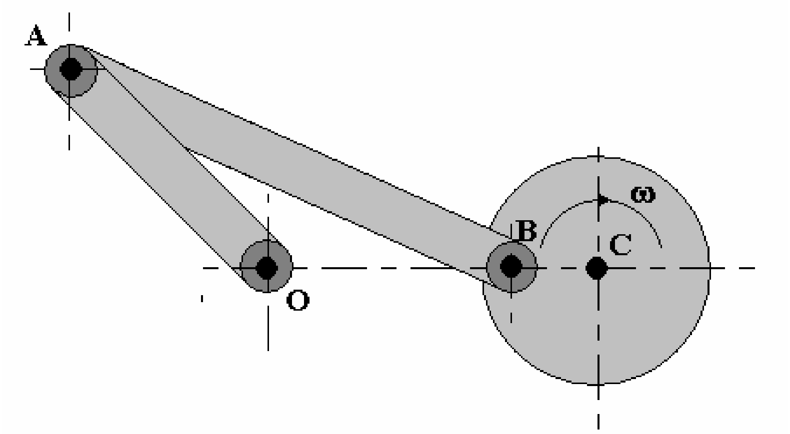


Figure 29

SOLUTION

The solution is best done graphically. First draw a line diagram of the mechanism to scale. It should look like this.

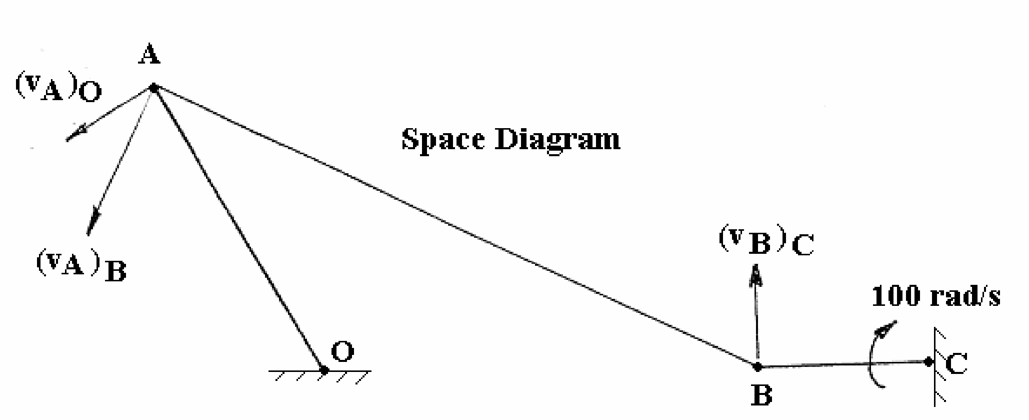


Figure 30

Next calculate the velocity of point B relative to C and construct the velocity diagram.

$(v_B)_C = \omega \times \text{radius} = 100 \times 0.025 = 2.5$ m/s

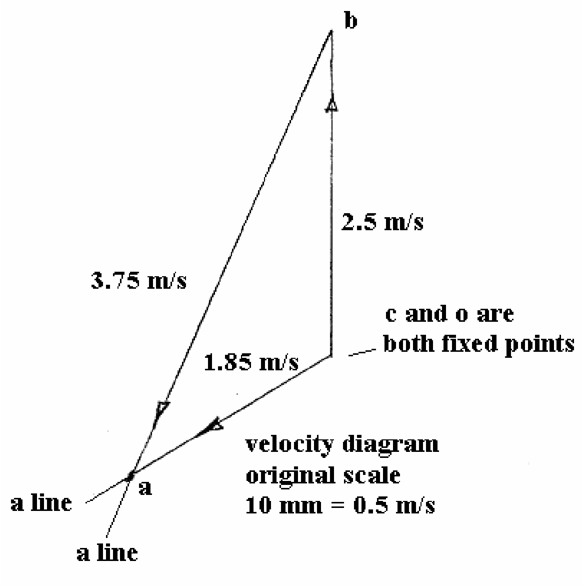


Figure 31

Scale the following velocities from the diagram.
$(v_A)_O = 1.85$ m/s {answer (i)} $(v_A)_B = 3.75$ m/s
Angular velocity = tangential velocity/radius
For link AB, $\omega = 3.75/0.1 = 37.5$ rad/s. {answer (i)}
Next calculate all the accelerations possible.
Radial acceleration of BC = $\omega^2 \times BC = 100^2 \times 0.025 = 250$ m/s^2. {answer (ii)}
Radial acceleration of AB = $v^2/AB = 3.75^2/0.1 = 140.6$ m/ s^2. {answer (ii)}
Check same answer from $\omega^2 \times AB = 37.5^2 \times 0.1 = 140.6$ m/ s^2.
Radial Acceleration of OA is $v^2/OA = 1.85^2/0.05 = 68.45$ m/ s^2. {answer (ii)}
Construction of the acceleration diagram gives the result shown.

