SOLID MECHANICS

TUTORIAL – FLYWHEELS AND TURNING MOMENT DIAGRAMS

This work covers elements of the syllabus for the Edexcel module 21722P HNC/D Mechanical Principles OUTCOME 4.

On completion of this short tutorial you should be able to do the following.

- Explain the need for flywheels in machines subjected to an erratic torque.
- Define the coefficient of fluctuation of speed.
- Define the coefficient of fluctuation of energy.
- Calculate the maximum and minimum speeds of a machine subjected to an erratic torque.
- Calculate the size of a flywheel needed to keep a machine speed within specified limits.
- Derive the effective moment of inertia for a geared system.

It is assumed that the student is already familiar with the following concepts.

- Angular motion.
- Moment of inertia.
- Angular kinetic energy.

All these above may be found in the pre-requisite tutorials.

CONTENTS

1. INTRODUCTION

This tutorial is about the use of flywheels to smooth out the rotation of machines subjected to an erratic torque. This is a problem with machines such as piston engines, gas compressors, reciprocating pumps and press tools where the torque on the shaft goes through a cyclic change. For example when a press tool goes through the pressing stage, the motor would tend to slow down and then during the idle stage it will tend to speed up. Flywheels are used to smooth out the motion and keep the variations in shaft speed to within acceptable limits.

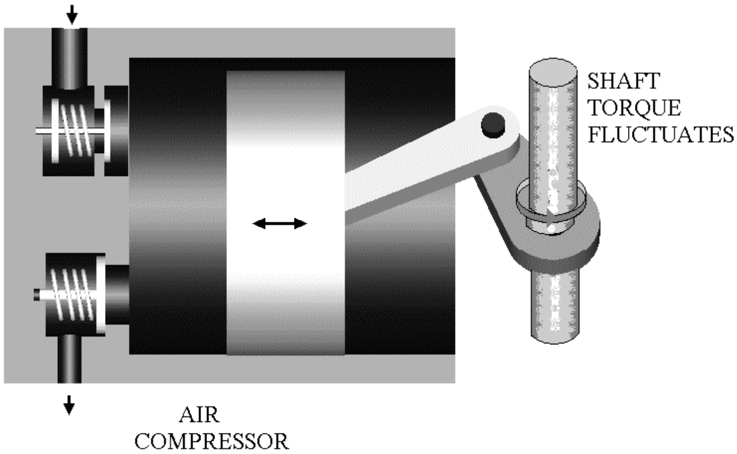

Figure 1

2. BASIC ANGULAR RELATIONSHIPS

A flywheel is essentially a device for storing angular kinetic energy for which the formula is

$$K.E. = I\,\omega^2/2$$

I is the moment of inertia given by the formulae $I = Mk^2$
ω is the angular velocity in rad/s
k is the radius of gyration in metres
M is the mass of the wheel

For a plain disc $I = MR^2/2$ where R is the outer radius

When a rotating body changes speed, the angular acceleration is related to the moment of inertia and the applied torque by the formula $T = I\alpha$

α is the angular acceleration in rad/s^2

When an increase in torque occurs, the flywheel will speed up and absorb energy. The greater the moment of inertia, the more energy it will absorb. The result is that it speeds up less than it would do with a smaller moment of inertia. When the torque decreases, the flywheel will slow down but the inertia of the system will limit the amount it slows.

When a torque is applied to a body and it rotates, the work done is the product of torque T Nm and angle θ radian.

$$\text{Work} = T\,\theta$$

This leads us on to torque – angle diagrams otherwise known as turning moment diagrams.

3. TORQUE – ANGLE DIAGRAMS

Consider a graph of torque against crank angle for a typical 4 stroke engine.

