

# **SOLID MECHANICS GOVERNORS**

**Outcome Explain the principles, and explain use and application of different governors and cams.**

This module is part of the City and Guilds Degree Course - Module C130

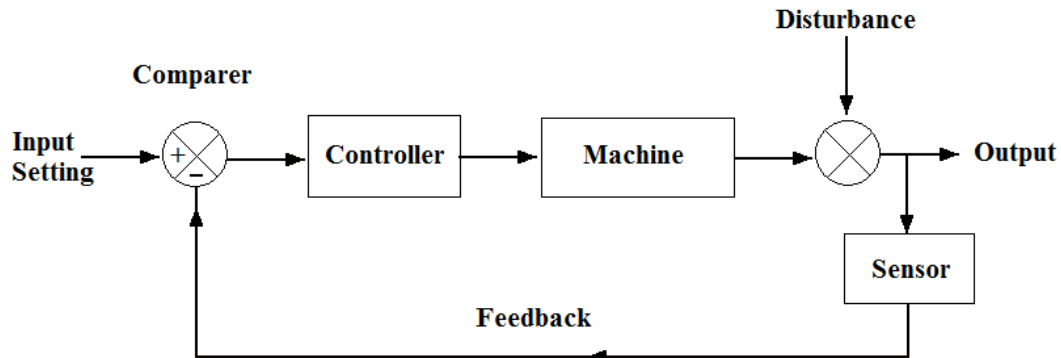
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## BASIC THEORY

This section of the syllabus seems to be rather old fashioned as the use of mechanical governors has largely been replaced by electronic analogue and digital systems.

The purpose of a governor is usually to regulate the speed of a rotating machine to keep it within a defined range. In general the term could be applied to any control system and it seems a good idea to start by taking a system based approach.



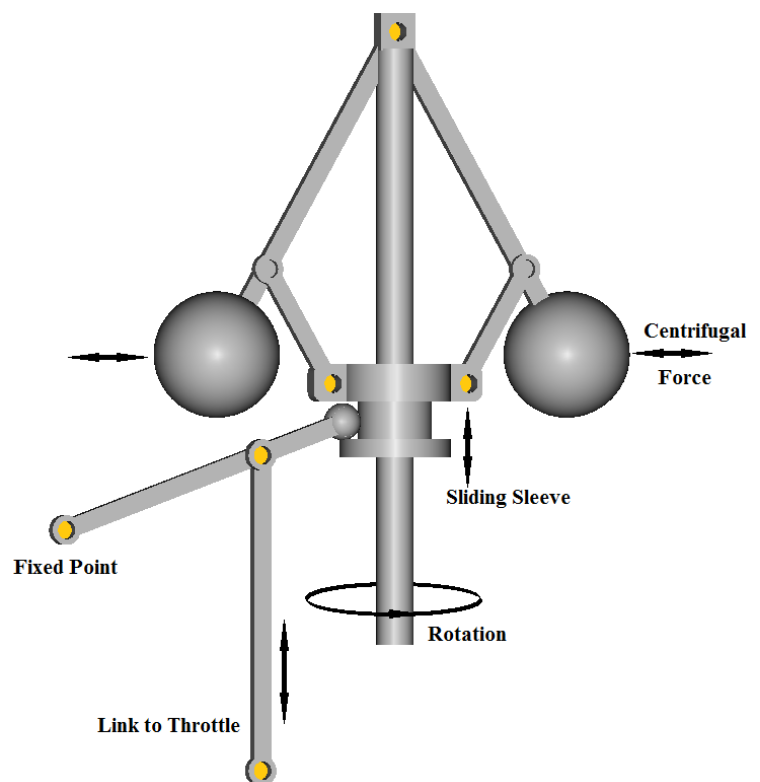
For any controlled system such as a machine there must be an input setting and ideally the output will be the same as the input setting. However due to many factors the output can be affected by disturbances that are added or subtracted to the output. In order to keep these to a minimum, the output must be sensed and processed and fed back to a comparing device which adjusts the controller so that it counteracts the disturbance.

Clearly there is a lot more to this that can only be understood by studying control systems. In modern machines the sensing, comparing and controlling are likely to be electronic digital control based around a computer. A prime example of this is the engine management system of a modern vehicle engine. We are not going to study this but look at purely mechanical systems which nevertheless contains all the same basic principles.

## MECHANICAL GOVERNORS

Internal combustion engines have long been used for stationary applications such as driving air compressors, pumps, electric generators and so on. The speed of the engine needs to be constant and so a governor is needed to adjust the controls according to the output speed. The same applies to external combustion engines such as gas turbines and steam turbines. The control action basically is that of adjusting a throttle of some kind that varies the flow of fuel, air or steam.

The fly ball or *conical governor* illustrated (designed by James Watt) is a complete control system. The sensing is done by the balls under the action of centrifugal force. This moves a sleeve and linkage to adjust the throttle. The mechanism settles down at the design speed but if the speed increases the balls fly out, the sleeve slides up and the throttle linkage acts to reduce the speed until equilibrium is restored. If the speed drops the reverse happens. See the video and others on U Tube.



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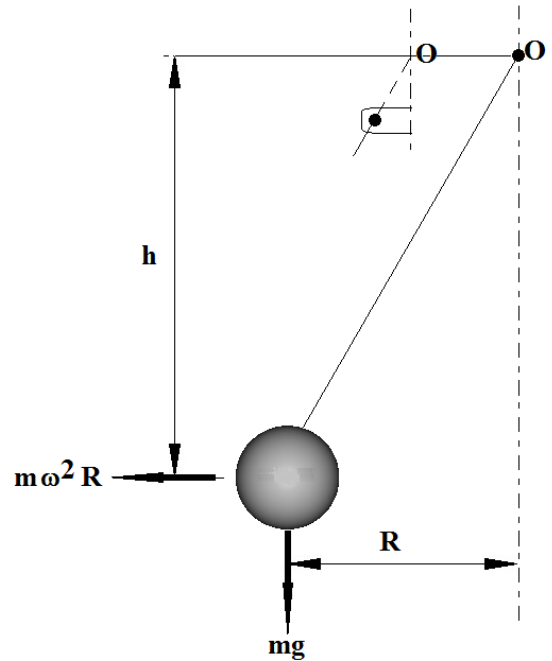
There are different configurations of this governor but all are basically the same. Consider one ball pivoted about O. When balanced the torques about O are:

Centrifugal  $m\omega^2 R h$  and this acts clockwise  
 Weight  $mgR$  and this acts anticlockwise

Equate and:

$$h = \frac{g}{\omega^2}$$

If the pivot point is not on the axis of rotation then point O is the intersection point as shown and h the distance to that point. The analysis does not take into account the weight of the sleeve or the friction force. Note the mass of the ball does not affect the height.