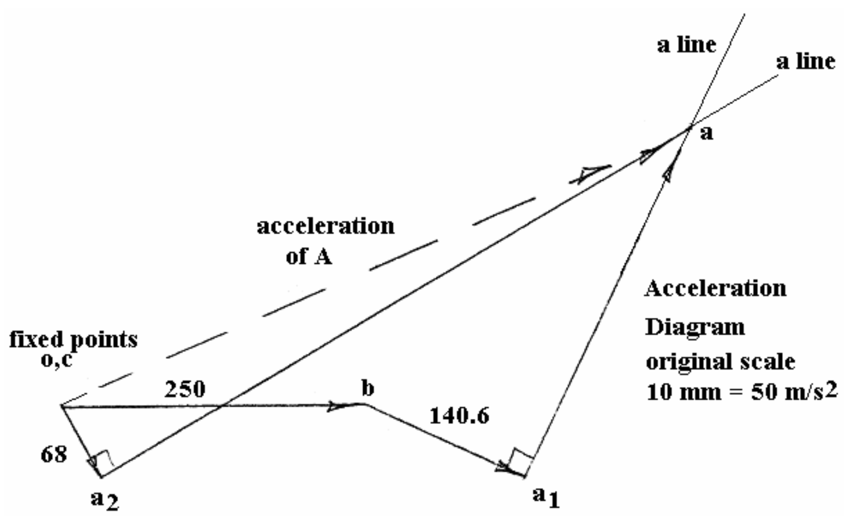


Figure 32

The acceleration of point A is the vector o- a shown as a dotted line. Scaling this we get 560 m/s^2. {answer (iii)}

SELF ASSESSMENT EXERCISE No.3

Solve the acceleration of the piston for each case shown. You should draw the space diagram out accurately first.

1.

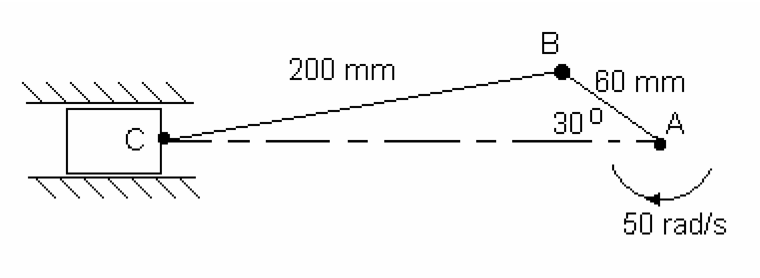


Figure 33

(Ans. 153 m/s)

2.

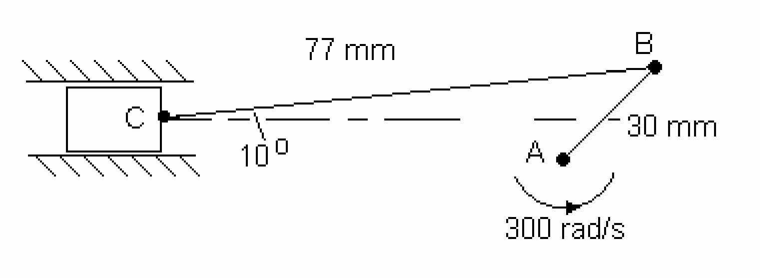


Figure 34

(Ans. 1650 m/s^2)

WORKED EXAMPLE No. 6

Find the angular acceleration of the link CD for the case shown.

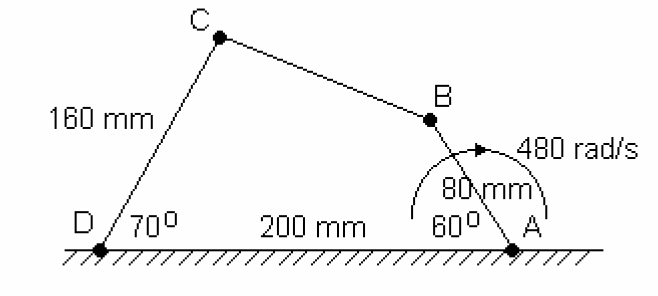


Figure 35

SOLUTION

First calculate or scale the length CB and find it to be 136 mm.

