SOLID MECHANICS

DYNAMICS

TUTORIAL – INERTIA FORCES IN MECHANISMS

This work covers elements of the syllabus for the Engineering Council Exam D225 – Dynamics of Mechanical Systems C103 Engineering Science.

This tutorial examines the relationship between inertia and acceleration.

On completion of this tutorial you should be able to

- Define inertia force.
- Calculate the force required to accelerate components in a mechanism.
- Consider the effect of friction on the acceleration.
- Use the system approach to solve forces in a mechanism.

It is assumed that the student is already familiar with the following concepts.

- Newton's Laws of Motion.
- The laws relating displacement, velocity and acceleration.
- Coulomb's Laws of Friction.
- The representation of forces by a vector.
- The resolution of forces into two perpendicular components.
- The forms of mechanical power and energy.

All the above may be found in the pre-requisite tutorials.

1.1. FORCE, MASS AND ACCELERATION

We have all probably observed that objects with large masses require more force to accelerate them than objects with small masses. This is why, for example, large vehicles need larger engines than small vehicles. The reluctance of a body to speed up or slow down is in direct proportion to the mass and the effect is known as inertia. Newton showed that the relationship between Force (F), mass (m) and acceleration (a) is

$$\mathbf{F = m\,a}$$

From this relationship we get the definition of force. A force of 1 Newton is that required to accelerate a mass of 1 kg at 1 m/s^2. It follows that the Newton is a derived unit such that 1 N = 1 kg m/s^2. This should always be remembered when checking the units in a calculation.

WORKED EXAMPLE No.1

A road vehicle has a mass of 2 Tonnes. Calculate the force required to accelerate it at 1.2 m/s^2. Assume that there is no resistance to motion other than the inertia of the vehicle.

SOLUTION

Mass = 2000 kg

a = 1.2 m/s^2.

F = m a = 2000 x 1.2 = 2400 N or 2.4 kN.

When applying Newton's 2nd law we must be careful to ensure that F is the net force actually used to overcome inertia. This means that the applied force may be reduced by any forces opposing motion, usually friction.

Remember that the relationship between friction force (F) and reaction force (R) for dry friction is $F = \mu R$ where μ is the coefficient of friction. This is Coulomb's law of friction.

WORKED EXAMPLE No.2

A block of mass 800 kg rests on a horizontal surface. A horizontal force of 3 kN is applied to it. The coefficient of friction between the block and the surface is 0.3. Calculate the acceleration of the block.

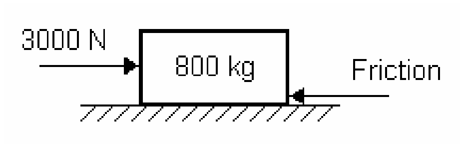

Figure 1

SOLUTION

R = weight of block = 800 x 9.81 = 7848 N

Frictional force opposing motion = μR = 0.3 x 7848 = 2354.4 N

Net force causing motion = 3000 - 2354.4 = 645.6 N.

From F = ma we have a = F/m = 645.6/800 = **0.807 m/s^2.**

WORKED EXAMPLE No.3

A piston of mass 10 kg has an acceleration of 20 m/s^2 as shown. At the instant considered the velocity is 5 m/s to the right. Motion is opposed by friction and by the force in the connecting link F_1 which is 1.19 kN. The coefficient of friction between the piston and the horizontal surface on which it slides is 0.3. Calculate the applied force F and the power being used.

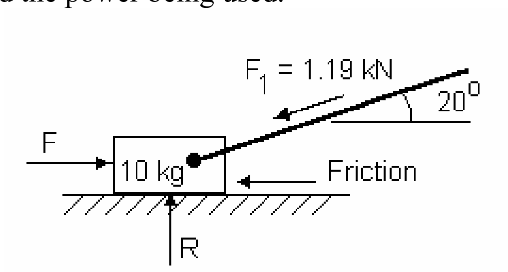

Figure 2

SOLUTION

First we must determine the total force opposing motion. To do this we must resolve F1 vertically and horizontally.

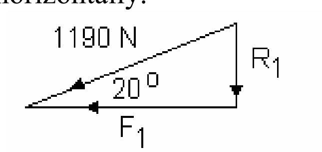

Figure 3

$R_1 = 1190 \sin 20^o = 407$ N

Weight = 10 x 9.81 = 98.1 N.

Total reaction force = R = 407 + 98.1 = 505.1 N.

Friction force = $\mu R = 0.3 \times 505.1 = 151.53$ N.

$F_1 = 1190 \cos 20^o = 1118$ N.

Total force opposing motion = F_1 + Friction = 1118 + 151.53 = 1269.5 N

Net force producing motion = F - 1269.5 = mass x acceleration.

