SOLID MECHANICS
DYNAMICS

GOVERNORS

Oncompletion of this tutorial you should be able to:

- **Explain the principles of Governors**
- **Explain the use and application of different governors**

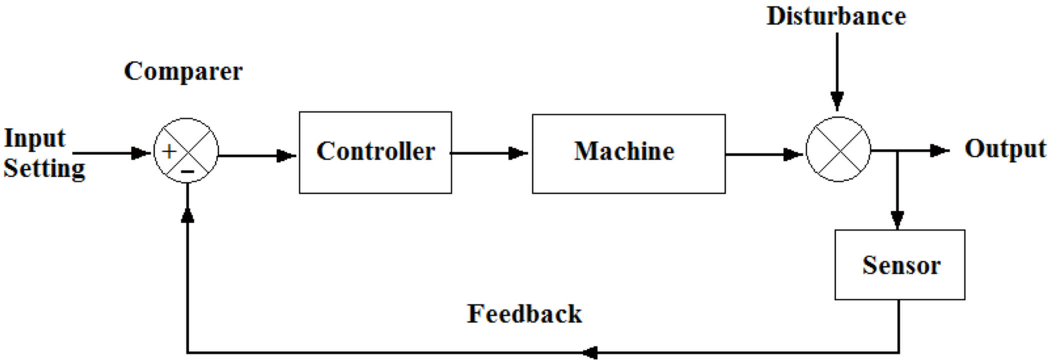
This module is part of the City and Guilds Degree Course - Module C130

Contents

1. *Basic Theory*

This section of the syllabus seems to be rather old fashioned as the use of mechanical governors has largely been replaced by electronic analogue and digital systems.

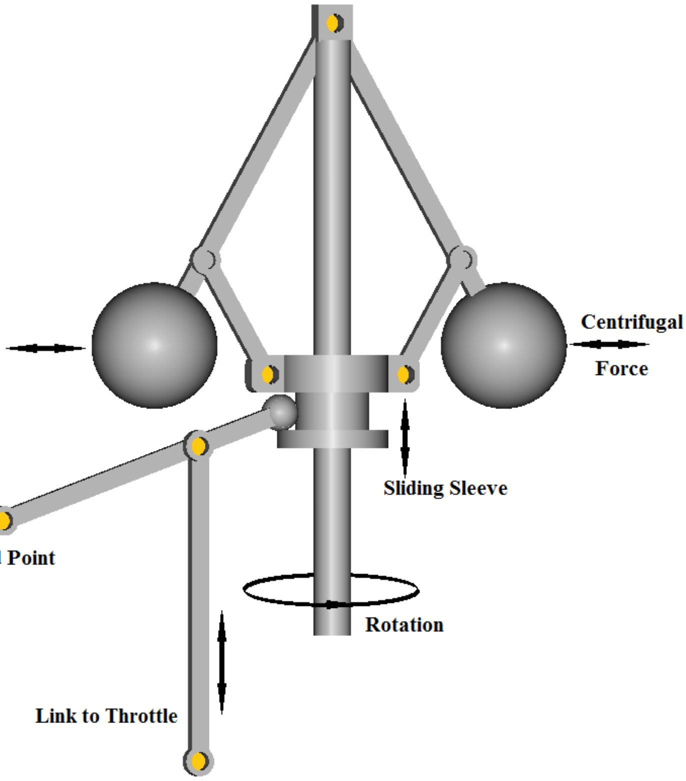
The purpose of a governor is usually to regulate the speed of a rotating machine to keep it within a defined range. In general the term could be applied to any control system and it seems a good idea to start by taking a system based approach.



For any controlled system such as a machine there must be an input setting and ideally the output will be the same as the input setting. However due to many factors the output can be affected by disturbances that are added or subtracted to the output. In order to keep these to a minimum, the output must be sensed and processed and fed back to a comparing device which adjusts the controller so that it counteracts the disturbance.

Clearly there is a lot more to this that can only be understood by studying control systems. In modern machines the sensing, comparing and controlling are likely to be electronic digital control based around a computer. A prime example of this is the engine management system of a modern vehicle engine. We are not going to study this but look at purely mechanical systems which never-the-less contains all the same basic principles.

2. *Mechanical Governors*



Internal combustion engines have long been used for stationary applications such as driving air compressors, pumps, electric generators and so on. The speed of the engine needs to be constant and so a governor is needed to adjust the controls according to the output speed. The same applies to external combustion engines such as gas turbines and steam turbines. The control action basically is that of adjusting a throttle of some kind that varies the flow of fuel, air or steam.

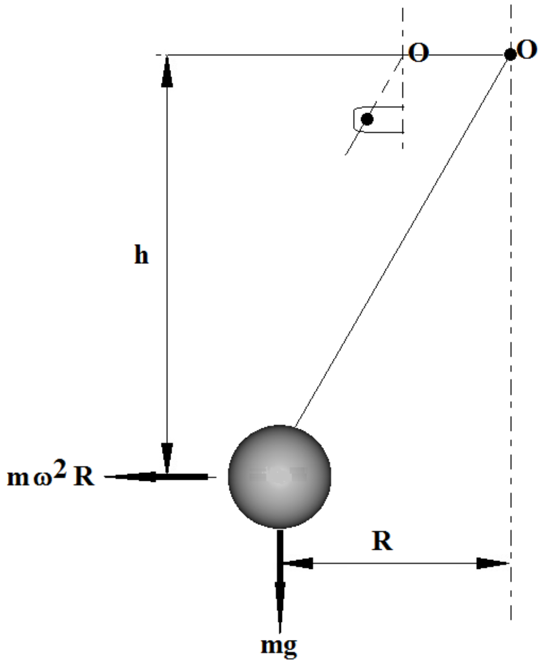
The fly ball or ***conical governor*** illustrated above (designed by James Watt) is a complete control system. The sensing is done by the balls under the action of centrifugal force. This moves a sleeve and linkage to adjust the throttle. The mechanism settles down at the design speed but if the speed increases the balls fly out, the sleeve slides up and the throttle linkage acts to reduce the speed until equilibrium is restored. If the speed drops the reverse happens. See the video and others on U Tube.

CLICK

There are different configurations of this governor but all are basically the same. Consider one ball pivoted about O. When balanced the torques about O are:

Centrifugal Force = $m\omega^2R$
Torque about O = $m\omega^2R$ h and this acts clockwise
Weight = mg
Restoring Torque = mgR and this acts anticlockwise
These are equal and opposite so we may equate

$$h = \frac{g}{\omega^2}$$



If the pivot point is not on the axis of rotation then point O is the intersection point as shown and h the distance to that point. The analysis does not take into account the weight of the sleeve or the friction force. Note the mass of the ball does not affect the height.

WORKED EXAMPLE No. 1

Calculate the value of h at a speed of 180 rev/min and the movement produced by an increase of 20 rev/min.