Next find the velocities and construct the velocity diagram. Start with link AB as this has a known constant angular velocity.

$(v_B)_A$ = ω x radius = 480 x 0.08 = 38.4 m/s

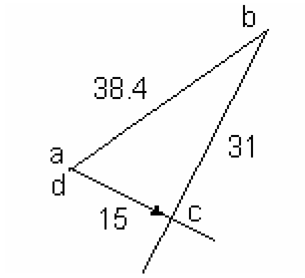


Figure 36

Next calculate all the accelerations possible.

The centripetal acceleration of B to A is $38.4^2/0.08$ = 18 432 m/s^2

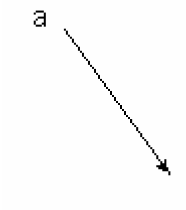
The centripetal acceleration of C to D is $15^2/0.16$ = 1406 m/s^2

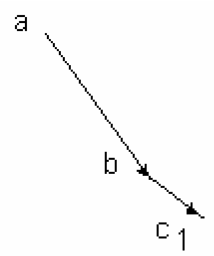
The centripetal acceleration of C to B is $31^2/0.136$ = 7066 m/s^2.

We cannot calculate any tangential accelerations at this stage.

The stage by stage construction of the acceleration diagram follows.

First draw the centripetal acceleration of B to A (Figure a). There is no tangential to add on).





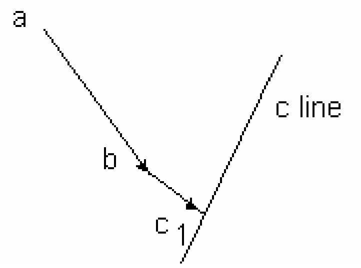
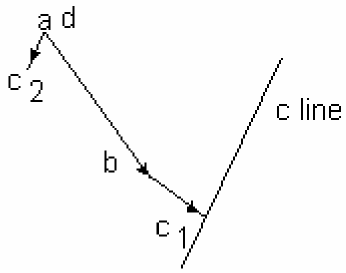


Figure 37a　　Figure 37b　　Figure 37c

Next add the centripetal acceleration of C to B (figure b)

Next draw the direction of the tangential acceleration of C to B of unknown length at right angles to the previous vector (figure c). Designate it as a c line.

We cannot proceed from this point unless we realise that points a and d are the same (there is no velocity or acceleration of D relative to A). Add the centripetal acceleration of C to D (figure d). This is 1406 m/s^2 in the direction of link CD. Designte it d c_2.



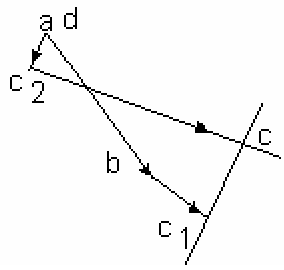


Figure 37d　　Figure 37e

Finally draw the tangential acceleration of C to D at right angles to the previous vector to intersect the c line (figure e).

From the diagram determine c_2 c to be 24 000 m/s^2. This is the tangential acceleration of C to D. The angular acceleration of the link DC is then:

$\alpha_{(CD)}$ = 24000/0.16 = 150 000 rad/s^2 in a clockwise direction.

Note that although the link AB rotates at constant speed, the link CD has angular acceleration.

WORKED EXAMPLE No. 7

The same arrangement exists as shown for example 5 except that the link AB is decelerating at 8000 rad/s^2 (i.e. in an anticlockwise direction). Determine the acceleration of the link CD.

SOLUTION

The problem is essentially the same as example 5 except that a tangential acceleration now exists for point B relative to point A. This is found from

$a_T = \alpha \times AB = 80000 \times 0.08 = 6400$ m/s^2

The direction is for an anticlockwise tangent. This is vector b_1 b which is at right angles to a b_1 in the appropriate direction. The new acceleration diagram looks like this.

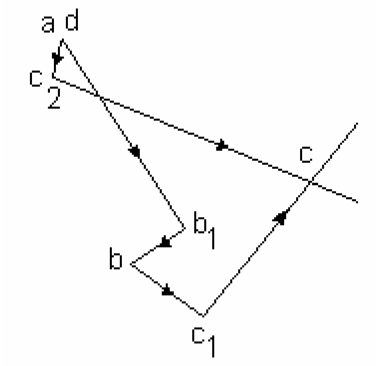


Figure 38

Scaling off the tangential acceleration c_2 c we get 19 300 m/s^2. Converting this into the angular acceleration we get

$\alpha = 19\ 300/0.16 = 120\ 625$ rad/s^2 in a clockwise direction.