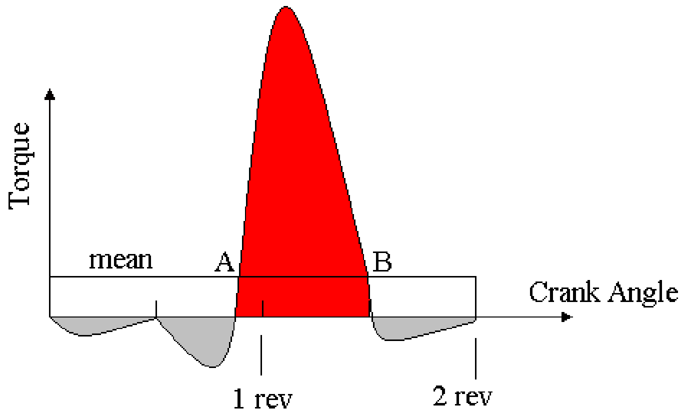

Figure 2

The complete cycle takes two revolutions divided into four equal strokes. One revolution is 2π radians so it takes 4π radians to complete the cycle. Only the red part of the diagram represents energy given out (the power stroke). The induction, compression and exhaust strokes require negative torque as energy is required to do these. If the machine is to be a successful engine, the area of the red part must be larger than the areas of the grey parts giving a positive mean torque. The area above and below the mean line must be equal by definition of mean. The area of the rectangle formed between the mean and zero must be the work output of the engine per cycle.

The total work done is angle x torque so in this case we have

$$W = T_m \times 4\pi = \text{net area } (A_{net})$$

$$T_m = A_{net}/4\pi$$

Note that the angle is not always 4π but whatever angle corresponds to the cycle. For example it would be 2π for a 2 stroke engine.

4. COEFFICIENT OF FLUCTUATION OF SPEED ϕ

Definition $\phi = (\omega_2 - \omega_1)/\omega$

ω_1 = smallest angular velocity. ω_2 = largest angular velocity.

ω = mean angular velocity = $(\omega_2 + \omega_1)/2$

WORKED EXAMPLE No.1

The speed of a shaft fluctuates from 500 to 600 rev/min. Calculate ϕ

SOLUTION

$\omega_1 = 2\pi N_1 = 2\pi(500/60) = 52.35$ rad/s

$\omega_2 = 2\pi N_2 = 2\pi(600/60) = 62.83$ rad/s

$\omega = (\omega_2+\omega_1)/2 = 57.59$ rad/s

$\phi = (\omega_2-\omega_1)/\omega = 0.1819$

Note we could use the speed in rev/min to find ϕ

Mean speed = $N_m = (500 + 600)/2 = 550$ rev/min

$\phi = (N_2 - N_1)/N_m = (100)/550 = 0.1819$

5. COEFFICIENT OF FLUCTUATION OF ENERGY β

Definition

$$\beta = \frac{\text{greatest fluctuation in kinetic energy}}{\text{work done}}$$

$$\beta = \frac{\frac{I\omega_2^2}{2} - \frac{I\omega_1^2}{2}}{W} = \frac{I}{2W}\left(\omega_2^2 - \omega_1^2\right) \text{Factorise the bracket}$$

$$\beta = \frac{I}{2W}(\omega_2 + \omega_1)(\omega_2 - \omega_1)$$

Since $(\omega_2 + \omega_1) = 2\omega$ and $(\omega_2 - \omega_1) = \phi\omega$ then $\beta = \frac{I}{2W}(2\omega)(\omega\phi)$

$\beta = \frac{I}{W}\omega^2\phi$ rearrange to make I the subject. $\quad I = \frac{\beta W}{\omega^2\phi}$

The only problem now is how to find the work W. In fact it is easier to find the greatest fluctuation in energy and equate it to βW as follows.

Consider the Torque angle diagram again. A and B are the points where the diagram passes through the mean. For convenience move the start of the cycle to point A.

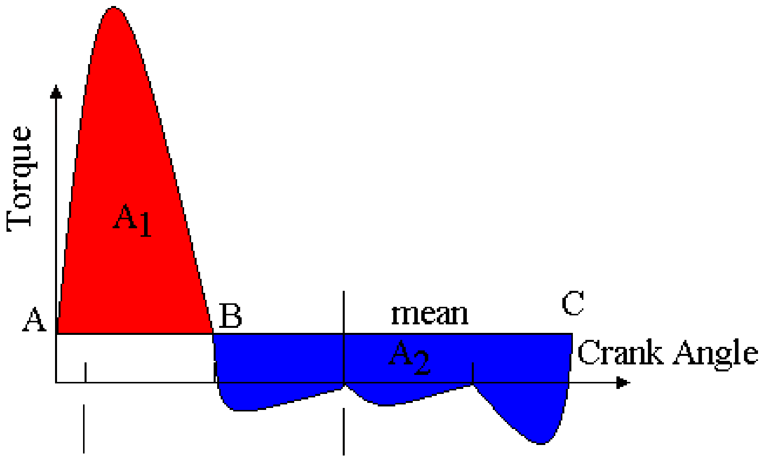

Figure 3

By definition of a mean value, the red area A_1 must be equal to the blue area A_2.