SOLUTION
At 180 rev/min

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 180}{60} = 6\pi \text{ rad/s}$$

$$h = \frac{g}{\omega^2} = \frac{9.81}{(6\pi)^2} = 0.0276 \text{ m or } 27.6 \text{ mm}$$

At 200 rev/min

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 200}{60} = 20.944\frac{\text{rad}}{\text{s}}$$

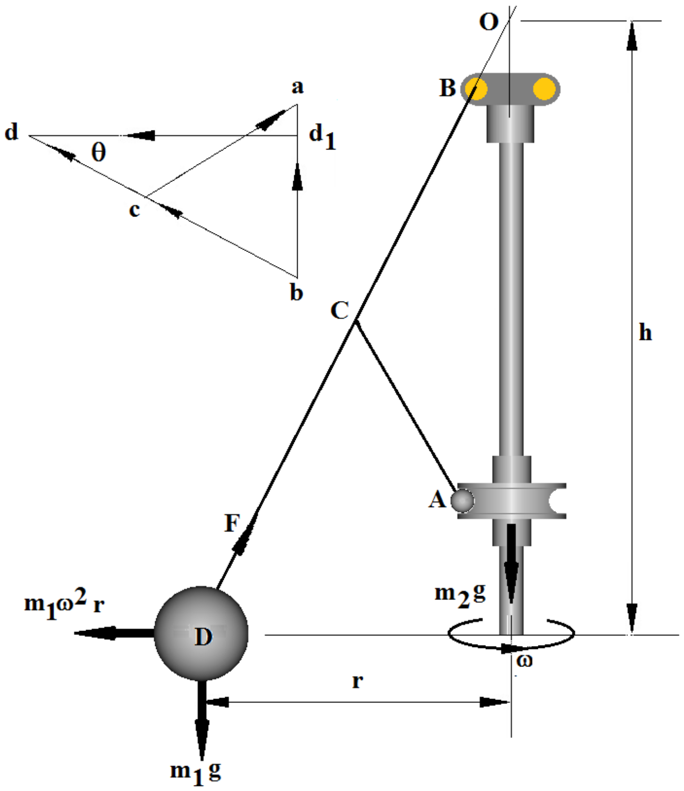
$$h = \frac{g}{\omega^2} = \frac{9.81}{(20.494)^2} = 0.02236 \text{ m or } 22.4 \text{ mm}$$

The movement is hence 5.2 mm

The movement of the ball needs to be changed into the movement of a sliding sleeve. There are many designs of mechanism for doing this. As with all control systems of this type, it is impossible to get exact control as there must be a change in speed to produce a change in the throttle setting. If the system is too sensitive then the phenomenon of "***hunting***" might occur where it first goes too fast, then too slow alternatively as it tries to find a settling point.

3. Watt Governor

The next diagram shows the layout of one half of the Watt governor. Consider the velocity diagram for the linkage A B C D (If you haven't studied velocity diagrams you need to do so in order to follow the derivation). The various parameters can be linked by considering the conservation of energy when movement occurs.



The velocity diagram is shown for the linkage and indicates the direction and relative movement of all the points. Except for the sliding sleeve these can only be at 90° to the links. Let the sleeve be sliding upwards so the movement of A relative to B can only be vertical.

Velocity of A relative to B is b a (up)
Velocity of A relative to C is c a
Velocity of C relative to B is b c
The velocity vectors can be added to give $\qquad$ b a = b c + c a

The velocity of D is an extension of b c
By relative proportions we can say:

$$\frac{\text{b c}}{\text{b d}} = \frac{\text{BC}}{\text{BD}}$$

Point d d_1 gives the horizontal component of the velocity of D and b d_1 the vertical component.

Work Method - When moving the work done per second by the various forces must all add up to zero. m_1 is the mass of the ball and m_2 is the mass of the sliding sleeve.

Work done by Centrifugal force is: $m_1\omega^2 r \times d\,d_1$
Work done by raising m_1 is: $m_1 g \times b\,d_1$
Work done by raising m_2 is: $\frac{1}{2} m_2 g \times b\,a$

Note that half the weight of the sleeve is supported by the other half of the linkage.
Balancing the work done we get

$$m_1\omega^2 r \times (d\,d_1) = m_1 g \times (b\,d_1) + \frac{m_2 g \times (b\,a)}{2}$$

$$m_1\omega^2 r = m_1 g\frac{(b\,d_1)}{(d\,d_1)} + \frac{m_2 g}{2}\frac{(b\,a)}{(d\,d_1)} = m_1 g\frac{r}{h} + \frac{m_2 g}{2}\frac{r}{h}\frac{(b\,a)}{(b\,d_1)}$$

$$m_1\omega^2 = \frac{m_1 g}{h} + \frac{m_2 g}{2h}\frac{(b\,a)}{(b\,d_1)}$$

Friction

From this one of the parameters can be calculated given the others. If there is friction between the sleeve and spindle this can be taken into account by adding or subtracting half the friction 'f' force to $m_2 g$

$$m_1\omega^2 r \times (d\,d_1) = m_1 g \times (b\,d_1) + \frac{(m_2 g \pm f) \times (b\,a)}{2}$$

The $\pm f$ is used because friction can work in opposite directions depending on whether the sleeve is moving up or down.

WORKED EXAMPLE No. 2

A Watt governor as illustrated previously has the following parameters

B is 37 mm from the vertical axis, A is 50 mm from the vertical axis.
AC is 160 mm, BC is 185 mm and BD is 300 mm.
The rotating ball is at a radius of 155 mm and has a mass of 2.75 kg.

Calculate the speed of the governor under the following conditions.

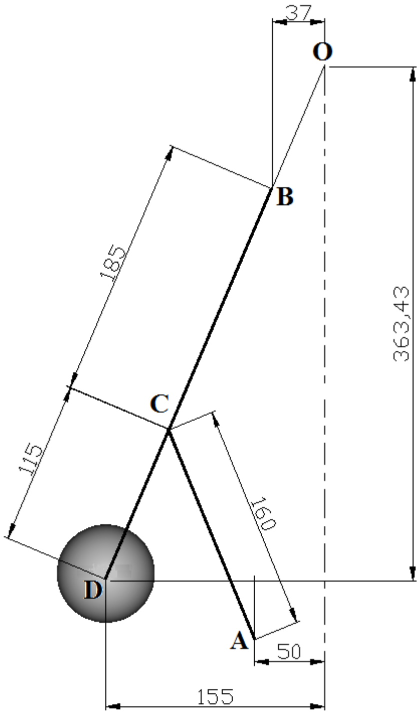
a) Ignoring the mass of the sleeve
b) If the sleeve has a mass of 800 g
c) If the friction force is 6 N