SELF ASSESSMENT EXERCISE No.4

1. The diagram shows a 4 bar chain. The link AB rotates at a constant speed of 5 rad/s in an anticlockwise direction. For the position shown, determine the angular acceleration of the link DC.

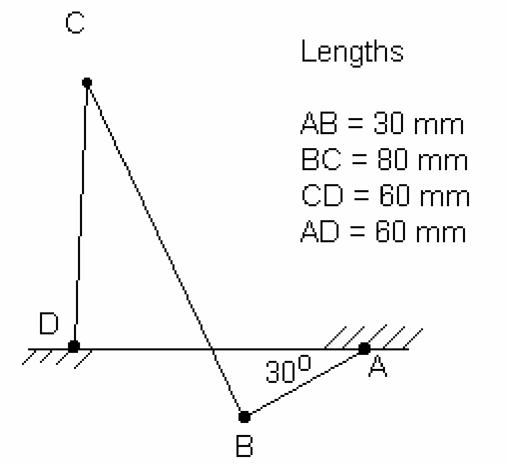


Figure 39

(Answer 30 rad/s^2 in an anticlockwise direction)

2. Repeat question 1 but this time the link AB is accelerating at 15 rad/s^2.
(Answer 15.3 rad/s^2 in an anticlockwise direction)

3. The diagram shows the instantaneous position of a mechanism in which member OA rotates anticlockwise with an angular velocity of 100 rad/s and angular acceleration of 10 000 rad/s^2 in the same direction. BD is a continuation of the rigid link AB. The links have the following lengths.
OA – 30 mm   BC – 90 mm   AD 168 mm   AB 1120 mm

Determine the linear the following.
i. The velocities of points A, B and D (1.5 m/s, 2.6 m/s and 2.7 m/s)
ii. The absolute linear accelerations of points A and B (424.26 m/s^2 and 440 m/s^2)

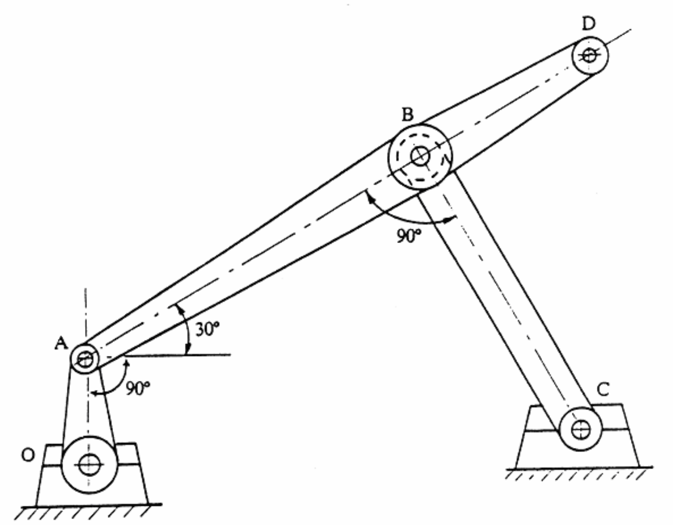


Figure 40

5. INERTIA FORCE

One of the reasons for finding the acceleration of links is to calculate the inertia force needed to accelerate or decelerate it. This is based on Newton's second law.

Force = mass x acceleration $F = M a$
And
Torque = moment of inertia x angular acceleration $T = I \alpha$

WORKED EXAMPLE No.8

A horizontal single cylinder reciprocating engine has a crank OC of radius 40 mm and a connecting rod PC 140 mm long as shown.
The crank rotates at 3000 rev/min clockwise. For the configuration shown, determine the velocity and acceleration of the piston.

The sliding piston has a mass of 0.5 kg and a diameter of 80 mm. The gas pressure acting on it is 1.2 MPa at the moment shown. Calculate the effective turning moment acting on the crank. Assume that the connecting rod and crank has negligible inertia and friction.