### WORKED EXAMPLE No.1

Calculate the value of h at a speed of 180 rev/min and the movement produced by an increase of 20 rev/min.

#### SOLUTION

At 180 rev/min

$$\omega = 2\pi N/60 = 2\pi \times 180/60 = 6\pi \text{ radian/s}$$

$$h = \frac{g}{\omega^2} = \frac{9.81}{(6\pi)^2} = 0.0276 \text{ m}$$

$$h = 27.6 \text{ mm}$$

At 200 rev/min

$$\omega = 2\pi N/60 = 2\pi \times 200/60 = 20.944 \text{ radian/s}$$

$$h = \frac{g}{\omega^2} = \frac{9.81}{(20.944)^2} = 0.02236 \text{ m}$$

$$h = 22.4 \text{ mm}$$

The movement is hence 5.2 mm

The movement of the ball needs to be changed into the movement of a sliding sleeve. There are many designs of mechanism for doing this. As with all control systems of this type, it is impossible to get exact control as there must be a change in speed to produce a change in the throttle setting. If the system is too sensitive then the phenomenon of "*hunting*" might occur where it first goes too fast, then too slow alternatively as it tries to find a settling point.

## WATT GOVERNOR

The next diagram shows the layout of one half of the Watt governor. Consider the velocity diagram for the linkage ABCD (If you haven't studied velocity diagrams you need to do so in order to follow the derivation). The various parameters can be linked by considering the conservation of energy when movement occurs.

The velocity diagram is shown for the linkage and indicates the direction and relative movement of all the points. Except for the sliding sleeve these can only be at  $90^\circ$  to the links. Let the sleeve be sliding upwards so the movement of A relative to B can only be vertical.

Velocity of A relative to B is  $ba$  (up)

Velocity of A relative to C is  $ca$

Velocity of C relative to B is  $bc$

The velocity vectors can be added to give:

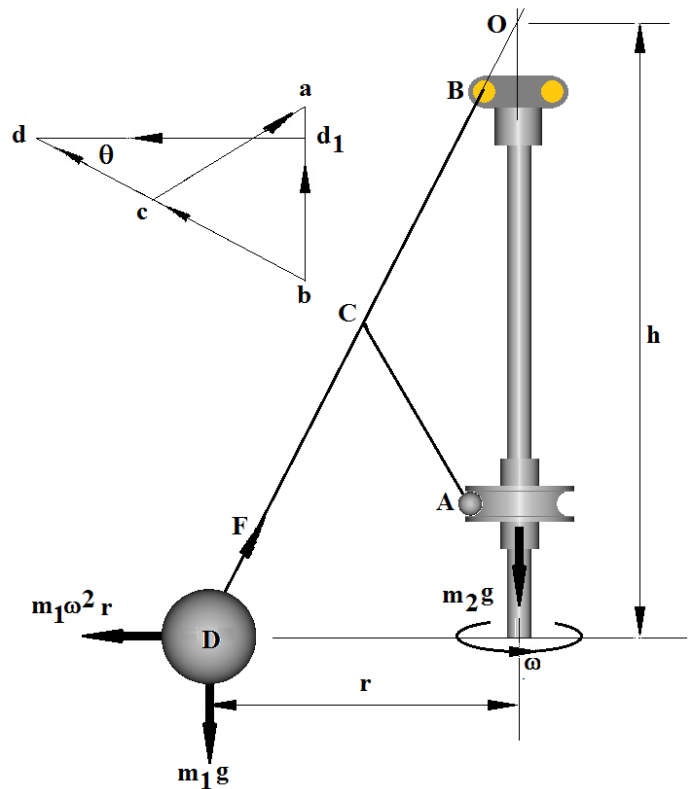
$$ba = bc + ca$$

The velocity of D is an extension of  $bc$ .

By relative proportions we can say:

$$\frac{bc}{bd} = \frac{BC}{BD}$$

Point  $dd_1$  gives the horizontal component of the velocity of D and  $bd_1$  the vertical component.



**WORK METHOD-** When moving the work done per second by the various forces must all add up to zero.

Work done by Centrifugal force is:  $m_1\omega^2 r \times dd_1$

Work done by raising  $m_1$  is:  $m_1 g \times bd_1$

Work done by raising  $m_2$  is:  $\frac{1}{2} m_2 g \times ba$

Note that half the weight of the sleeve is supported by the other half of the linkage.

Balancing the work done we get:

$$\begin{aligned} (m_1\omega^2 r) dd_1 &= (m_1 g) bd_1 + \frac{1}{2} (m_2 g) ba \\ m_1\omega^2 r &= m_1 g \frac{bd_1}{dd_1} + \frac{m_2 g}{2} \frac{ba}{dd_1} = m_1 g \frac{r}{h} + \frac{m_2 g}{2} \frac{r}{h} \left( \frac{ba}{bd_1} \right) \\ m_1\omega^2 &= \frac{m_1 g}{h} + \frac{m_2 g}{2h} \left( \frac{ba}{bd_1} \right) \end{aligned}$$

### **FRICTION**

From this one of the parameters can be calculated given the others. If there is friction between the sleeve and spindle this can be taken into account by adding or subtracting half the friction ' $f$ ' force to  $m_2 g$ .

$$(m_1\omega^2 r) dd_1 = (m_1 g) bd_1 + \frac{1}{2} (m_2 g \pm f) ba$$

The  $\pm f$  is used because friction can work in opposite directions depending on whether the sleeve is moving up or down.

## WORKED EXAMPLE No. 2

A Watt governor as illustrated previously has the following parameters:

B is 37 mm from the vertical axis, A is 50 mm from the vertical axis. AC is 160 mm, BC is 185 mm and BD is 300 mm. The rotating ball is at a radius of 155 mm and has a mass of 2.75 kg.

Calculate the speed of the governor under the following conditions.