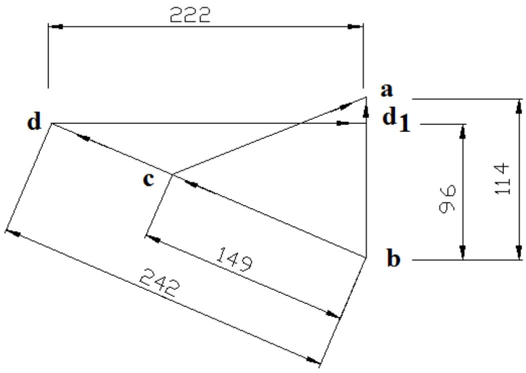
SOLUTION

The analytical method of solution would be quite difficult and it is easier to draw the linkage to scale and from that draw the velocity diagram. The scale drawing is shown with angles that can be deduced analytically. A tip on construction is to locate points B, D and C and draw an arc to find point A. The velocity diagram a b c d d_1 can be drawn to an arbitrary scale with the lines normal to those on the space diagram.

$$\frac{bc}{bd} = \frac{BC}{BD} = \frac{185}{300} = \frac{149}{bd}$$

149 mm is the length measured from the original diagram. From this bd = 242 mm and then the velocity diagram can be completed. Measuring bd_1 and dd_1 gives 96 and 222 mm in this case and ba is 114 mm.





a) Put $m_2 = 0$ and first after finding h = 363 mm we can use

$$\omega = \sqrt{\frac{g}{h}} = \sqrt{\frac{9.81}{0.363}} = 5.2\frac{\text{rad}}{\text{s}} \text{ or } 49.6 \text{ rev/min}$$

Solving by the Work method

$$m_1\omega^2 r \times (dd_1) = m_1 g \times (bd_1)$$

$$\omega = \sqrt{\frac{g\,(bd_1)}{r\,(dd_1)}} = \sqrt{\frac{9.81 \text{ x } 96}{0.155 \text{ x } 222}} = 5.23\text{rad/s or } 49.9 \text{ rev/min}$$

The slight difference is due to rounding off figures.

b) Taking the mass of the sleeve into account we have

$$m_1\omega^2 r \times (dd_1) = m_1 g \times (bd_1) + \frac{m_2 g(ba)}{2}$$

$$2.75\omega^2 \times 0.155 \times 222 = 2.75 \times 9.81 \times 96 + \frac{0.8 \times 9.81 \times 114}{2}$$

$$94.63\ \omega^2 = 2589.8 + 447.3$$
$$\omega = 5.66 \text{ rad/s or } 54 \text{ rev/min}$$

c) With friction

$$m_1\omega^2 r \times (dd_1) = m_1 g \times (bd_1) + \frac{(m_2 g \pm f) \times (b\,a)}{2}$$

$$2.75\omega^2 \times 0.155 \times 222 = 2.75 \times 9.81 \times 96 + \frac{(0.8 \times 9.81 \pm 6) \times 114}{2}$$

$$94.63\ \omega^2 = 2\,589.8 + 789$$

$$\omega = 5.98 \text{ rad/s or } 57 \text{ rev/min}$$

or

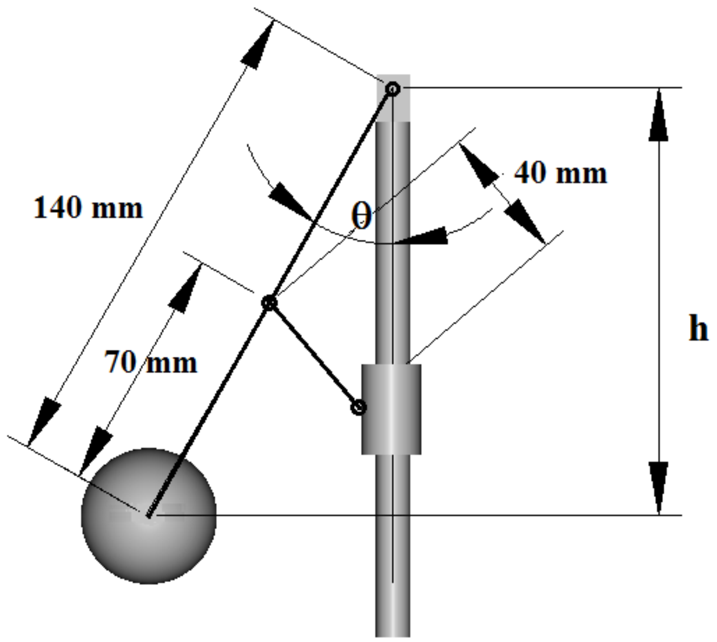
$$94.63\ \omega^2 = 2\,589.8 + 105.3$$

$$\omega = 5.35 \text{ rad/s or } 51 \text{ rev/min}$$

Remember you might get different values for dd_1, bd_1 and ba but you should get the same ratios. It is fine to leave these dimensions in mm as done here but r must be in metres as it is the actual radius.

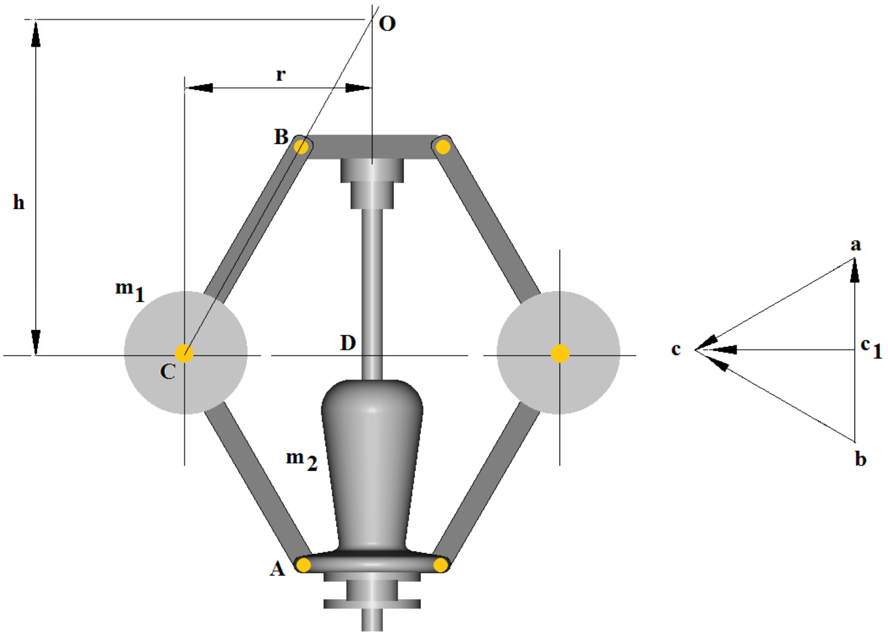
SELF ASSESSMENT EXERCISE No. 1

A Watt governor as illustrated must control the speed between 90 and 85 rev/min. Determine the movement of the sleeve. Assume no friction and ignore the weight of the sleeve.
(Answer 17 mm)



4. Porter Governor

It should be clear that the speed at which a governor operates depends on the downwards forces acting on it. If we increased the mass of the sleeve the governor would have to run faster to maintain the same position of the sleeve. An example is the ***Porter Governor*** (also known as the loaded Watt Governor) and this usually has equal arms and B and A are the same distance from the vertical axis. The velocity diagram is as shown. c_1c is the horizontal velocity of m_1 and bc_1 is the vertical component.