Let the energy at A be E_A.
At point B the energy has increased by an amount equal to A_1.
The energy at B is $\quad E_B = E_A + A_1$.
At point C the energy has been reduced by an amount equal to A_2.
The energy at C is $E_C = E_B - A_2 = E_A + A_1 - A_2 = E_A$.
After 1 cycle the energy must be returned to the starting value and obviously points A and C are the same point.

From the figures, we deduce the maximum fluctuation in energy. In this case the maximum energy was at B and the minimum at A or C.
The fluctuation is $E_A - E_B = A_1$.
Hence $\beta W = A_1$ in this case.

WORKED EXAMPLE No.2

Find βW for the torque - angle diagram shown. The enclosed areas are
$A_1 = 400$ J $A_2 = 800$ J $A_3 = 550$ J $A_4 = 150$ J

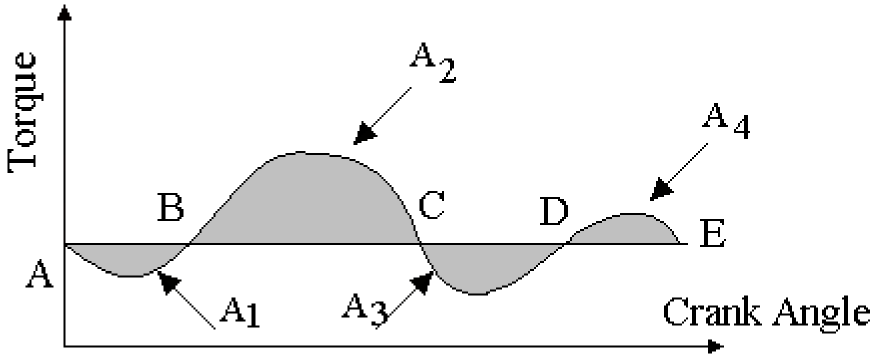

Figure 4

Go on to find the moment of inertia for a flywheel which will keep the speed within the range 410 to 416 rev/min.

Find the mass of a suitable flywheel with a radius of gyration of 0.5 m

SOLUTION

First find the greatest fluctuation in energy.
Energy at A = E_A
Energy at B = $E_B = E_A - A_1 = E_A - 400$
Energy at C = $E_C = E_B + A_2 = E_A - 400 + 800 = E_A + 400$
Energy at D = $E_D = E_C - A_3 = E_A + 400 - 550 = E_A - 150$
Energy at E = $E_E = E_D + A_4 = E_A - 150 + 150 = E_A$

If the last figure is not equal to E_A then there would be an error.
The largest energy value is $E_A + 400$ and the smallest value is $E_A - 400$ so the greatest fluctuation is from +400 to -400 giving 800 Joules.
Equate to βW $\beta W = 800$ Joules.
Now find the coefficient of fluctuation in speed.
Mean speed N = (410 + 416)/2 = 413 rev/min.
There is no need to convert this into radian in order to find ϕ
$\phi = (\omega_2 - \omega_1)/\omega = (N_2 - N_1)/N = (416 - 410)/413 = 0.01452$
We need the mean in radian/s so $\omega = 413 \times (2\pi/60) = 43.25$ rad/s
Now find the moment of inertia of the flywheel using

$$I = \frac{\beta W}{\omega^2 \varphi} = \frac{800}{43.25^2 \times 0.01452} = 29.44 \text{ kg m}^2$$

Now find the mass of the flywheel.
$I = mk^2$ $29.44 = m \times 0.5^2 = 0.25m$
$m = 29.44/0.25 = 117.8$ kg

SELF ASSESSMENT EXERCISE No.1

1. Flywheels are used to smooth out fluctuating torques such as produced on the crank of piston engines. The diagram shows a torque - angle diagram for a certain machine. The speed of the shaft must be maintained between 490 and 510 rev/min.