F - 1269.5 = 10 x 20 = 200 N

F = 200 + 1269.5 = 1469.5 N

Power = Force x velocity = 1469.5 x 5 = 7348 Watts.

SELF ASSESSMENT EXERCISE No.1

1. A piston is accelerated from left to right at 15 m/s^2 with a force F. The piston has a mass of 5 kg and the coefficient of friction between it and the cylinder walls is 0.25. The force in the connecting rod opposing motion is 2000 N at 15^o to the horizontal. Calculate the applied force F acting on the piston. The motion is also from left to right. (Answer 2149 N)

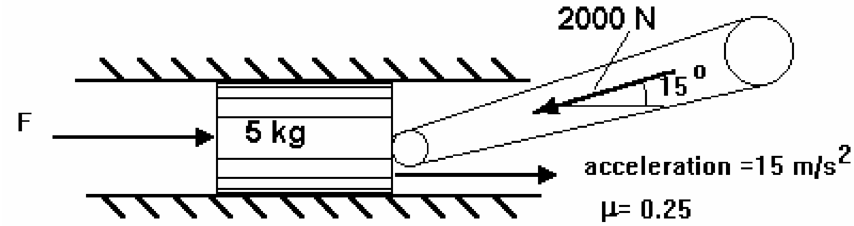

Figure 4

Now we will consider a complete piston, connecting rod and crank system as commonly used in reciprocating mechanisms. You will need to revise triangles of forces covered in earlier tutorials. We will go straight to a worked example.

WORKED EXAMPLE No.4

The diagram shows a piston, connecting rod and crank mechanism. A force is applied as shown at point B which opposes motion. The piston is made to move from right to left by a force F. Determine the force in the connecting rod BC.

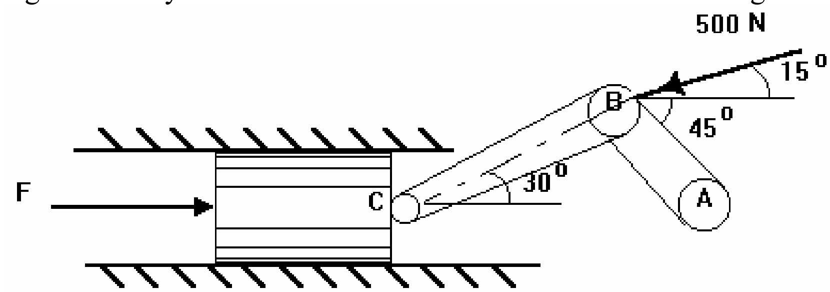

Figure 5

SOLUTION

First we must draw a triangle of forces for point B assuming that all the forces are in equilibrium.

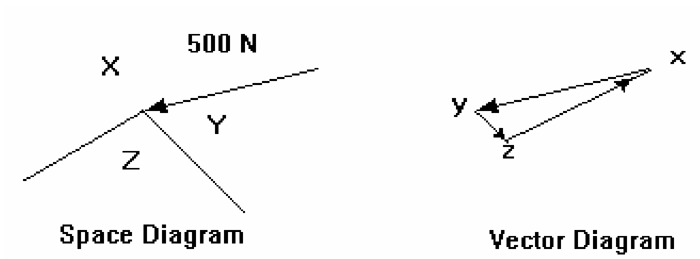

Figure 6

The connecting rod force is xz. By drawing to scale or by other means, this force is 448 N.

SELF ASSESSMENT EXERCISE No. 2

1. The same system described in the worked example has the following additional information. The mass of the piston is 2 kg. The coefficient of friction between it and the cylinder walls is 0.28. The piston accelerates and moves from left to right at 50 m/s^2.

   Determine the force F acting on the piston. (Answer 556 N)

2. A piston, crank and connecting rod system is shown in the diagram below. The coefficient of friction between the piston and cylinder walls is 0.3. Determine the force F acting on the piston when it accelerates from left to right at 30 m/s^2.
   (Answer 1333N)

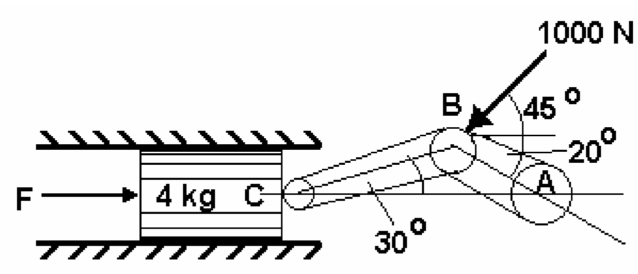

Figure 7

1.2 THE SYSTEM APPROACH

The system approach enables us to solve power inputs and outputs to the mechanism. We draw a boundary around the system and apply the law of conservation of energy to it. In this instance we interpret this as saying the difference between the power entering and leaving the system must be the power wasted within the system. The power is wasted as frictional heat which warms up the component parts. Consider such a system.