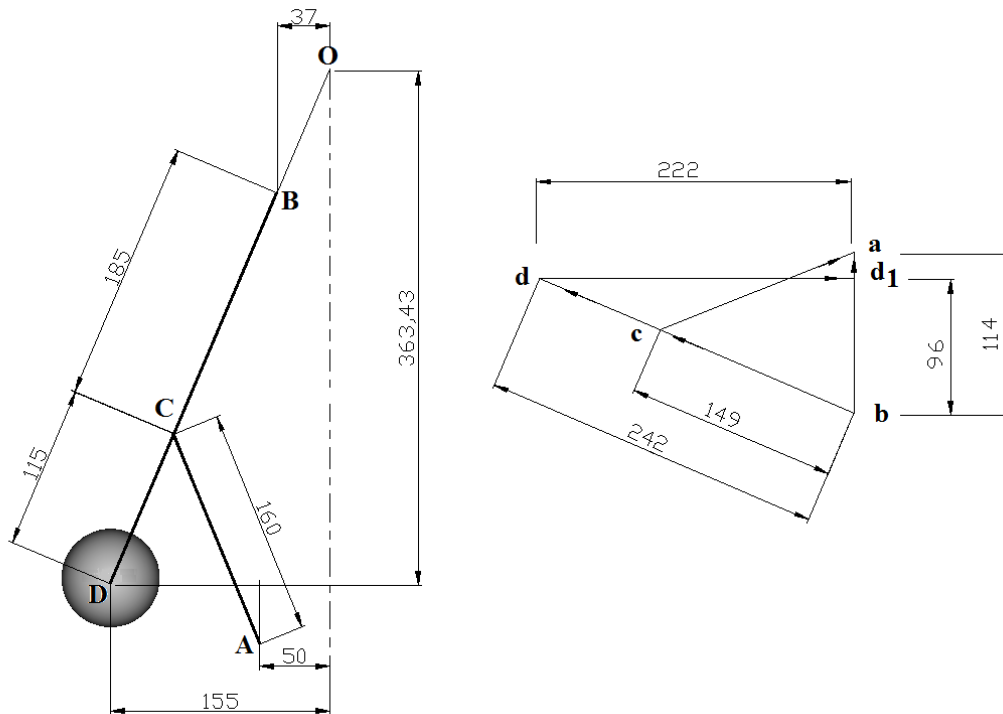
- Ignoring the mass of the sleeve
- If the sleeve has a mass of 800 g
- If the friction force is 6 N

## SOLUTION

The analytical method of solution would be quite difficult and it is easier to draw the linkage to scale and from that draw the velocity diagram. The scale drawing is shown with angles that can be deduced analytically. A tip on construction is to locate points B, D and C and draw an arc to find point A. The velocity diagram  $a b c d d_1$  can be drawn to an arbitrary scale with the lines normal to those on the space diagram.

$$\frac{bc}{bd} = \frac{BC}{BD} = \frac{185}{300} = \frac{149}{bd}$$

149 mm is the length measured from the original diagram. From this  $bd = 242$  mm and then the velocity diagram can be completed. Measuring  $bd_1$  and  $dd_1$  gives 96 and 222 mm in this case and  $ba$  is 114 mm.



- a) Put  $m_2 = 0$  and first after finding  $h = 363$  mm we can use:

$$\omega = \sqrt{\frac{g}{h}} = \sqrt{\frac{9.81}{0.363}} = 5.2 \frac{\text{rad}}{\text{s}} \text{ or } 49.6 \text{ rev/min}$$

Solving by the Work method:

$$(m_1 \omega^2 r) \times dd_1 = (m_1 g) bd_1$$

$$\omega = \sqrt{\frac{g bd_1}{r dd_1}} = \sqrt{\frac{9.81 \times 96}{0.155 \times 222}} = 5.23 \frac{\text{rad}}{\text{s}} \text{ or } 49.9 \text{ rev/min}$$

The slight difference is due to rounding off figures.

b) Taking the mass of the sleeve into account we have:

$$(m_1 \omega^2 r) \times dd_1 = (m_1 g) bd_1 + \frac{1}{2} (m_2 g) ba$$

$$2.75 \omega^2 \times 0.155 \times 222 = (2.75 \times 9.81 \times 96) + (1/2)(0.8 \times 9.81 \times 114)$$

$$94.63 \omega^2 = 2589.8 + 447.3$$

$$\omega = 5.66 \text{ rad/s or } 54 \text{ rev/min}$$

c) With friction:

$$(m_1 \omega^2 r) \times dd_1 = (m_1 g) bd_1 + \frac{1}{2} (m_2 g + f) ba$$

$$2.75 \omega^2 \times 0.155 \times 222 = (2.75 \times 9.81 \times 96) + \{(1/2)(0.8 \times 9.81 \pm 6)\} \times 114$$

$$94.63 \omega^2 = 2589.8 + 789$$

$$\omega = 5.98 \text{ rad/s or } 57 \text{ rev/min}$$

or

$$94.63 \omega^2 = 2589.8 + 105.3$$

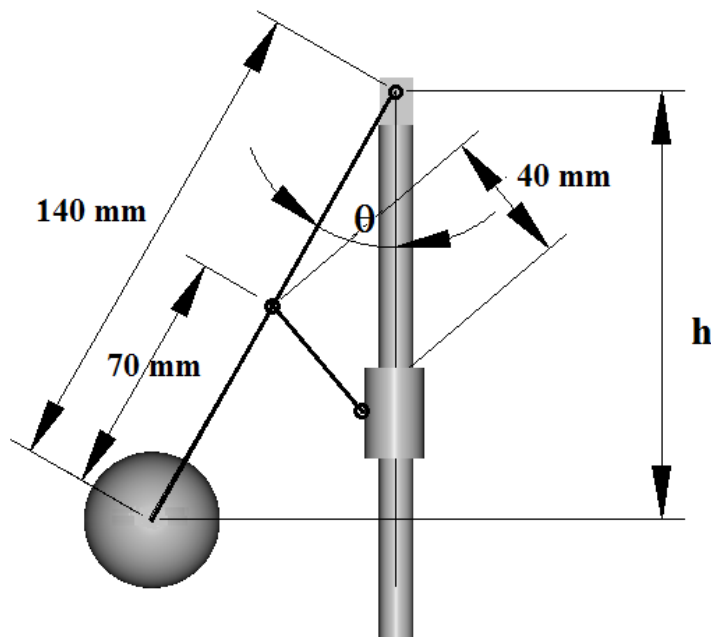
$$\omega = 5.35 \text{ rad/s or } 51 \text{ rev/min}$$

Remember you might get different values for  $dd_1$ ,  $bd_1$  and  $ba$  but you should get the same ratios. It is fine to leave these dimensions in mm as done here but  $r$  must be in metres as it is the actual radius.