- ***Calculate the moment of inertia of a suitable flywheel.*** ***(1.185 kgm^2)***
- ***Calculate the mass required if the radius of gyration is to be 0.3 m.***
(13.17 kg)

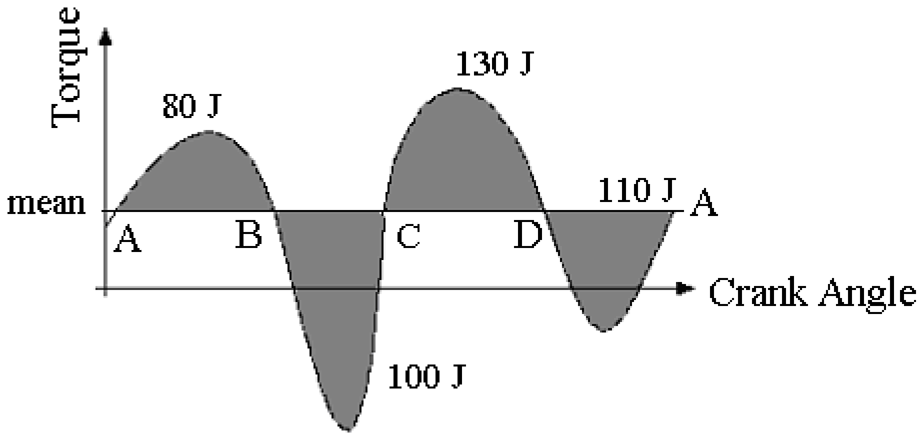

Figure 5

2. A press tool machine uses a flywheel in the form of a solid steel disc 0.7 m diameter and 0.1 m thick rotating with a mean speed of 60 rev/min. The density of the steel is 7830 kg/m^3. The press is operated once every 5 revolutions and a torque of 2000 Nm is exerted on the tool for a duration of 0.05 seconds. The energy absorbed is replaced during the rest of the cycle.

Sketch the torque angle diagram and determine the following.

- ***The mass of the flywheel (301.33 kg)***
- ***The moment of inertia (18.46 kg m^2)***
- ***The work done during the pressing operation (628.3J)***
- ***The coefficient of fluctuation of speed (0.8623)***
- ***The maximum and minimum speeds of the flywheel***
(85.9 and 34.1 rev/min).

6. GEARED SYSTEMS

If a load is driven through a gear box, the affect of the inertia is dramatically altered. Consider a motor coupled to a load through a speed changing device such as a gear box. In this case we shall ignore damping and the inertia of the gears.

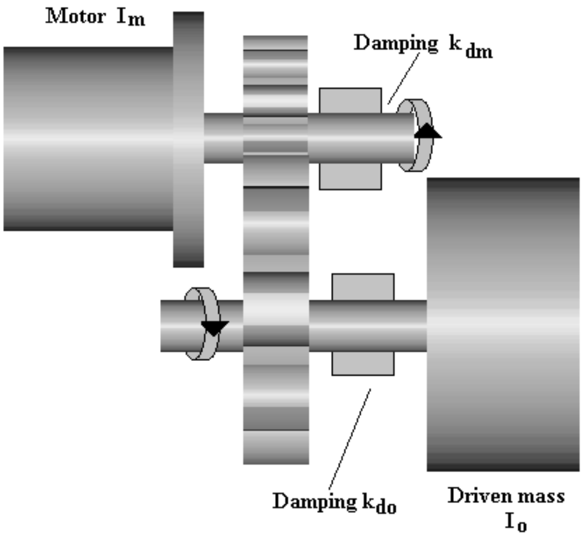

Figure 6

θ_m is the motor rotation and θ_o the output rotation. The gear ratio is $G_r = \theta_o/\theta_m$

Please note that this definition is adopted to conform to control system theory where a transfer function is defined as Output/Input. However, many authors adopt the definition of gear ratio as Input/Output so take care when comparing this to other text.