Balancing the rate of work we have:

$$m_1\omega^2 r \times (c_1c) = m_1 g \times (bc_1) + \frac{m_2 g \times (ba)}{2}$$

For the ***symmetrical layout*** shown it follows that

$$\frac{c_1c}{bc_1} = \frac{OD}{CD} = \frac{h}{r} \text{ and } \frac{ba}{bc_1} = 2$$

Substituting and rearranging gives

$$m_1\omega^2\, r\,\frac{c_1c}{bc_1} = m_1 g\,\frac{bc_1}{bc_1} + \frac{m_2 g}{2}\,\frac{ba}{bc_1}$$

$$m_1\omega^2\, r\,\frac{h}{r} = m_1 g + \frac{m_2 g}{2} \text{ x } 2 = m_1 g + m_2 g$$

$$\omega^2 = \left(1 + \frac{m_2}{m_1}\right)\frac{g}{h}$$

Friction

If friction is involved then:

$$m_1\omega^2 r \times (c_1 c) = m_1 g \times (b\,c_1) + \frac{(m_2 g \pm f) \times (ba)}{2}$$

Substituting and rearranging gives

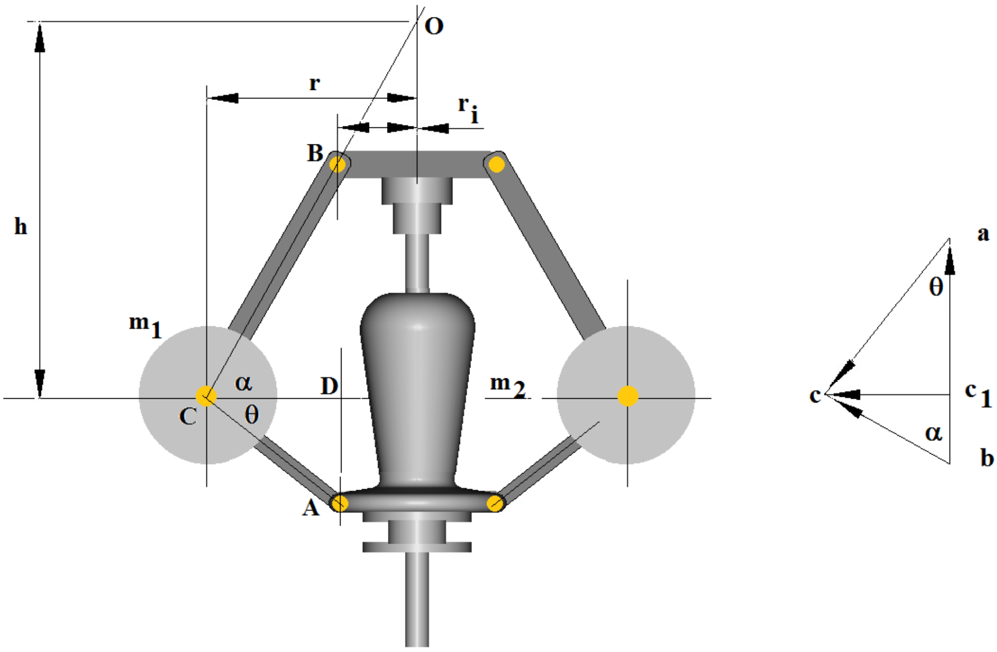
$$m_1\omega^2\, r\, \frac{c_1 c}{bc_1} = m_1 g\, \frac{bc_1}{bc_1} + \frac{m_2 g \pm f}{2}\, \frac{ba}{bc_1}$$

$$m_1\omega^2\, r\, \frac{h}{r} = m_1 g + \frac{m_2 g \pm f}{2} \times 2$$

$$\omega^2 h = g + \frac{m_2 g \pm f}{m_1}$$

$$\omega^2 = \frac{g}{h} + \frac{m_2 g \pm f}{hm_1} = \frac{g}{h} + \frac{m_2 g}{hm_1} \pm \frac{f}{hm_1} = \frac{g}{h}\left(1 + \frac{m_2}{m_1} \pm \frac{f}{gm_1}\right)$$

Unequal Links



If the links are not the same length then the ratio ba/bc_1 must be deduced. With points A and B at the same radius r_i a bit of geometry show

$$\tan\theta = \frac{AD}{CD} = \frac{c_1 c}{ac_1} \qquad ac_1 = \frac{CD}{AD}\, c_1 c$$

$$\tan\alpha = \frac{BD}{CD} = \frac{c_1 c}{bc_1} \qquad bc_1 = \frac{CD}{BD} c_1 c$$

$$q = \frac{ba}{bc_1} = \frac{bc_1 + ac_1}{bc_1} = 1 + \frac{ac_1}{bc_1} = 1 + \frac{\frac{CD}{AD}}{\frac{CD}{BD}} = 1 + \frac{BD}{AD}$$

$$q = 1 + \frac{BD}{AD} = 1 + \frac{\sqrt{(BC)^2 - (r - r_i)^2}}{\sqrt{(AC)^2 - (r - r_i)^2}}$$

For equal links AC = BC so q = 2

Redevelop the equations for speed

$$m_1\omega^2\, r\, \frac{c_1c}{bc_1} = m_1g + \frac{m_2g \pm f}{2}\, \frac{ba}{bc_1}$$

$$\tan(\alpha) = \frac{h}{r} = \frac{c_1c}{bc_1}$$

$$m_1\omega^2\, h = m_1g + \frac{m_2g \pm f}{2} \times q$$

$$\omega^2\, h = g + \frac{m_2g \pm f}{2m_1} \times q$$

$$\omega^2 = \frac{g}{h} + \frac{m_2g \pm f}{2hm_1} \times q$$

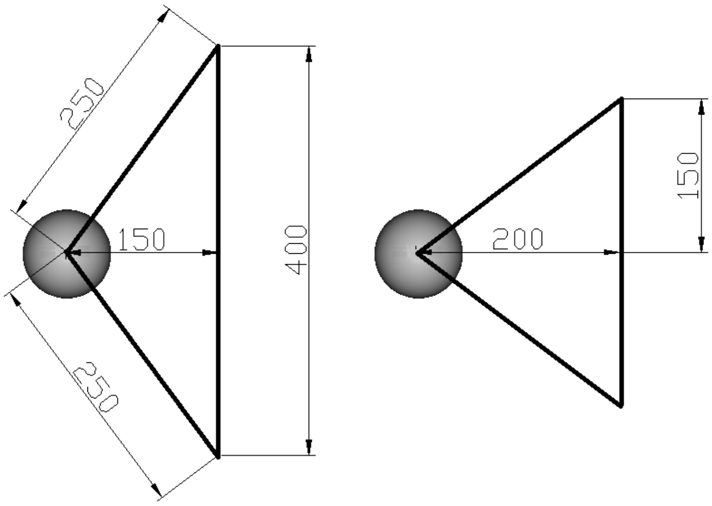
WORKED EXAMPLE No. 3

A Porter governor similar to the one illustrated previously has the linkage pivoted on the vertical axis. All links are 250 mm long. The rotating masses are 2.75 kg. The sliding mass is 12.75 kg. The radius to the centre of the rotating masses is 150 mm when they just start to rise and 200 mm when running at the maximum speed.