### SELF ASSESSMENT EXERCISE No.1

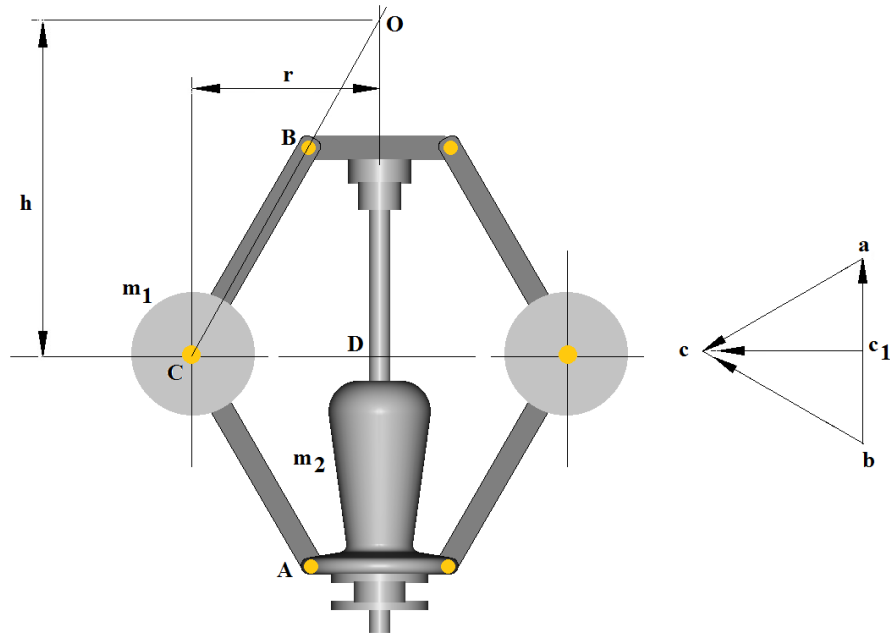
A Watt governor as illustrated must control the speed between 90 and 85 rev/min. Determine the movement of the sleeve. Assume no friction and ignore the weight of the sleeve.

(Answer 17 mm)



## PORTER GOVERNOR

It should be clear that the speed at which a governor operates depends on the downwards forces acting on it. If we increased the mass of the sleeve the governor would have to run faster to maintain the same position of the sleeve. An example is the *Porter Governor* (also known as the loaded Watt Governor) and this usually has equal arms and B and A are the same distance from the vertical axis. The velocity diagram is as shown.  $c_1c$  is the horizontal velocity of  $m_1$  and  $bc_1$  is the vertical component.



Balancing the rate of work we have:  $(m_1\omega^2 r) c_1c = (m_1g) bc_1 + \frac{1}{2} (m_2g) ba$

For the *symmetrical layout* shown it follows that:

$$\frac{c_1c}{bc_1} = \frac{OD}{CD} = \frac{h}{r} \text{ and } \frac{ba}{bc_1} = 2$$

Substituting and rearranging gives:

$$m_1\omega^2 r \frac{c_1c}{bc_1} = m_1g \frac{bc_1}{bc_1} + \frac{m_2g}{2} \frac{ba}{bc_1}$$

$$m_1\omega^2 r \frac{h}{r} = m_1g + \frac{m_2g}{2} \times 2 = m_1g + m_2g$$

$$\omega^2 = \left(1 + \frac{m_2}{m_1}\right) \frac{g}{h}$$

### **FRICTION**

If friction is involved then:  $(m_1\omega^2 r) c_1c = (m_1g) bc_1 + \frac{1}{2} (m_2g \pm f) ba$

Substituting and rearranging gives:

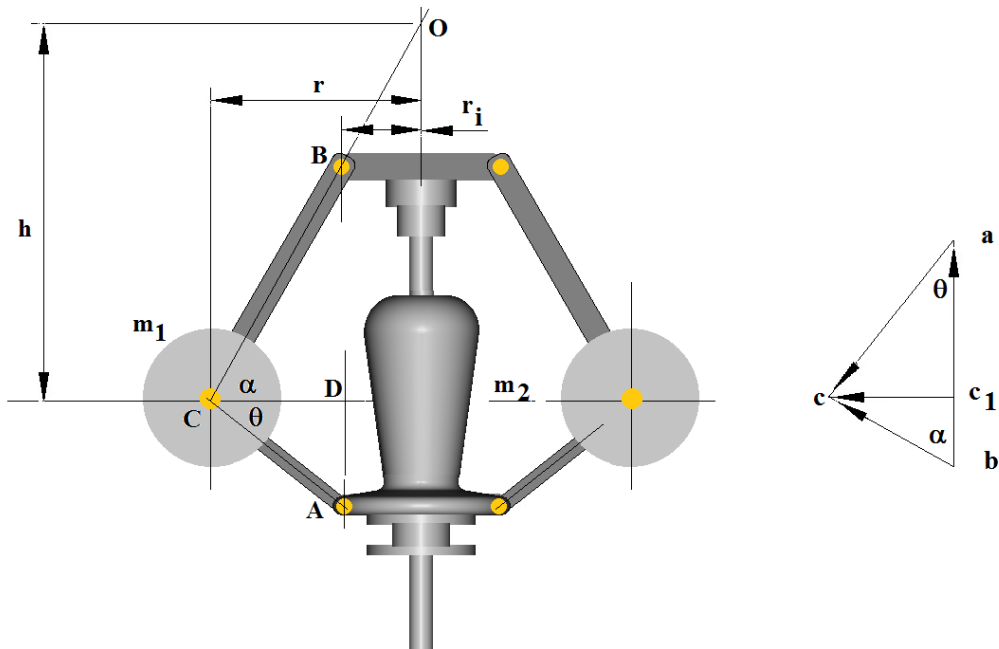
$$m_1\omega^2 r \frac{c_1c}{bc_1} = m_1g \frac{bc_1}{bc_1} + \frac{m_2g \pm f}{2} \frac{ba}{bc_1}$$

$$m_1\omega^2 r \frac{h}{r} = m_1g + \frac{m_2g \pm f}{2} \times 2$$

$$\omega^2 h = g + \frac{m_2g \pm f}{m_1}$$

$$\omega^2 = \frac{g}{h} + \frac{m_2g \pm f}{hm_1} = \frac{g}{h} + \frac{m_2g}{hm_1} \pm \frac{f}{hm_1} = \frac{g}{h} \left(1 + \frac{m_2}{m_1} \pm \frac{f}{gm_1}\right)$$

## UNEQUAL LINKS



If the links are not the same length then the ratio  $ba/bc_1$  must be deduced. With points A and B at the same radius  $r_i$  a bit of geometry show:

$$\tan\theta = \frac{AD}{CD} = \frac{c_1c}{ac_1} \quad ac_1 = \frac{CD}{AD} c_1c$$

$$\tan\alpha = \frac{BD}{CD} = \frac{c_1c}{bc_1} \quad bc_1 = \frac{CD}{BD} c_1c$$

$$q = \frac{ba}{bc_1} = \frac{bc_1 + ac_1}{bc_1} = 1 + \frac{ac_1}{bc_1} = 1 + \frac{\frac{CD}{AD} c_1c}{\frac{CD}{BD} c_1c} = 1 + \frac{BD}{AD}$$

$$q = 1 + \frac{BD}{AD} = 1 + \frac{\sqrt{(BC)^2 - (r - r_i)^2}}{\sqrt{(AC)^2 - (r - r_i)^2}}$$

