

SOLID MECHANICS

DYNAMICS

PRE-REQUISITE STUDIES TUTORIAL 1

LINEAR AND ANGULAR DISPLACEMENT, VELOCITY AND ACCELERATION

This tutorial is essential for anyone studying the group of tutorials on beams.

- Essential pre-requisite knowledge for Edexcel HNC Mechanical Principles UNIT 21722P.
- Essential pre-requisite knowledge for the Engineering Council Diploma Exams D209 Mechanics of Solids and D225 Dynamics of Mechanical Systems.
- Essential pre-requisite knowledge for the Engineering Council Certificate Exam C105 Mechanical and Structural engineering.
- Covers part of the syllabus for the Engineering Council Certificate Exam C103 Engineering Science.

On completion of this tutorial you should be able to

- Define linear motion.
- Explain the relationship between distance, velocity, acceleration and time.
- Define angular motion.
- Explain the relationship between angle, angular velocity, angular acceleration and time.
- Explain the relationship between linear and angular motion.

1. LINEAR MOTION

1.1 MOVEMENT or DISPLACEMENT

This is the distance travelled by an object and is usually denoted by x or s . Units of distance are metres.

1.2 VELOCITY

This is the distance moved per second or the rate of change of distance with time. Velocity is movement in a known direction so it is a vector quantity. The symbol is v or u and it may be expressed in calculus terms as the first derivative of distance with respect to time so that $v = dx/dt$ or $u = ds/dt$. The units of velocity are m/s.

1.3. SPEED

This is the same as velocity except that the direction is not known and it is not a vector and cannot be drawn as such.

1.4. AVERAGE SPEED OR VELOCITY

When a journey is undertaken in which the body speeds up and slows down, the average velocity is defined as TOTAL DISTANCE MOVED/TIME TAKEN.

1.5. ACCELERATION

When a body slows down or speeds up, the velocity changes and acceleration or deceleration occurs. Acceleration is the rate of change of velocity and is denoted with a . In calculus terms it is the first derivative of velocity with time and the second derivative of distance with time such that $a = dv/dt = d^2x/dt^2$. The units are m/s^2 .

Note that all bodies falling freely under the action of gravity experience a downwards acceleration of $9.81 m/s^2$.

WORKED EXAMPLE No.1

A vehicle accelerates from 2 m/s to 26 m/s in 12 seconds. Determine the acceleration. Also find the average velocity and distance travelled.

SOLUTION

$$a = \Delta v/t = (26 - 2)/12 = 2 m/s^2$$

$$\text{Average velocity} = (26 + 2)/2 = 14 m/s$$

$$\text{Distance travelled} = 14 \times 12 = 168 m$$

SELF ASSESSMENT EXERCISE No.1

1. A body moves 5000 m in 25 seconds. What is the average velocity?
(Answer 200 m/s)
2. A car accelerates from rest to a velocity of 8 m/s in 5 s. Calculate the average acceleration.
(Answer 1.6 m/s²)
3. A train travelling at 20 m/s decelerates to rest in 40 s. What is the acceleration?
(Answer -0.5 m/s²)

1.6. GRAPHS

It is very useful to draw graphs representing movement with time. Much useful information may be found from the graph.

1.6.1. DISTANCE - TIME GRAPHS

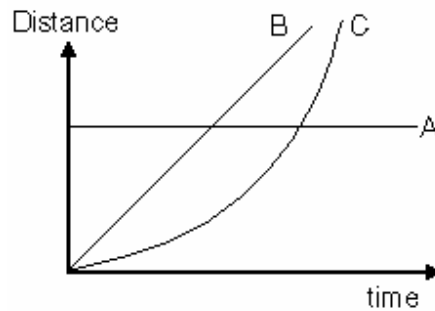


Figure 1

Graph A shows constant distance at all times so the body must be stationary.

Graph B shows that every second, the distance from start increases by the same amount so the body must be travelling at a constant velocity.

Graph C shows that every passing second, the distance travelled is greater than the one before so the body must be accelerating.