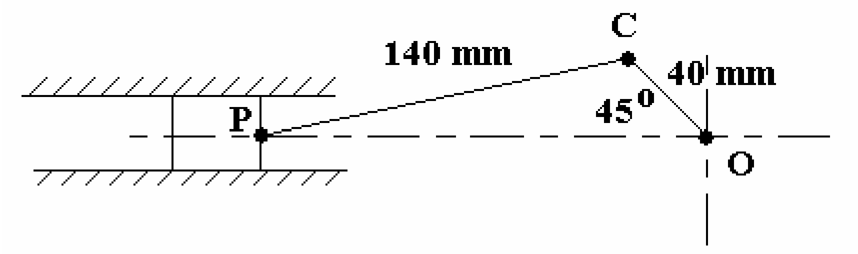


Figure 41

SOLUTION

Draw the space diagram to scale.

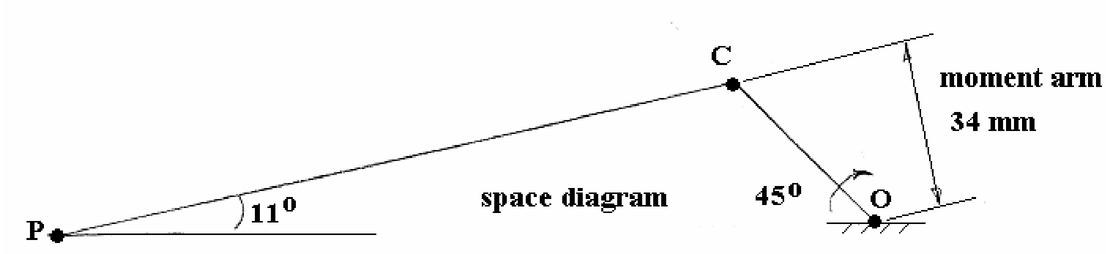


Figure 42

The moment arm should be scaled and found to be 34 mm (measured at right angles to the connecting rod PC.

Calculate the velocity of C relative to O.
$\omega = 2\pi N/60 = 2\pi \times 3000/60 = 314.16$ rad/s
$(v_C)_O = \omega \times \text{radius} = 314.16 \times 0.04 = 12.57$ m/s

Draw the velocity diagram.

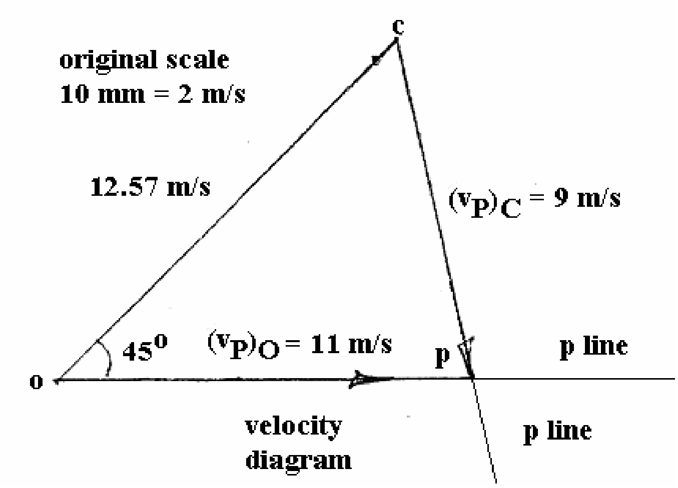


Figure 43

From the velocity diagram we find the velocity of the piston is 11 m/s.
Next calculate all the accelerations possible.
Point C only has a radial acceleration towards O
Radial acceleration of C is v^2/radius = $12.57^2/0.04$ = 3950 m/s^2
Point P has radial and tangential acceleration relative to C.
Tangential acceleration is unknown.
Radial acceleration = $(v_P)_C{}^2/CP = 9^2/0.14 = 578.57$ m/s^2
Now draw the acceleration diagram and it comes out like this.

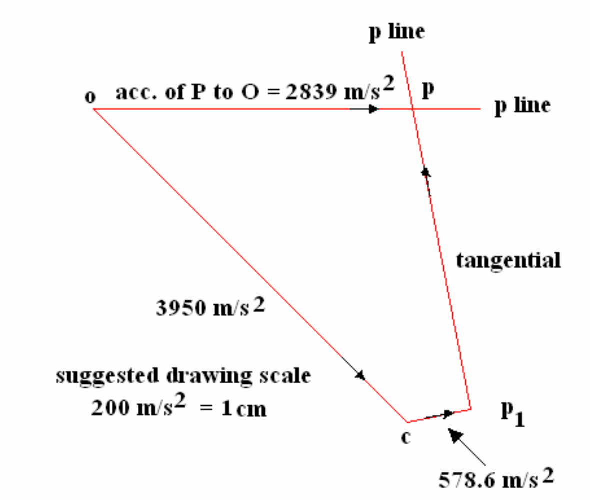


Figure 44

The acceleration of the piston is 2839 m/s^2.