For equal links  $AC = BC$  so  $q = 2$

Redevelop the equations for speed

$$m_1\omega^2 r \frac{c_1c}{bc_1} = m_1g + \frac{m_2g \pm f}{2} \frac{ba}{bc_1}$$

$$\tan(\alpha) = \frac{h}{r} = \frac{c_1c}{bc_1}$$

$$m_1\omega^2 h = m_1g + \frac{m_2g \pm f}{2} \times q$$

$$\omega^2 h = g + \frac{m_2g \pm f}{2m_1} \times q$$

$$\omega^2 = \frac{g}{h} + \frac{m_2g \pm f}{2hm_1} \times q$$



### WORKED EXAMPLE No. 3

A Porter governor similar to the one illustrated previously has the linkage pivoted on the vertical axis. All links are 250 mm long. The rotating masses are 2.75 kg. The sliding mass is 12.75 kg. The radius to the centre of the rotating masses is 150 mm when they just start to rise and 200 mm when running at the maximum speed.

Calculate the operating range of speed.

### SOLUTION

It is easy to construct the diagram below and deduce that  $h = 200$  mm and 150 mm

When the mass starts to lift:

$$\omega^2 = \left(1 + \frac{m_2}{m_1}\right) \frac{g}{h}$$

$$\omega^2 = \left(1 + \frac{12.75}{2.75}\right) \frac{9.81}{0.2}$$

$$\omega = 16.63 \text{ rad/s or } 158.8 \text{ rev/min}$$

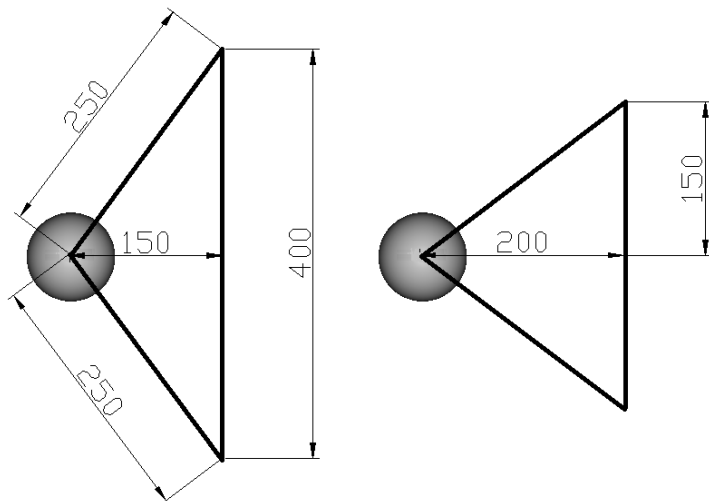
At maximum speed:

$$\omega^2 = \left(1 + \frac{m_2}{m_1}\right) \frac{g}{h}$$

$$\omega^2 = \left(1 + \frac{12.75}{2.75}\right) \frac{9.81}{0.15}$$

$$\omega = 19.2 \text{ rad/s or } 183.3 \text{ rev/min}$$

The speed range is 24.5 rev/min



## CONTROLLING FORCES

It should be clear that friction in the sleeve produces two different speeds for the same position of the sleeve and this produces a dead band in the action of the governor. The speed would have to drop or increase significantly before movement reverses.

The *sensitivity* of the governor control depends on the relative movement of the balls and the link attached to the throttle (or other control device). The speed range of a governor is limited by stops on the spindle. The definition is:

$$\text{sensitivity} = \frac{\text{mean speed}}{\text{speed range}}$$

A governor with excessive sensitivity will produce a large movement of the sleeve when a small change in load occurs on the engine (or other machine) and will over compensate causing the system to hunt. A governor with a zero range of speed or infinite sensitivity is called **ISOCHRONOUS**. This is impossible to achieve because a change in the position of the sleeve is required in order to compensate for a change in load.

## CONTROLLING FORCE DIAGRAM

This is produced by plotting the centrifugal force and controlling force against speed.

The centrifugal force is

$$F = m_1 \omega^2 r$$

For the Porter governor with equal links we had the equation:

$$\omega^2 = \left(1 + \frac{m_2}{m_1}\right) \frac{g}{h}$$

Substitute for  $\omega^2$  and:

$$m_1 \omega^2 r = m_1 r \left(1 + \frac{m_2}{m_1}\right) \frac{g}{h}$$
$$m_1 \omega^2 r = m_1 g \left(1 + \frac{m_2}{m_1}\right) \frac{r}{h} = g(m_1 + m_2) \frac{r}{h}$$

The controlling force is:

$$F_2 = g(m_1 + m_2) \frac{r}{h}$$

If friction  $f$  is taken into account then the controlling force is:

$$F_2 = g \left(m_1 + m_2 \mp \frac{f}{g}\right) \frac{r}{h}$$

Note that the controlling force depends on the ratio  $r/h$  and this is defined by the geometry of the governor.

### WORKED EXAMPLE No. 4

Draw the controlling force diagram for the governor in worked example No.3 for the speed range 140 to 190 rev/min. Assume there is no friction in the sleeve.

If the friction force in the sleeve is 13 N plot the upper and lower range of the controlling force.

### SOLUTION

For this example  $r = 150$  and the links are 250 mm long.

For the centrifugal force calculate two points for each speed at a given radius as the graphs are straight lines.

$$F = m_1 \omega^2 r = m_1 \left( \frac{2\pi N}{60} \right)^2 r = 2.75 N^2 (0.010966) r = 0.0302 N^2 r$$

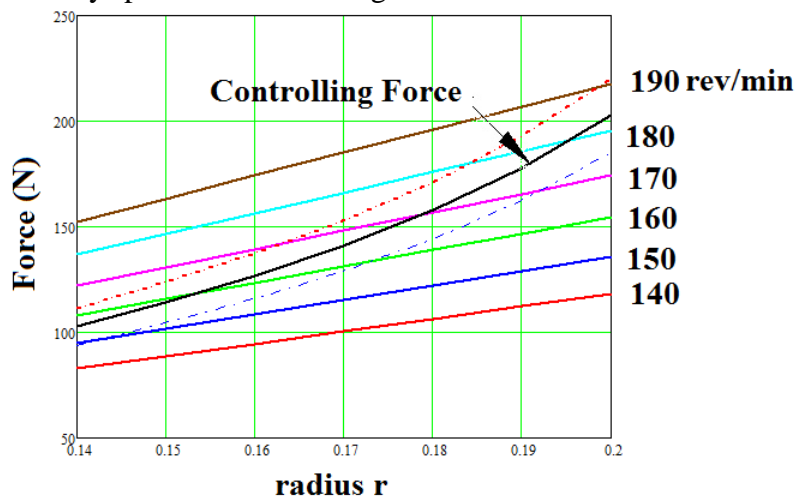
Controlling Force

For this example  $r = 150$  and the links are 250 mm long.