Calculate the operating range of speed.

SOLUTION

It is easy to construct the diagram below and deduce that h = 200 mm and 150 mm



When the mass starts to lift:

$$\omega^2 = \left(1+\frac{m_2}{m_1}\right)\frac{g}{h} = \left(1+\frac{12.75}{2.75}\right)\frac{9.81}{0.2}$$

$$\omega = 16.63 \text{ rad/s or } 158.8 \text{ rev/min}$$

At maximum speed:

$$\omega^2 = \left(1+\frac{m_2}{m_1}\right)\frac{g}{h}$$

$$\omega^2 = \left(1+\frac{12.75}{2.75}\right)\frac{9.81}{0.15}$$

ω = 19.2 rad/s or 183.3 rev/min

The speed range is 24.5 rev/min

5. *Controlling Forces*

It should be clear that friction in the sleeve produces two different speeds for the same position of the sleeve and this produces a dead band in the action of the governor. The speed would have to drop or increase significantly before movement reverses.

The ***sensitivity*** of the governor control depends on the relative movement of the balls and the link attached to the throttle (or other control device). The speed range of a governor is limited by stops on the spindle. The definition is:

$$\text{sensitivity} = \frac{\text{mean speed}}{\text{speed range}}$$

A governor with excessive sensitivity will produce a large movement of the sleeve when a small change in load occurs on the engine (or other machine) and will over compensate causing the system to hunt. A governor with a zero range of speed or infinite sensitivity is called ***Isochronous***. This is impossible to achieve because a change in the position of the sleeve is required in order to compensate for a change in load.

Controlling Force Diagram

This is produced by plotting the centrifugal force and controlling force against speed.
The centrifugal force is $F = m_1 \omega^2 r$

For the Porter governor with equal links we had the equation

$$\omega^2 = \left(1 + \frac{m_2}{m_1}\right)\frac{g}{h}$$

Substitute for ω^2 and:

$$m_1\omega^2 r = m_1 r\left(1 + \frac{m_2}{m_1}\right)\frac{g}{h}$$

$$m_1\omega^2 r = m_1 g\left(1 + \frac{m_2}{m_1}\right)\frac{r}{h} = g(m_1 + m_2)\frac{r}{h}$$

The controlling force is:

$$F_2 = g(m_1 + m_2)\frac{r}{h}$$

If friction *f* is taken into account then the controlling force is

$$F_2 = g\left(m_1 + m_2 \mp \frac{f}{g}\right)\frac{r}{h}$$

Note that the controlling force depends on the ratio r/h and this is defined by the geometry of the governor.

WORKED EXAMPLE No. 4

Draw the controlling force diagram for the governor in worked example No.3 for the speed range 140 to 190 rev/min. Assume there is no friction in the sleeve.
If the friction force in the sleeve is 13 N plot the upper and lower range of the controlling force.

SOLUTION

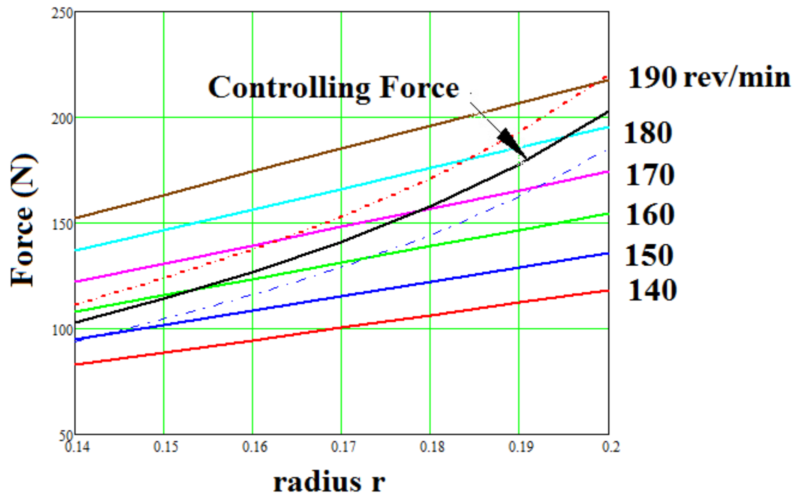
For this example r = 150 and the links are 250 mm long.
For the centrifugal force calculate two points for each speed at a given radius as the graphs are straight lines.

$$F = m_1\omega^2 r = m_1\left(\frac{2\pi N}{60}\right)^2 r = 2.75N^2(0.010966)r = 0.0302\ N^2 r$$

Controlling Force
For this example r = 150 and the links are 250 mm long.
From the right angle triangle h = $\sqrt{(250^2 - r^2)}$ The Controlling Force is

$$F_2 = g\left(m_1 + m_2 \mp \frac{f}{g}\right)\frac{r}{h} = 9.81\left(2.75 + 12.75 \mp \frac{13}{9.81}\right)\frac{150}{h}$$

The data to be plotted is evaluated and shown below. The plot shows the upper and lower limits of the controlling force at any speed within the range.



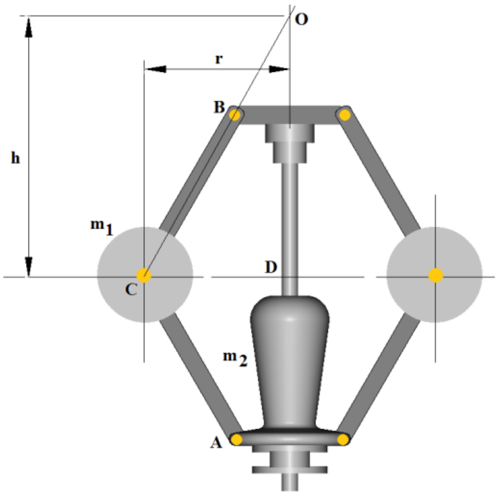
Plot Data

N rev/min	F@ r = 0.15 m F = $0.0302\ N^2\ r$	F@ r = 0.2 m
190	163.3 N	218 N
180	146.6	195.7
170	130.7	174.6
160	115.8	154.6
150	101.8	135.9
140	88.7 N	118.4 N