1.6.2. VELOCITY - TIME GRAPHS

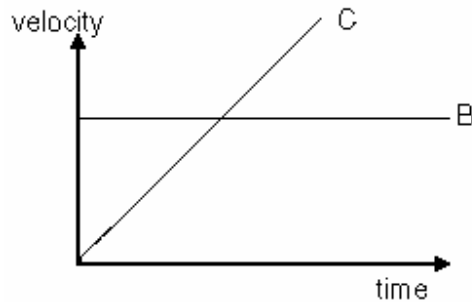


Figure 2

Graph B shows that the velocity is the same at all moments in time so the body must be travelling at constant velocity.

Graph C shows that for every passing second, the velocity increases by the same amount so the body must be accelerating at a constant rate.

1.7 SIGNIFICANCE OF THE AREA UNDER A VELOCITY-TIME GRAPH

The average velocity is the average height of the v-t graph. By definition, the average height of a graph is the area under it divided by the base length.

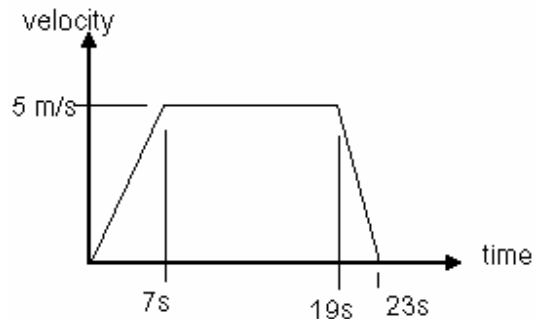


Figure 3

Average velocity = total distance travelled/time taken

Average velocity = area under graph/base length

Since the base length is also the time taken it follows that the area under the graph is the distance travelled. This is true whatever the shape of the graph. When working out the areas, the true scales on the graphs axis are used.

WORKED EXAMPLE No.2

Find the average velocity and distance travelled for the journey depicted on the graph above. Also find the acceleration over the first part of the journey.

SOLUTION

Total Area under graph = A + B + C

Area A = $5 \times 7 / 2 = 17.5$ (Triangle)

Area B = $5 \times 12 = 60$ (Rectangle)

Area C = $5 \times 4 / 2 = 10$ (Triangle)

Total area = $17.5 + 60 + 10 = 87.5$

The units resulting for the area are $\text{m/s} \times \text{s} = \text{m}$

Distance travelled = 87.5 m

Time taken = 23 s

Average velocity = $87.5 / 23 = 3.8 \text{ m/s}$

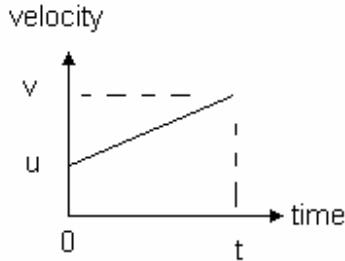
Acceleration over part A = change in velocity/time taken = $5/7 = 0.714 \text{ m/s}^2$.

SELF ASSESSMENT EXERCISE No.2

1. A vehicle travelling at 1.5 m/s suddenly accelerates uniformly to 5 m/s in 30 seconds. Calculate the acceleration, the average velocity and distance travelled.
(Answers 0.117 m/s², 3.25 m/s and 97.5 m)
2. A train travelling at 60 km/h decelerates uniformly to rest at a rate of 2 m/s². Calculate the time and distance taken to stop.
(Answers 8.33 s and 69.44 m)
3. A shell fired in a gun accelerates in the barrel over a length of 1.5 m to the exit velocity of 220 m/s. Calculate the time taken to travel the length of the barrel and the acceleration of the shell.
(Answers 0.01364 s and 16133 m/s²)

1.8. STANDARD FORMULAE

Consider a body moving at constant velocity u . Over a time period t seconds it accelerates from u to a final velocity v . The graph looks like this.



Distance travelled = s
 $s = \text{area under the graph} = ut + (v-u)t/2$
 $s = ut + (v - u)t/2 \dots\dots\dots(1)$
 $s = ut + t/2 v - ut/2$
 $s = vt/2 + ut/2$
 $s = t/2 (v + u)$

Figure 4

The acceleration = $a = (v - u)/t$ from which $(v - u) = at$
 Substituting this into equation (1) gives

$$s = ut + at^2/2$$

Since $v = u + \text{the increase in velocity}$
 $v = u + at$ and squaring we get $v^2 = u^2 + 2a[at^2/2 + ut]$
 $v^2 = u^2 + 2as$

WORKED EXAMPLE No.3

A missile is fired vertically with an initial velocity of 400 m/s. It is acted on by gravity. Calculate the height it reaches and the time taken to go up and down again.