From the right angle triangle  $h = \sqrt{(250^2 - r^2)}$  The Controlling Force is:

$$F_2 = g \left( m_1 + m_2 \mp \frac{f}{g} \right) \frac{r}{h} = 9.81 \left( 2.75 + 12.75 \mp \frac{13}{9.81} \right) \frac{150}{h}$$

The data to be plotted is evaluated and shown below. The plot shows the upper and lower limits of the controlling force at any speed within the range.



Plot Data

N rev/min		$F@ r = 0.15 \text{ m} \quad F = 0.0302 N^2 r$		$F@ r = 0.2 \text{ m}$
190		163.3 N		218 N
180		146.6		195.7
170		130.7		174.6
160		115.8		154.6
150		101.8		135.9
140		88.7 N		118.4 N
Radius mm	$h = \sqrt{(250^2 - r^2)}$	Controlling Force $g(m_1 + m_2) \frac{r}{h}$	$g \left( m_1 + m_2 + \frac{f}{g} \right) \frac{r}{h}$	$g \left( m_1 + m_2 - \frac{f}{g} \right) \frac{r}{h}$
140	207	102.8 N	111.6 N	94 N
150	200	114	123.8	104.3
160	192	126.7	137.5	115.8
170	183	141	153	129
180	173	157.8	171.2	144.3
190	162	177.8	193	162.6
200	150	202.7	220.1	185.4

## SELF ASSESSMENT EXERCISE No.2

Q1

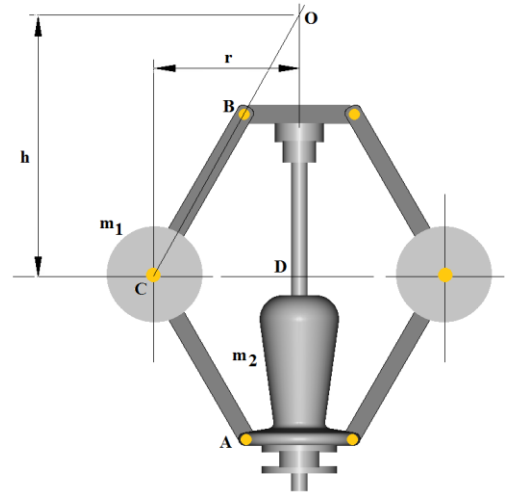
A Porter Governor as shown has rotating masses of 2 kg. All the arms are 120 mm long. The sliding mass is 3 kg. The friction force is 4 N. Calculate the speed when the masses rotate on a radius of 60 mm. The pivot points are at 10 mm radius

Answer 135 to 125 rev/min

Q2

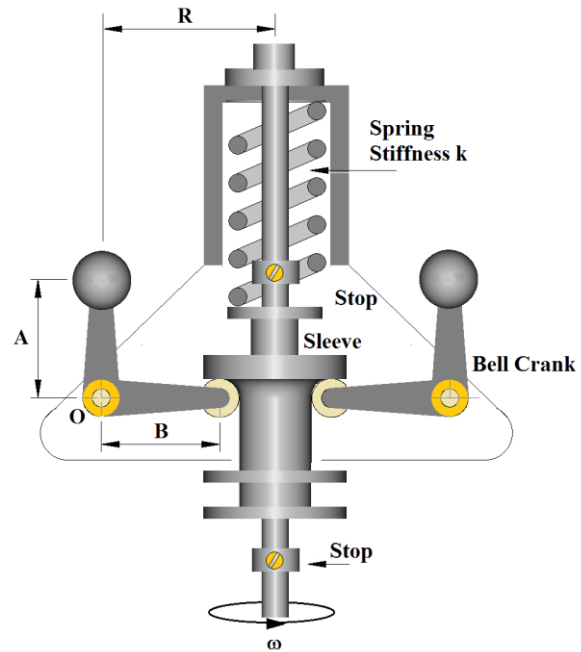
Repeat Q1 but for the case where AC is 60 mm

Answer 178 to 161 rev/min



## HARTNELL GOVERNOR

This is a governor with a spring so we will examine this to see how the spring affects the controlling force. The balls of mass 'm' are carried on a bell crank with arms of length A and B as shown. The sleeve is held down against the crank by a spring that can be adjusted. As the balls fly outwards under centrifugal force the sleeve is raised and compresses the spring. As the sleeve moves the radius of the balls from the axis of rotation 'R' will change and so the weight of the ball comes into play either adding to the spring force or reducing the spring force depending which way they move.



### **GEOMETRY**

When the crank is at the mid point it is horizontal and vertical.  $x_0$  is the position of the sleeve and  $R_0$  is the radius of the balls. Rotating the crank  $\theta$  the sleeve moves to position  $x$  and the balls to radius  $R$ .

$$\sin(\theta) = \frac{R - R_0}{A} = \frac{x - x_0}{B}$$

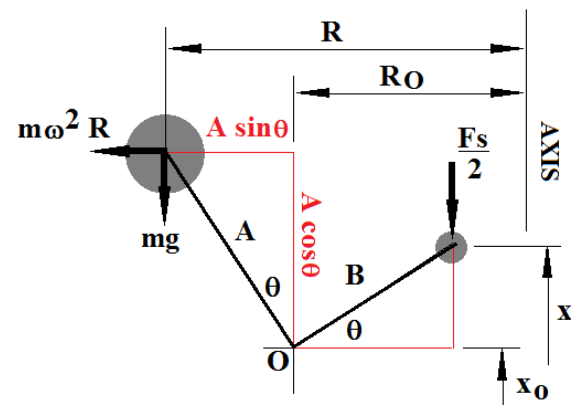
A change in angle will produce changes in  $x$  and  $R$

$$\sin(\theta + \Delta\theta) = \frac{R - R_0 + \Delta R}{A} = \frac{x - x_0 + \Delta x}{B}$$

$$\frac{R - R_0}{A} + \frac{\Delta R}{A} = \frac{x - x_0}{B} + \frac{\Delta x}{B}$$

$$\sin(\theta) + \frac{\Delta R}{A} = \sin(\theta) + \frac{\Delta x}{B}$$

$$\frac{\Delta R}{A} = \frac{\Delta x}{B} \quad \Delta x = \frac{B}{A} \Delta R$$



### **SPRING FORCE**

$F_s$  = spring force. The change in spring force is  $\Delta F_s$ . The spring rate is hence:

$$k = \frac{\Delta F_s}{\Delta x} = \frac{\Delta F_s}{\Delta R} \frac{A}{B} \quad \Delta F_s = k \Delta R \frac{B}{A}$$

### **INITIAL COMPRESSION**

$x_i$  is the initial compression of the spring on the bottom stop

$x_T$  is the total spring compression  $x_T = x_i + x$

The spring force is hence  $F_s = k(x_i + x)$

### **MOMENT BALANCE**

Moments about O gives:

$$m\omega^2 R A \cos(\theta) = \frac{F_s}{2} B \cos(\theta) - mg A \sin(\theta)$$

In practice it is found that the term  $mgA \sin(\theta)$  is small and may be left out.

$$m\omega^2 R = \frac{F_s}{2} \frac{B}{A} \quad \omega^2 = \frac{F_s}{2mR} \frac{B}{A}$$

$m\omega^2 R$  is the controlled force and  $\frac{F_s}{2} \frac{B}{A}$  is the control force.