Radius mm	$h = \sqrt{(250^2 - r^2)}$	Controlling Force $g(m_1 + m_2)\frac{r}{h}$	$g\left(m_1 + m_2 + \frac{f}{g}\right)\frac{r}{h}$	$g\left(m_1 + m_2 - \frac{f}{g}\right)\frac{r}{h}$
140	207	102.8 N	111.6 N	94 N
150	200	114	123.8	104.3
160	192	126.7	137.5	115.8
170	183	141	153	129
180	173	157.8	171.2	144.3
190	162	177.8	193	162.6
200	150	202.7	220.1	185.4

SELF ASSESSMENT EXERCISE No. 2

Q1
A Porter Governor as shown has rotating masses of 2 kg.
All the arms are 120 mm long.
The sliding mass is 3 kg. The friction force is 4 N.
Calculate the speed when the masses rotate on a radius of 60 mm. The pivot points are at 10 mm radius

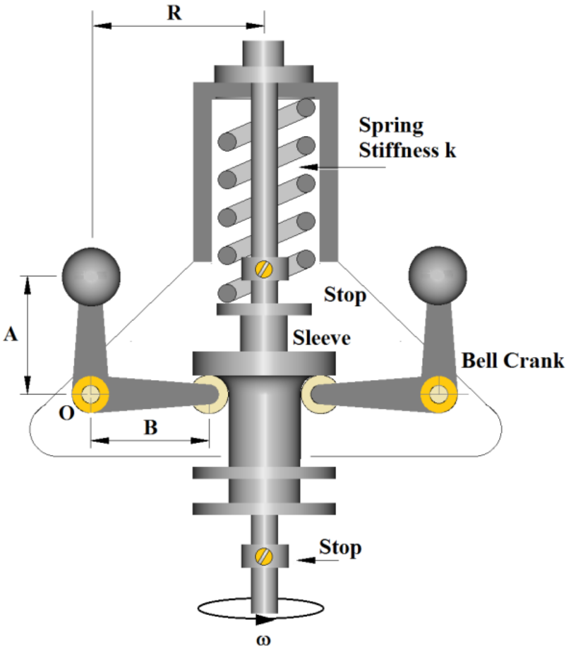


Answer 135 to 125 rev/min

Q2
Repeat Q1 but for the case where AC is 60 mm

Answer 178 to 161 rev/min

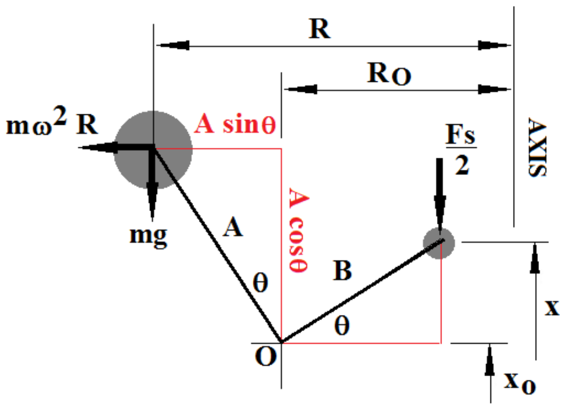
6. *Hartnell Governor*



This is a governor with a spring so we will examine this to see how the spring affects the controlling force. The balls of mass 'm' are carried on a bell crank with arms of length A and B as shown. The sleeve is held down against the crank by a spring that can be adjusted. As the balls fly outwards under centrifugal force the sleeve is raised and compresses the spring. As the sleeve moves the radius of the balls from the axis of rotation 'R' will change and so the weight of the ball comes into play either adding to the spring force or reducing the spring force depending which way they move.

Geometry

When the crank is at the mid point it is horizontal and vertical. x_O is the position of the sleeve and R_O is the radius of the balls. Rotating the crank θ the sleeve moves to position x and the balls to radius R.



$$\sin(\theta) = \frac{R - R_O}{A} = \frac{x - x_O}{B}$$

A change in angle will produce changes in x and R

$$\sin(\theta + \Delta\theta) = \frac{R - R_O + \Delta R}{A} = \frac{x - x_O + \Delta x}{B}$$

$$\frac{R - R_O}{A} + \frac{\Delta R}{A} = \frac{x - x_O}{B} + \frac{\Delta x}{B}$$

$$\sin(\theta) + \frac{\Delta R}{A} = \sin(\theta) + \frac{\Delta x}{B}$$

$$\frac{\Delta R}{A} = \frac{\Delta x}{B} \qquad \Delta x = \frac{B}{A}\Delta R$$

Spring Force

F_S = spring force. The change in spring force is Δ F_S. The spring rate is hence

$$k = \frac{\Delta F_s}{\Delta x} = \frac{\Delta F_s}{\Delta R}\frac{A}{B} \qquad \Delta F_s = k\Delta R\frac{B}{A}$$

Initial Compression

x_i is the initial compression of the spring on the bottom stop
x_T is the total spring compression $\quad x_T = x_i + x$
The spring force is hence $\quad F_S = k(x_i + x)$

Moment Balance

Moments about O gives:

$$m\omega^2 R\,A\cos(\theta) = \frac{F_S}{2} B\cos(\theta) - mg\,A\sin(\theta)$$

In practice it is found that the term mgA sin(θ) is small and may be left out.

$$m\omega^2 R = \frac{F_S}{2}\frac{B}{A} \qquad \omega^2 = \frac{F_S}{2mR}\frac{B}{A}$$

$m\omega^2 R$ is the controlled force and $\frac{F_S}{2}\frac{B}{A}$ is the control force.

We can develop the control force expression as follows

$$\Delta x = \frac{B}{A}\Delta R$$

$$\frac{F_S}{2}\frac{B}{A} = \frac{kx_T}{2} \times \frac{B}{A} = \frac{k(x_i + \Delta x)}{2} \times \frac{B}{A} = \frac{k\left(x_i + \frac{B\Delta R}{A}\right)}{2} \times \frac{B}{A} = \frac{k}{2}\frac{B}{A}\left[x_i + \Delta R\frac{B}{A}\right]$$

Note ΔR is any change in radius corresponding to a change in the sleeve position Δx.

Isochronous Condition

When the governor is ***isochronous*** the change in the controlled force is the same as the change in control force for any change in radius.