Since this is a fixed number and is not a function of time, the speed and acceleration are also in the same ratio.

$$G_r = \omega_o/\omega_m \quad \omega \text{ is the angular velocity}$$

$$G_r = \alpha_o/\alpha_m \quad \alpha \text{ is the angular acceleration.}$$

The power transmitted by a shaft is given by Power = ωT. If there is no power lost, the output and input power must be equal so it follows that

$$\omega_m T_m = \omega_o T_o \text{ hence}$$

$$T_m = \omega_o T_o / \omega_m = G_r T_o$$

Consider the inertia torque due the inertia on the output shaft I_o.

$$T_o = I_o\alpha_o = I_o \alpha_m \text{ x } G_r$$

$$T_m = T_o \text{ x } G_r = I_o \alpha_m \text{ x } G_r^2$$

This is the output inertia torque referred to the motor. In addition we have the inertia torque of the motor itself $I_m \alpha_m$. This must be added so:

$$T_m = I_m\alpha_m + I_o \alpha_m \text{ x } G_r^2$$

$$T_m = \alpha_m (I_m + G_r^2 I_o)$$

$$I_e = (I_m + G_r^2 I_o)$$

I_e is the effective moment of inertia I_e referred to the motor shaft.

WORKED EXAMPLE No.3

A motor rotates at 600 rev/min and has a moment of inertia of 0.1 kg m^2. It drives a load through a gearbox at 300 rev/min with a moment of inertia of 0.2 kg m^2. The torque on the load shaft varies sinusoidally with amplitude of 200 Nm

The speed of the load is to be regulated to ± 2% of the mean by adding a flywheel to the motor shaft. Calculate this moment of inertia to be added.

SOLUTION

The coefficient of fluctuation of speed $\phi = \Delta\omega/\omega$ = 4 % or 0.04
The coefficient of fluctuation of energy β
β = max fluctuation in energy/work done per cycle βW = max fluctuation in energy $\beta W = I\omega^2\phi$
ω = mean angular velocity.
Load speed = 300 rev/min $\omega = 2\pi \times 300/60 = 10\pi$ rad/s
W = work done per cycle. In this case we will choose ½ revolution as the cycle since it is repeated every ½ revolution.

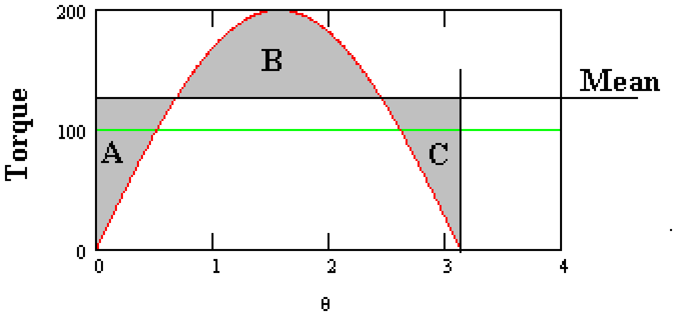

Figure 7

$$W = \int_0^\pi 200\sin\theta\, d\theta = -200\left[\cos\theta\right]_0^\pi = 400 \text{ J}$$

Mean Torque = W/base = 400/π =127.471 Nm
The angle where the graph cuts the mean is $\sin^{-1}$(127.32/200) = 39.54° (0.69 rad) and 180° - 39.54° = 140.46° (2.451 rad).
The area represented by B is found from

$$W = \int_{0.69}^{2.45} 200\sin\theta\, d\theta = 200\left[\cos\theta\right]_{0.69}^{2.45} = 308.295 \text{ J}$$

It follows that areas A and C are both half this and equal to 154.15 J
The maximum fluctuation of energy is from (mean - 154.15) to (mean + 154.15) so βW = 308.471
$\beta = I\omega^2\phi/W$ so $I = \beta W/\omega^2\phi = 308.471/(31.41^2 \times 0.04)$
I = 7.814 kg m^2

This is the moment of inertia that needs to be seen at the output shaft. The input is the load and the output is the motor. The gear ratio is 2 from the load end.

$I_e = I_L + Gr^2 I_M = 0.2 + 2^2 (0.1 + I) = 7.814$ kg m^2
I = (7.614/4) - 0.1 = 1.8 kg m^2

SELF ASSESSMENT EXERCISE No.2

A motor drives a load through a reduction gear box with a ratio of 4. The Motor produces a fluctuating torque of 500 sin θ Nm every half revolution. The moment of inertia of the motor is 0.5 kg m^2 and for the load is 1.0 kg m^2. The speed of the load is to be regulated to 250 rev/min ± 1% by adding a flywheel to the motor shaft. Calculate this moment of inertia.
(6.47 kg m^2)