We can develop the control force expression as follows:

$$\Delta x = \frac{B}{A} \Delta R$$

$$\frac{F_S B}{2 A} = \frac{k x_T}{2} x \frac{B}{A} = \frac{k(x_i + \Delta x)}{2} x \frac{B}{A} = \frac{k\left(x_i + \frac{B \Delta R}{A}\right)}{2} x \frac{B}{A} = \frac{k B}{2 A} \left[ x_i + \Delta R \frac{B}{A} \right]$$

Note  $\Delta R$  is any change in radius corresponding to a change in the sleeve position  $\Delta x$ .

### ***ISOCHRONOUS CONDITION***

When the governor is *isochronous* the change in the controlled force is the same as the change in control force for any change in radius.

Change in Controlled Force

Centrifugal Force =  $m\omega^2 R$  Differentiate:

$$\frac{dF}{dR} = m\omega^2 = \text{constant} \quad \text{hence} \quad \frac{\Delta F}{\Delta R} = m\omega^2$$

Consider the control force.

$$F = \frac{F_S B}{2 A}$$

Substitute:  $x = (R - R_0)(B/A)$  and  $F_S = k x_T = k(x_i + x)$

$$F = \frac{k\{x_i + x\}B}{2A} = \frac{kB}{2A} \left\{ x_i + \frac{B}{A} (R - R_0) \right\} = \frac{kB}{2A} \left\{ x_i + \frac{B}{A} R - \frac{B}{A} R_0 \right\}$$

Differentiate:

$$\frac{dF}{dR} = \frac{k B^2}{2 A^2}$$

This is constant so:

$$\frac{\Delta F}{\Delta R} = \frac{k B^2}{2 A^2}$$

For isochronism:

$$m\omega^2 = \frac{k B^2}{2 A^2} \quad \text{or} \quad k = \frac{2 A^2 m \omega^2}{B^2}$$

### WORKED EXAMPLE No.5

The diagram shows a Hartnell Governor with no friction. The stops are  $\pm 30$  mm from the position shown. The speed is to be set to obtain 250 rev/min and 265 rev/min at the lower and upper stops respectively. Calculate the spring rate based on the total movement. Show that the mid position does not give the mean speed. Calculate the initial spring compression needed.

### SOLUTION

From the diagram  $A=150$  mm  $B=130$  mm  $R_0=180$  mm

*At the higher speed*

$$\omega = \frac{265}{60} \times 2\pi = 27.75 \frac{\text{rad}}{\text{s}} \quad \theta = \sin^{-1}(30/130) = 13.34^\circ$$

$$R = 180 + 150 \sin(13.34^\circ) = 214.6 \text{ mm}$$

$$\frac{F_S}{2} \frac{B}{A} = m\omega^2 R + mg \tan(\theta)$$

$$\frac{F_S}{2} \frac{0.13}{0.15} = 5.5 \times 27.75^2 \times 0.2146 + 5.5 \times 9.81 \tan(13.34^\circ)$$

$$0.433F_S = 908.9 + 12.79 \quad F_S = 2128 \text{ N}$$

*At the lower speed*

$$\omega = \frac{250}{60} \times 2\pi = 26.18 \frac{\text{rad}}{\text{s}} \quad \theta = \sin^{-1}(-30/130) = -13.34^\circ$$

$$R = 180 + 150 \sin(-13.34) = 145.4 \text{ mm}$$

$$\frac{F_S}{2} \frac{B}{A} = m\omega^2 R + mg \tan(\theta)$$

$$\frac{F_S}{2} \frac{0.13}{0.15} = 5.5 \times 26.18^2 \times 0.1454 + 5.5 \times 9.81 \tan(-13.34^\circ)$$

$$0.433F_S = 548 - 12.79 \quad F_S = 1236 \text{ N}$$

$$k = \frac{\Delta F_S}{\Delta x} = \frac{2128 - 1236}{60} = 14.86 \text{ N/mm}$$

The initial compression is hence  $1236/14.86 = 83.18$  mm

At the mid position  $B$  is horizontal  $\theta = 0$   $R = 180$  mm

$$F_S = \text{mean force} = (1236 + 2128)/2 = 1682 \text{ N}$$

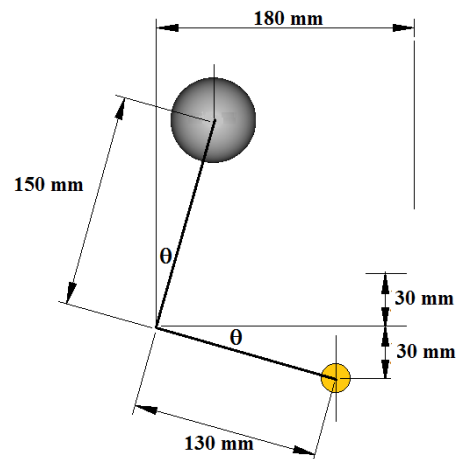
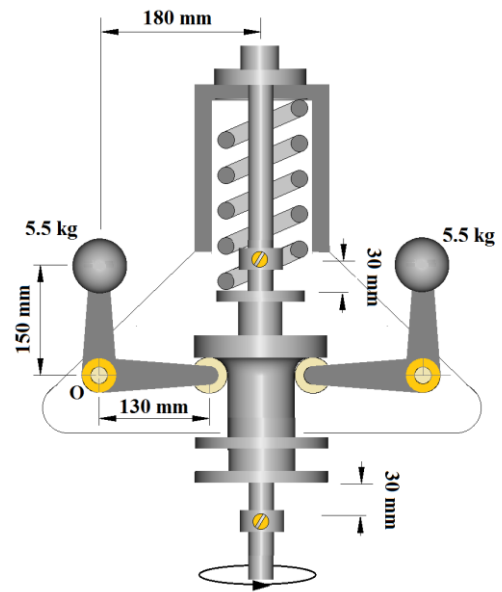
$$\frac{F_S}{2} \frac{B}{A} = m\omega^2 R \quad \frac{1682}{2} \times \frac{0.13}{0.15} = 5.5 \times \omega^2 \times 0.18$$

$$\omega = 27.13 \text{ rad/s} \quad N = 259.1 \text{ rev/in}$$

Based on the upper and lower limits the mean speed is

$$N = \frac{265 + 250}{2} = 257.5 \text{ rev/min}$$

The mean speed is not obtained at the mid position.