Change in Controlled Force
Centrifugal Force = $m\omega^2 R$ Differentiate

$$\frac{dF}{dR} = m\omega^2 = \text{constant} \qquad \text{hence } \frac{\Delta F}{\Delta R} = m\omega^2$$

Consider the control force.

$$F = \frac{F_S}{2}\frac{B}{A}$$

Substitute

$$x = (R - R_o)\frac{B}{A} \text{ and } F_s = kx_t = k(x_1 - x)$$

$$F = \frac{k\{x_i + x\}B}{2A} = \frac{kB}{2A}\left\{x_i + \frac{B}{A}(R - R_O)\right\} = \frac{kB}{2A}\left\{x_i + \frac{B}{A}R - \frac{B}{A}R_O\right\}$$

Differentiate:

$$\frac{dF}{dR} = \frac{k}{2}\frac{B^2}{A^2}$$

This is constant so

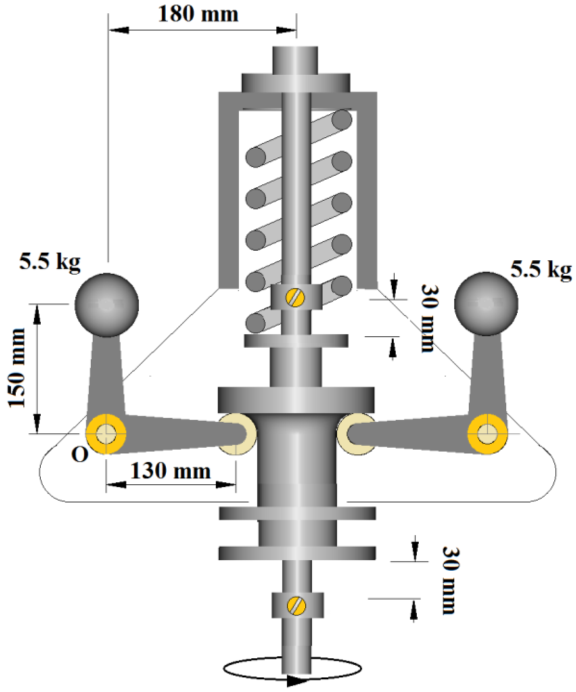
$$\frac{\Delta F}{\Delta R} = \frac{k}{2}\frac{B^2}{A^2}$$

For isochronism:

$$\boldsymbol{m\omega^2 = \frac{k}{2}\frac{B^2}{A^2} \text{ or } k = \frac{2A^2 m\omega^2}{B^2}}$$

WORKED EXAMPLE No. 5

The diagram shows a Hartnell Governor with no friction.



The stops are ± 30 mm from the position shown. The speed is to be set to obtain 250 rev/min and 265 rev/min at the lower and upper stops respectively.
Calculate the spring rate based on the total movement
Show that the mid position does not give the mean speed.
Calculate the initial spring compression needed.

SOLUTION

From the diagram A =150 mm B=130 mm R_O=180 mm
At the higher speed

$$\omega = \frac{265}{60} \text{x } 2\pi = 27.75 \frac{\text{rad}}{\text{s}}$$

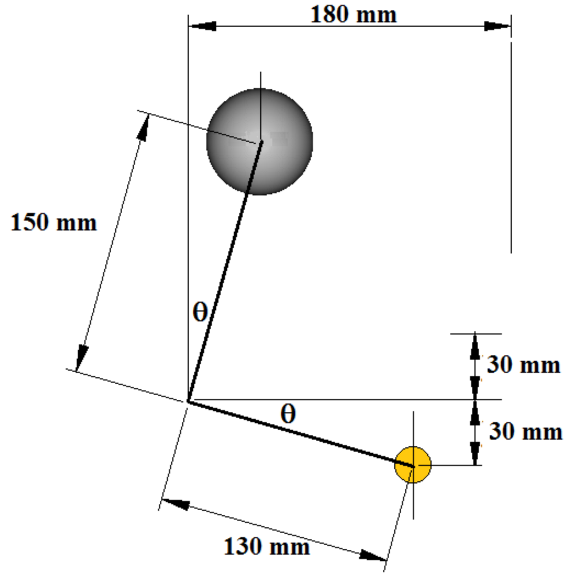
$$\theta = \sin^{-1}(30/130) = 13.34^o$$

$$R = 180 + 150 \sin(13.34^o) = 214.6 \text{ mm}$$

$$\frac{F_S}{2}\frac{B}{A} = m\omega^2 R + mg\tan(\theta)$$

$$\frac{F_S}{2}\frac{0.13}{0.15} = 5.5 \text{ x } 27.75^2 \times 0.2146 + 5.5 \times 9.81\tan(13.34^o)$$

$$0.433F_S = 908.9 + 12.79 \quad F_S = 2\ 128 \text{ N}$$



At the lower speed

$$\omega = \frac{250}{60} \times 2\pi = 26.18 \text{ rad/s}$$

$$\theta = \sin^{-1}(-30/130) = -13.34^{\circ}$$

$$R = 180 + 150 \sin(-13.34) = 145.4 \text{ mm}$$

$$\frac{F_S}{2}\frac{B}{A} = m\omega^2 R + mg\tan(\theta)$$

$$\frac{F_S}{2}\frac{0.13}{0.15} = 5.5 \times 26.18^2 \times 0.1454 + 5.5 \times 9.81\tan(-13.34^{o})$$

$$0.433F_S = 548 - 12.79 \qquad F_S = 1\,236 \text{ N}$$

$$k = \frac{\Delta F_S}{\Delta x} = \frac{2128 - 1236}{60} = 14.86 \text{ N/mm}$$

The initial compression is hence 1 236/14.86 =83.18 mm

At the mid position B is horizontal $\theta = 0$ R = 180 mm
F_S = mean force = (1 236 + 2 128)/2 = 1 682 N

$$\frac{F_S}{2}\frac{B}{A} = m\omega^2 R \qquad\qquad \frac{1682}{2} \times \frac{0.13}{0.15} = 5.5 \times \omega^2 \times 0.18$$

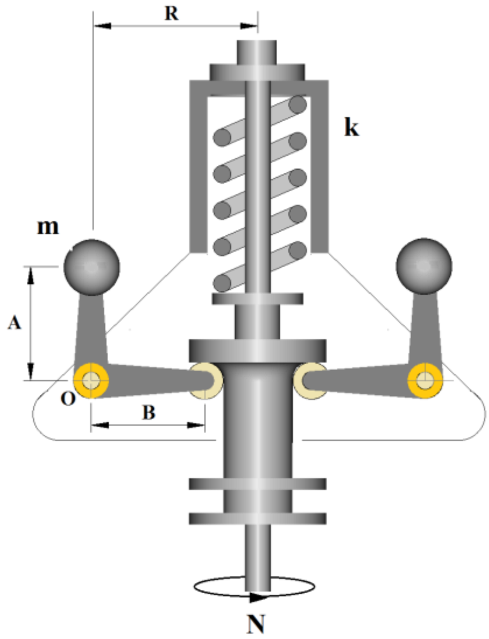
$$\omega = 27.13 \text{ rad/s} \qquad N = 259.1 \text{ rev/in}$$

Based on the upper and lower limits the mean speed is

$$N = \frac{265 + 250}{2} = 257.5 \text{ rev/min}$$

The mean speed is not obtained at the mid position.

WORKED EXAMPLE No. 6



The diagram shows a Hartnell Governor for which the data is: A = 150 mm B = 100 mm R = 130 mm
m = 5.5 kg k = 15 N/mm
The sleeve is shown at the mid position and can move ±25 mm between stops.
Calculate the isochronous speed and draw the control force diagram to confirm it. Draw the diagram using speed range of 180 to 280 rev/min and initial compressions of 35, 60 and 80 mm.