Now we can solve the forces.

Pressure force = p x area = $1.2 \times 10^6 \times \pi \times 0.08^2/4$ = 6032 N and this acts left to right.

Inertia force acting on the piston = M a = 0.5 x 2839 = 1419.5 N and this must be provided by the pressure force so the difference is the force exerted on the connecting rod.
Net Force = 6032 – 1419.5 = 4612.5 N.

The connecting rod makes an angle of 11° to the line of the force (angle scaled from space diagram). This must be resolved to find the force acting along the line of the connecting rod.

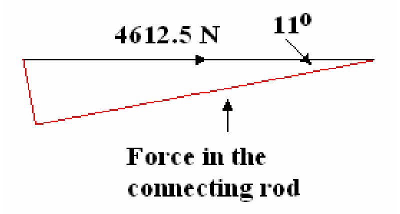


Figure 45

The force in the connecting rod is 4612.5 cos 11° = 4528 N.

This acts at a radius of 34 mm from the centre of the crank so the torque provided by the crank is

T = 4528 x 0.034 = 154 N m.

SELF ASSESSMENT EXERCISE No.5

1. The piston in the mechanism shown has a mass of 0.8 kg. Determine its acceleration and the inertia force needed for the position shown.

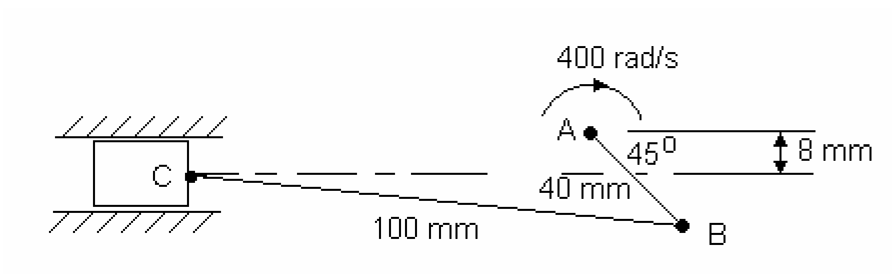


Figure 46

(Ans. 4000 m/s^2 and 3200 N)

6. CORIOLIS ACCELERATION

Consider a link rotating at ω rad/s and accelerating at α rad/s^2. On the link is a sliding element moving away from the centre of rotation at velocity v_R = dR/dt (positive if getting larger)

The link has a tangential velocity $v_T = \omega R$

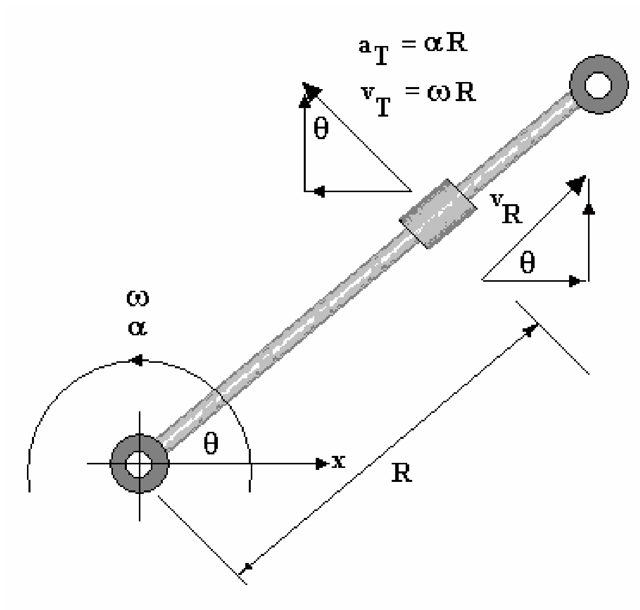
The component of this velocity in the x direction is
$v_T \sin\theta = v_T \sin \omega R$

The velocity v_R also has a component in the x direction
And this is $v_R \cos\theta$

The total velocity in the x direction of the sliding link is

$v_x = -v_T \sin\theta + v_R \cos\theta$

$v_x = -(dR/dt) \cos \omega t + \omega R \sin \omega t$



The acceleration in the x direction is a_x

Figure 47

$$a_x = \frac{dv_x}{dt} = -\frac{d^2R}{dt^2}\cos(\omega t) + \frac{dR}{dt}\omega\sin(\omega t) + \omega^2 R\cos(\omega t) + \frac{dR}{dt}\omega\sin(\omega t) + \frac{d\omega}{dt}R\sin(\omega t)$$

$$a_x = -\frac{d^2R}{dt^2}\cos(\omega t) + 2\frac{dR}{dt}\omega\sin(\omega t) + R\omega^2\cos(\omega t) + R\frac{d\omega}{dt}\sin(\omega t)$$

$$a_x = -\frac{d^2R}{dt^2}\cos\theta + 2\,\omega\sin\theta\frac{dR}{dt} + R\,\omega^2\cos\theta + \alpha\,R\sin\theta$$