SOLUTION

$$u = 400 \text{ m/s} \qquad v = 0 \qquad a = -g = -9.81 \text{ m/s}^2$$

$$v^2 = u^2 + 2as \qquad 0 = 400^2 + 2(-9.81)s \quad s = 8155 \text{ m}$$

$$s = (t/2)(u + v) \qquad 8155 = t/2(400 + 0)$$

$$t = 8155 \times 2/400 = 40.77 \text{ s}$$

To go up and down takes twice as long. $t = 81.54 \text{ m/s}$

WORKED EXAMPLE No.4

A lift is accelerated from rest to 3 m/s at a rate of 1.5 m/s^2 . It then moves at constant velocity for 8 seconds and then decelerates to rest at 1.2 m/s^2 . Draw the velocity - time graph and deduce the distance travelled during the journey. Also deduce the average velocity for the journey.

SOLUTION

The velocity - time graph is shown in fig.5.

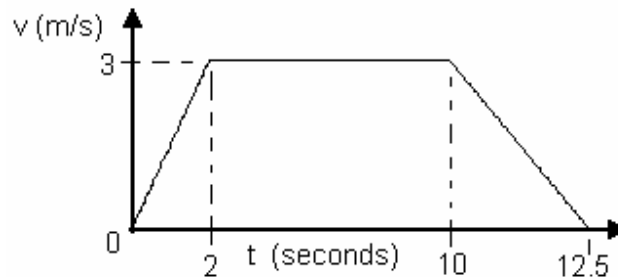


Figure 5

The first part of the graph shows uniform acceleration from 0 to 3 m/s. The time taken is given by

$$t_1 = 3/1.5 = 2 \text{ seconds.}$$

The distance travelled during this part of the journey is

$$x_1 = 3 \times 2/2 = 3 \text{ m}$$

The second part of the journey is a constant velocity of 3 m/s for 8 seconds so the distance travelled is

$$x_2 = 3 \times 8 = 24 \text{ m}$$

The time taken to decelerate the lift over the third part of the journey is

$$t_3 = 3/1.2 = 2.5 \text{ seconds.}$$

the distance travelled is

$$x_3 = 3 \times 2.5/2 = 3.75 \text{ m}$$

The total distance travelled is $3 + 24 + 3.75 = 30.75 \text{ m}$.

The average velocity = distance/time = $30.75/12.5 = 2.46 \text{ m/s}$.

SELF ASSESSMENT EXERCISE No.3

The diagram shows a distance-time graph for a moving object. Calculate the velocity.

(Answer 1.82 m/s)

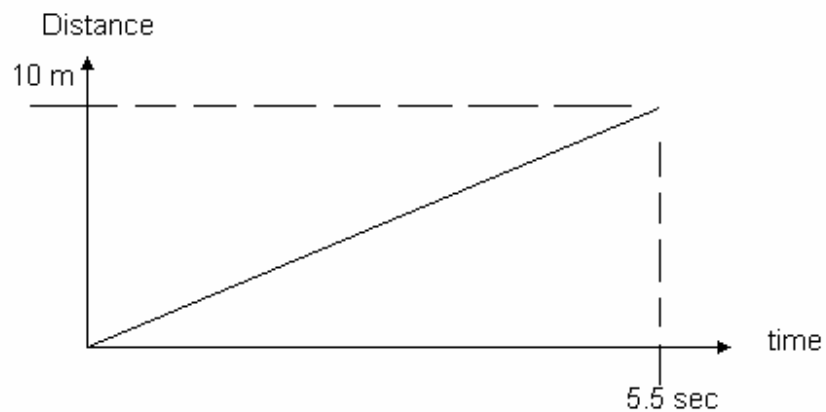


Figure 5

2. The diagram shows a velocity time graph for a vehicle. Calculate the following.

i. The acceleration from O to A.

ii. The acceleration from A to B.

iii. The distance travelled.

iv. The average velocity.