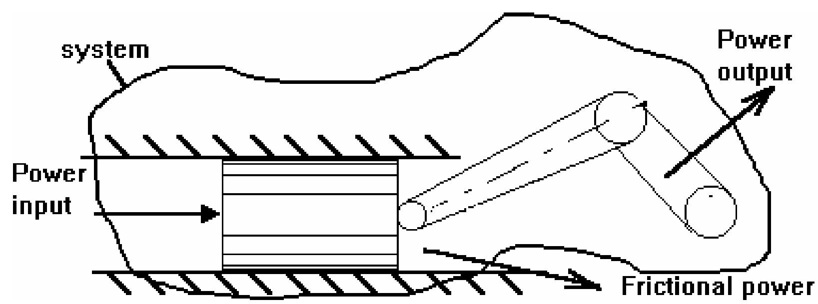

Figure 8

The power input at the piston is Force x Velocity.

The frictional power is friction Force x velocity.

The power output from crank may hence be deduced by the following equation.

Power Output = Power input - frictional power loss.

WORKED EXAMPLE No.5

The system shown in the previous diagram has the following data.

Piston mass	8 kg
Coefficient of friction	0.3
Force in connecting rod	1000 N
Force pushing on piston	2000 N
Angle of connecting rod	30^o to the horizontal
Piston velocity	3 m/s from left to right

Calculate the power output from the crank and the acceleration of the piston.

SOLUTION

First resolve the force in the connecting rod vertically and horizontally.

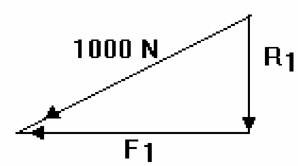

Figure 9

$R_1 = 1000 \sin 30^o = 500$ N

$F_1 = 1000 \cos 30^o = 866$ N

Weight of piston = 8 x 9.81 = 78.5 N

Total reaction force R = 500 + 78.5 = 578.5 N

Friction force opposing movement = μR = 0.3 x 578.5 = 173.5 N

Frictional power out = Force x velocity = 173.5 x 3 = 520.5 Watts.

Power input = Input force x velocity = 2000 x 3 = 6000 Watts.

Power out at crank = 6000 - 520.5 = 5479.5 Watts

Net force on piston = 2000 - F_1 - friction = 2000 - 866 - 173.5 =960.5 N

Acceleration = Net Force/mass = 960.5/8 = 120 m/s^2.

SELF ASSESSMENT EXERCISE No. 3

The diagram shows part of a mechanism consisting of a piston of mass 10 kg inside a cylinder, and two light links AB and BC pin-jointed at their ends. The friction coefficient between the piston and cylinder is 0.3. For the configuration shown the piston is moving to the right with a velocity of 5 m/s and an acceleration of 20 m/s^2 and is driven by the cylinder pressure which exerts a total force F. A resisting force of 1 kN is applied at joint B in the direction shown.

Sketch free body-diagrams for the piston and for the joint at B showing all relevant forces and determine

i) The magnitude and nature of the force in each link.

ii) The magnitude of the force F. (1851 N)

iii) The rate of doing work on the piston. (9255 W)

iv) The rate of doing work at joint B. (8310 W)

Assume that both links are of negligible mass and that all the joints are frictionless.

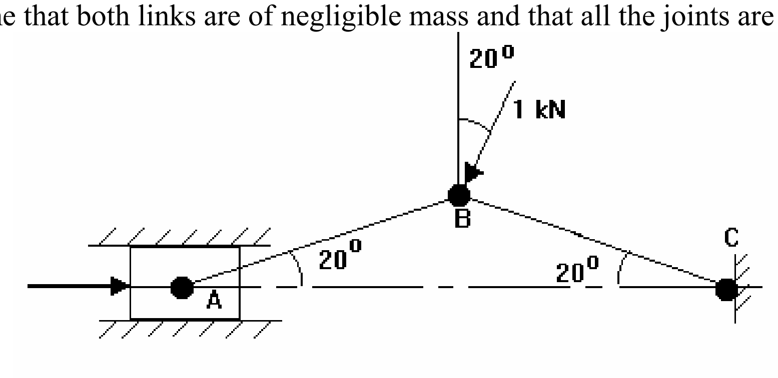

Figure 10

Hint... the rate of doing work on the piston is F x velocity.
the rate of doing work at joint B is the power out at the crank.
frictional power must be considered.