SOLUTION

Isochronous Speed - note that k must be in N/m

$$\omega = \sqrt{\frac{k}{2}\frac{B^2}{mA^2}} = \sqrt{\frac{15000}{2} x \frac{100^2}{5.5\ x 150^2}} = 24.62 \text{ rad/s}$$

$$N = \omega\ x\ \frac{60}{2\pi} = 235 \text{ rev/min}$$

Control force diagram.
At the upper stop R = 130 + (A/B) x 25 R_1 = 130 + 1.5 x 25 = 167.5 mm
At the lower stop R = 130 - (A/B) x 25 R_1 = 130 - 1.5 x 25 = 92.5 mm
The centrifugal force is $m\omega^2R = 5.5\ (2\pi N/60)^2 \ x\ R = 0.0603N^2R$

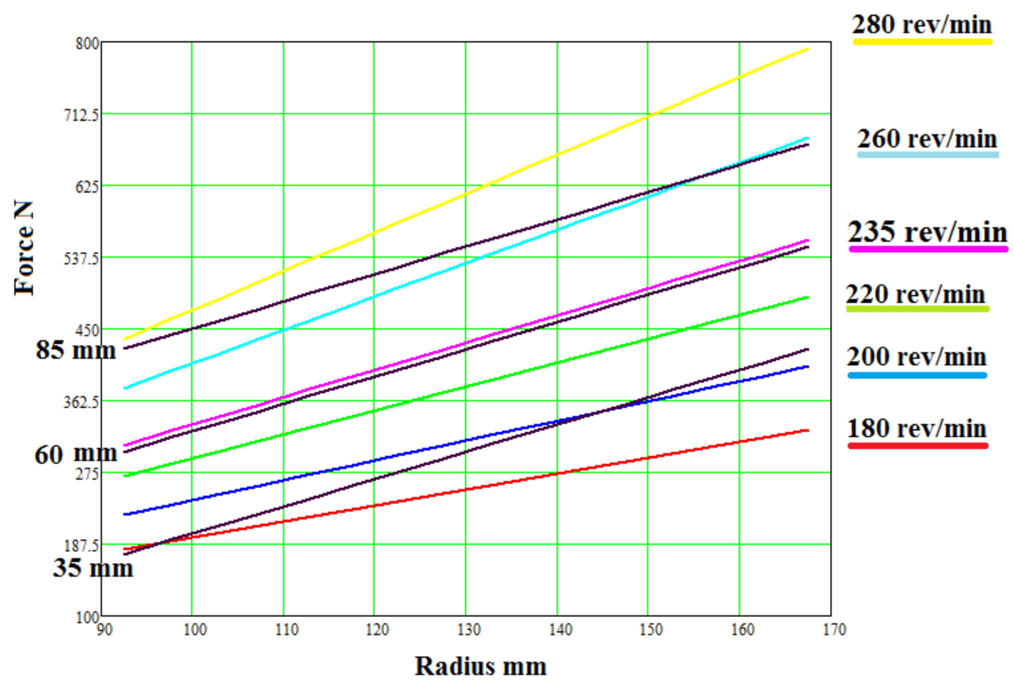
Work out the control force for a few values of initial compression.
The initial compression is at R = 92.5 mm A = 150 mm B = 100 mm k = 15 N/mm
The control force is

$$F = \frac{F_S}{2}\frac{B}{A}$$

x is the sleeve movement from the bottom stop
x_i is the initial compression of the spring on the bottom stop
x_T is the total spring compression $x_T = x_i + x$
$x = \Delta R(B/A) = \Delta R(2/3)$
F_S is the actual spring force $F_S = k\,x_T = k\,(x_i + x)$

The control force is:

$$F = \frac{kx_i}{2}\frac{B}{A} + k\frac{\Delta R}{2}\frac{B^2}{A^2} = \frac{15x_i}{2}\frac{100}{150} + 15\frac{\Delta R}{2}\frac{100^2}{150^2} = 5x_i + \frac{30\Delta R}{9}$$



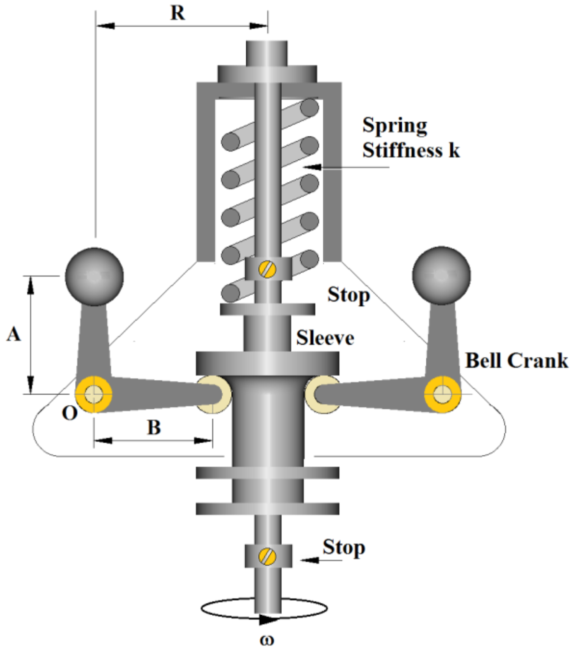
The plots show that 235 rev/min produces a parallel plot so it is isochronous.

PLOT DATA

Speed rev/min	$m\omega^2R = 0.0603N^2R$	
	R = 0.0925 m	R = 0.1675 m
180	180.8 N	327.3 N
200	223.2 N	404.1 N
220	246 N	445.5 N
235	308 N	558 N
260	321 N	582 N
280	377 N	683 N

x_i	Controlling Force $F = 5x_i + \frac{30\Delta R}{9}$	
	R = 92.5 mm $\Delta R = 0$	R = 167.5 mm $\Delta R = 75$ mm
35 mm	$x_T = 35$ mm F = 175 N	$x_T = 50$ mm F = 425 N
60 mm	$x_T = 60$ mm F = 300 N	$x_T = 125$ mm F = 550 N
85 mm	$x_T = 80$ mm F = 425 N	$x_T = 200$ mm F = 675 N

SELF ASSESSMENT EXERCISE No.3



1. A Hartnell Governor as shown has the following data.
A = 200 mm, B = 100 mm and R = 115 mm.
The balls have a mass of 4 kg.
The stops are ±20 mm from the position shown.

i. Calculate the spring rate to give an isochronous speed of 300 rev/in.
Answer 31.6 N/mm

ii. What is the initial compression required to make this speed occur at the mid position?
Answer 8.7 mm

2. The same governor as Q1 has a spring rate of 40 N/mm and the initial compression is 10 mm. Calculate the speeds at the upper and lower stops.
Answer 158.1 rev/min and 334.8 rev/min

3. Repeat Q2 with an initial compression of 100 mm
Answer 479.4 rev/min and 563.3 rev/min