(Answers 2.857 m/s^2 , -2.4 m/s^2 , 35 m and 5.83 m/s)

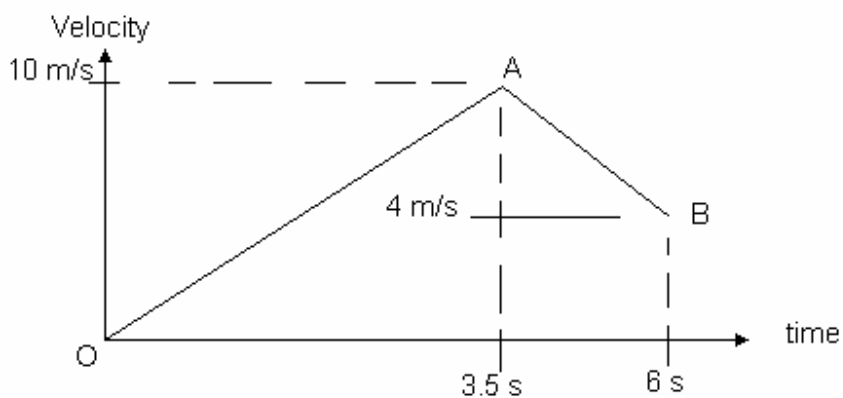


Figure 6

3. The diagram shows a velocity-time graph for a vehicle. Calculate the following.

i. The acceleration from O to A.

ii. The acceleration from B to C.

iii. The distance travelled.

iv. The average velocity.

(Answers 5 m/s^2 , -2 m/s^2 , 85 m and 7.08 m/s)

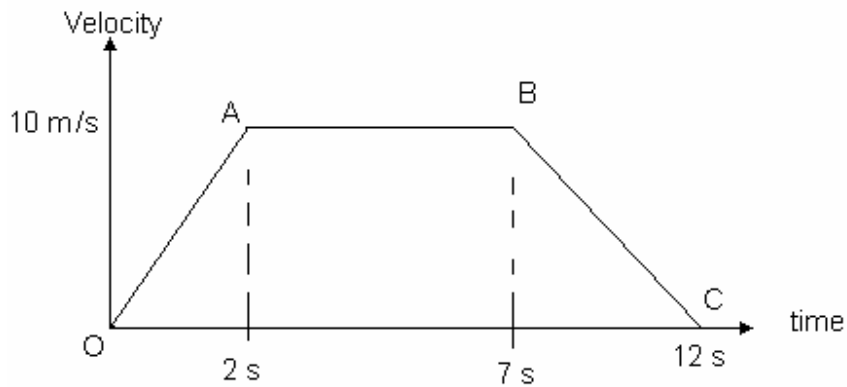
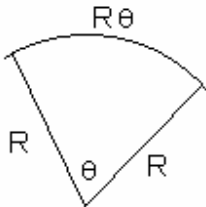


Figure 7

2. ANGULAR MOTION

2.1 ANGLE θ

Angle has no units since it is a ratio of arc length to radius. We use the names revolution, degree and radian. Engineers use radian. Consider the arc shown.



The length of the arc is $R\theta$ and the radius is R .

The angle is the ratio of the arc length to the radius.

$\theta = \text{arc length} / \text{radius}$ hence it has no units but it is called radians.

If the arc length is one radius, the angle is one radian so a radian is defined as the angle which produces an arc length of one radius.

Figure 8

2.2 ANGULAR VELOCITY ω

Angular velocity is the rate of change of angle per second. Although rev/s is commonly used to measure angular velocity, we should use radians/s (symbol ω). Note that since a circle (or revolution) is 2π radian we convert rev/s into rad/s by $\omega = 2\pi N$.