When θ = 90°

$$a_x = a_T = 2\,\omega\frac{dR}{dt} + \alpha\,R$$

$$a_T = 2\,\omega\,v_R + \alpha\,R$$

The tangential acceleration is not simply α R as is the case for a constant radius but an extra term of 2ωvR is added and this term is called the Coriolis acceleration and must be taken into consideration when solving problems with changing radius.

WORKED EXAMPLE No.9

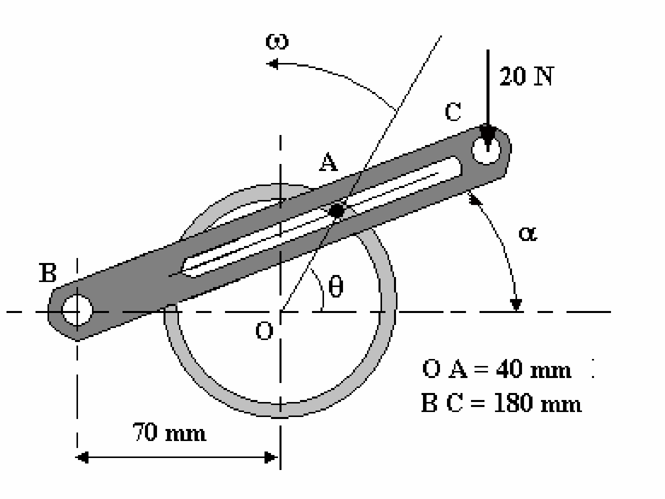


Figure 48

The diagram shows part of a quick return mechanism. The pin A slides in the slot when the disc is rotated. Calculate the angular velocity and acceleration of link BC when $\theta = 60^o$ and $\omega = 100$ rad/s.

SOLUTION

The tangential velocity of A relative to O is $\omega R = 100 \times 0.04 = 4$ m/s. The velocity diagram is constructed as shown.

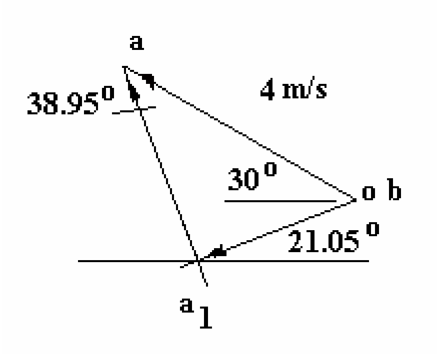


Figure 49

The tangential velocity of pin A relative to B is $(V_{A1})_B = a - a_1 = 4 \cos(38.95^o) = 3.11$ m/s
The radial velocity of A relative to B is $(V_A)_B = 4 \sin(38.95^o) = 2.515$ m/s
The length of BA is easily calculated from the diagram.

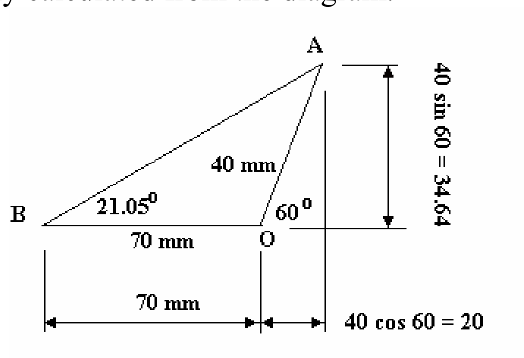


Figure 50

$BA = \sqrt{(34.64^2 + 90^2)} = 96.44$ mm

The angular velocity link BC = 3.11/BA = 32.2 rad/s

ANALYTICAL METHOD

The angle of link BC is

$$\alpha = \tan^{-1}\left[\frac{40\sin\theta}{70+40\cos\theta}\right] = \tan^{-1}\left[\frac{\sin\theta}{7/4+\cos\theta}\right]$$

The angular velocity is dα/dt and the tools for doing the differentiation are given in the question as follows.

Let $x = \left[\frac{\sin\theta}{7/4+\cos\theta}\right]$ $\quad \frac{d\alpha}{dx} = \frac{1}{1+x^2} = \frac{1}{1+\left(\frac{\sin\theta}{7/4+\cos\theta}\right)^2}$ $\quad \frac{dx}{d\theta} = \frac{1+7/4\cos\theta}{(7/4+\cos\theta)^2}$

$$\frac{d\alpha}{dt} = \frac{d\alpha}{dx}\frac{dx}{d\theta} = \frac{1}{1+\left(\frac{\sin\theta}{7/4+\cos\theta}\right)^2}\frac{1+7/4\cos\theta}{(7/4+\cos\theta)^2}$$ put $\theta = 60^o$ and evaluate

$d\alpha/d\theta = 0.316$

$\theta = \omega t$ so $d\theta = \omega\, dt$ $\quad dt = d\theta/\omega$ so $d\alpha/dt = 0.316 \times \omega = 31.6$ rad/s which is close to the answer found before.

Next construct the acceleration diagram.

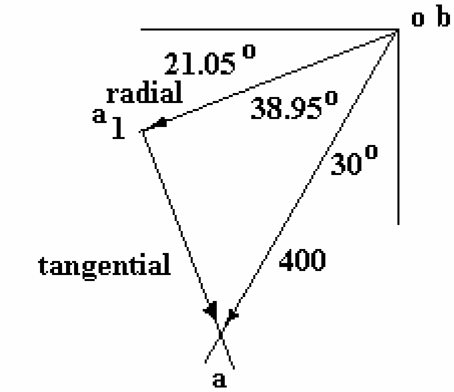


Figure 51

Link O A only has centripetal acceleration inwards

$(a_A)_O = \omega^2 R = 100^2 \times 0.04 = 400$ m/s^2

The pin A has a tangential acceleration and Coriolis acceleration normal to the link. It has centripetal acceleration and radial acceleration towards the centre of rotation. B. The diagram can be constructed without calculating them.

The Corioilis acceleration is $2\omega v$ where $\omega = 32.2$ and v is the radial velocity = 2.515 m/s

The Coriolis term is hence 161.97 m/s^2

The tangential acceleration of A relative to B is

$a_1 - a = 400 \sin 38.95 = 251.46$ m/s^2

Part of this is the Coriolis so the tangential acceleration is 251.46 - 161.97 = 89.49 m/s^2

The angular acceleration of link AC is α = 89.49 /BA = 89.49/0.09644 = 928 rad/s^2

The direction is negative (clockwise) so it is decelerating.

SELF ASSESSMENT EXERCISE No.6

A link OA is 80 mm long and rotates at a constant speed of 50 rad/s. A sliding link attached to it slides on link BC and makes BC rotate about B as shown. Calculate the angular velocity and acceleration of BC when angle $\theta = 70^{\circ}$.

(22.8 rad/s and -76.2 rad/s^2)

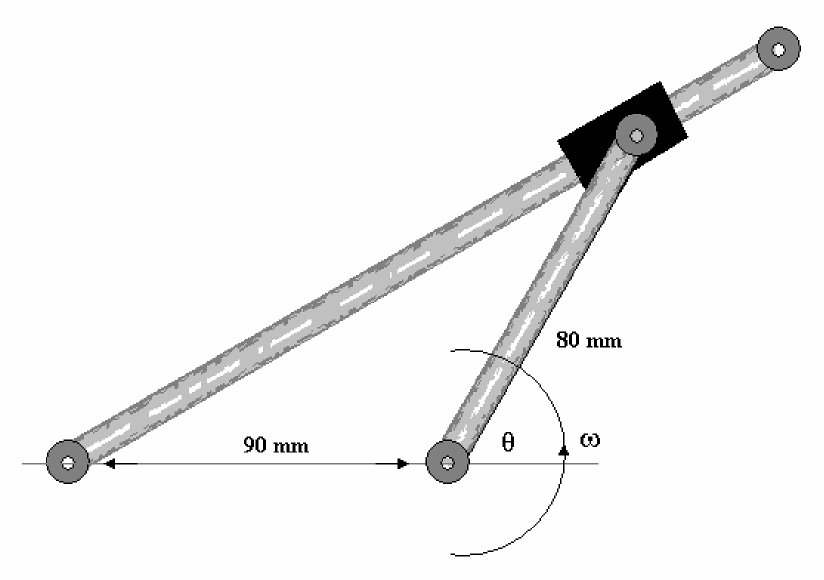


Figure 52