Also note that since one revolution is 2π radian and 360° we convert degrees into radian as follows. θ radian = degrees $\times 2\pi/360 = \text{degrees} \times \pi/180$

DEFINITION

angular velocity = $\omega = \text{angle rotated}/\text{time taken} = \theta/t$

EXAMPLE No.5

A wheel rotates 200° in 4 seconds. Calculate the following.

i. The angle turned in radians?

ii. The angular velocity in rad/s

SOLUTION

$$\theta = (200/180)\pi = 3.49 \text{ rad.}$$

$$\omega = 3.49/4 = 0.873 \text{ rad/s}$$

SELF ASSESSMENT EXERCISE No.4

1. A wheel rotates 5 revolutions in 8 seconds. Calculate the angular velocity in rev/s and rad/s. (Answers 0.625 rev/s and 3.927 rad/s)
2. A disc spins at 3000 rev/min. Calculate its angular velocity in rad/s. How many radians has it rotated after 2.5 seconds? (Answers 314.2 rad/s and 785.4 rad)

2.3 ANGULAR ACCELERATION α

Angular acceleration (symbol α) occurs when a wheel speeds up or slows down. It is defined as the rate of change of angular velocity. If the wheel changes its velocity by $\Delta\omega$ in t seconds, the acceleration is

$$\alpha = \Delta\omega / t \text{ rad/s}^2$$

WORKED EXAMPLE No.6

A disc is spinning at 2 rad/s and it is uniformly accelerated to 6 rad/s in 3 seconds. Calculate the angular acceleration.

SOLUTION

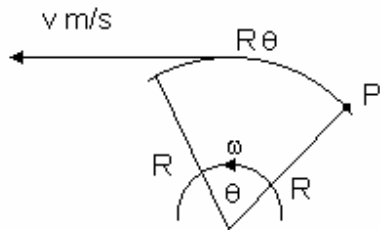
$$\alpha = \Delta\omega/t = (\omega_2 - \omega_1)/t = (6 - 2)/3 = 1.33 \text{ rad/s}^2$$

SELF ASSESSMENT EXERCISE No.5

1. A wheel at rest accelerates to 8 rad/s in 2 seconds. Calculate the acceleration. (Answer 4 rad/s²)
2. A flywheel spins at 5000 rev/min and is decelerated uniformly to 2000 rev/min in 12 seconds. Calculate the acceleration in rad/s². (Answer -26.2 rad/s²)

2.4 LINK BETWEEN ANGULAR AND LINEAR MOTION

Consider a point moving on a circular path as shown.



The length of the arc = s metres.
Angle of the arc is θ radians

The link is $s = R\theta$

Figure 9

Suppose the point P travels the length of the arc in time t seconds. The wheel rotates θ radians and the point travels a distance of $R\theta$.

The velocity along the circular path is $v = R\theta/t = R\omega$

Next suppose that the point accelerates from angular velocity ω_1 to ω_2 .
The velocity along the curve also changes from v_1 to v_2 .

Angular acceleration = $\alpha = (\omega_2 - \omega_1)/t$

Substituting $\omega = v/R$

$\alpha = (v_2/R - v_1/R)/t = a/R$ hence $a = R\alpha$

It is apparent that to change an angular quantity into a linear quantity all we have to do is multiply it by the radius.

WORKED EXAMPLE No.7

A car travels around a circular track of radius 40 m at a velocity of 8 m/s. Calculate its angular velocity.

SOLUTION

$$v = \omega R$$

$$\omega = v/R = 8/40 = 0.2 \text{ rad/s}$$

2.5 EQUATIONS OF MOTIONS

The equations of motion for angular motion are the same as those for linear motion but with angular quantities replacing linear quantities.

replace s or x with θ

replace v or u with ω

replace a with α

LINEAR	ANGULAR
$s = ut + at^2/2$	$\theta = \omega_1 t + \alpha t^2/2$
$s = (u + v)t/2$	$\theta = (\omega_1 + \omega_2)t/2$
$v^2 = u^2 + 2as$	$\omega_2^2 = \omega_1^2 + 2\alpha\theta$

Also remember that the angle turned by a wheel is the area under the velocity - time graph.

SELF ASSESSMENT EXERCISE No.6

1. A wheel accelerates from rest to 3 rad/s in 5 seconds. Sketch the graph and determine the angle rotated.
(Answer 7.5 radian).
2. A wheel accelerates from rest to 4 rad/s in 4 seconds. It then rotates at a constant speed for 3 seconds and then decelerates uniformly to rest in 5 seconds. Sketch the velocity time graph and determine
 - i. The angle rotated. (30 radian)
 - ii. The initial angular acceleration. (1 rad/s²)
 - iii. The average angular velocity. (2.5 rad/s)