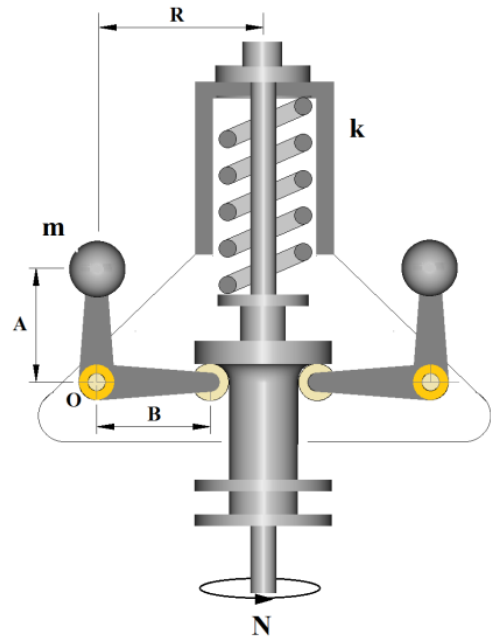


### WORKED EXAMPLE No. 6

The diagram shows a Hartnell Governor for which the data is:  $A = 150 \text{ mm}$   $B = 100 \text{ mm}$   $R = 130 \text{ mm}$   
 $m = 5.5 \text{ kg}$   $k = 15 \text{ N/mm}$

The sleeve is shown at the mid position and can move  $\pm 25 \text{ mm}$  between stops.

Calculate the isochronous speed and draw the control force diagram to confirm it. Draw the diagram using speed range of 180 to 280 rev/min and initial compressions of 35, 60 and 80 mm.



### SOLUTION

Isochronous Speed - note that  $k$  must be in N/m

$$\omega = \sqrt{\frac{k B^2}{2 m A^2}} = \sqrt{\frac{15000}{2} \times \frac{100^2}{5.5 \times 150^2}} = 24.62 \text{ rad/s}$$

$$N = \omega \times \frac{60}{2\pi} = 235 \text{ rev/min}$$

Control force diagram.

$$\text{At the upper stop } R = 130 + (A/B) \times 25 \quad R_1 = 130 + 1.5 \times 25 = 167.5 \text{ mm}$$

$$\text{At the lower stop } R = 130 - (A/B) \times 25 \quad R_1 = 130 - 1.5 \times 25 = 92.5 \text{ mm}$$

$$\text{The centrifugal force is } m\omega^2 R = 5.5 (2\pi N/60)^2 \times R = 0.0603 N^2 R$$

Work out the control force for a few values of initial compression.

The initial compression is at  $R = 92.5 \text{ mm}$   $A = 150 \text{ mm}$   $B = 100 \text{ mm}$   $k = 15 \text{ N/mm}$

The control force is

$$F = \frac{F_S}{2} \frac{B}{A}$$

$x$  is the sleeve movement from the bottom stop

$x_i$  is the initial compression of the spring on the bottom stop

$x_T$  is the total spring compression  $x_T = x_i + x$

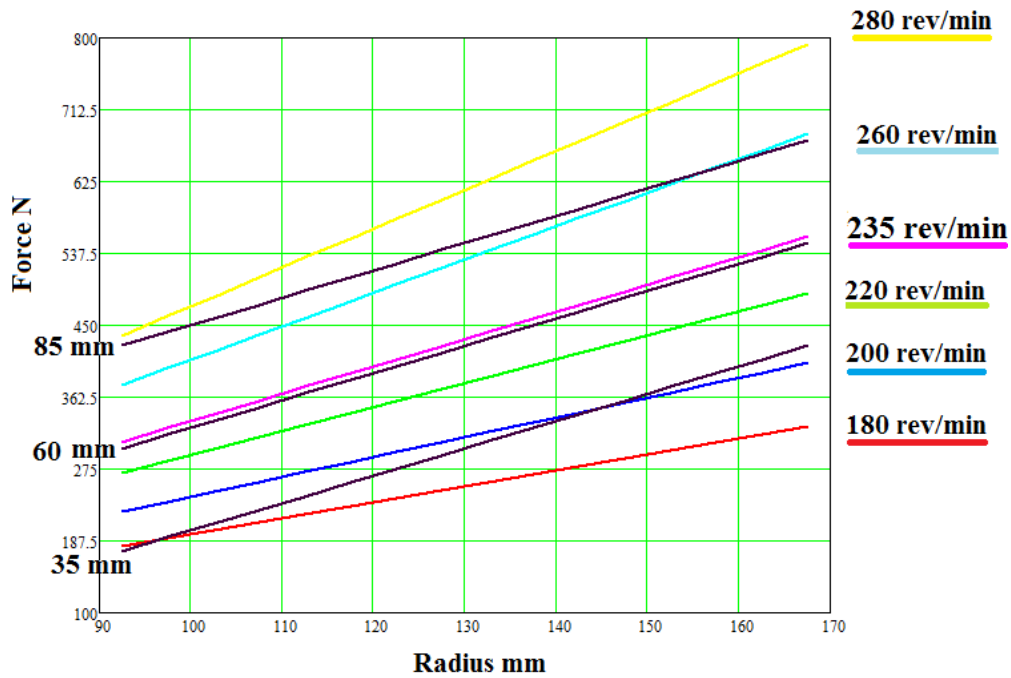
$$x = \Delta R (B/A) = \Delta R (2/3)$$

$$F_S \text{ is the actual spring force } F_S = k x_T = k (x_i + x)$$

The control force is:

$$F = \frac{k x_i}{2} \frac{B}{A} + k \frac{\Delta R}{2} \frac{B^2}{A^2} = \frac{15 x_i}{2} \frac{100}{150} + 15 \frac{\Delta R}{2} \frac{100^2}{150^2} = 5 x_i + \frac{30 \Delta R}{9}$$





The plots show that 235 rev/min produces a parallel plot so it is isochronous.

#### PLOT DATA

Speed rev/min	$m\omega^2R = 0.0603N^2R$	
	R = 0.0925 m	R = 0.1675 m
180	180.8 N	327.3 N
200	223.2 N	404.1 N
220	246 N	445.5 N
235	308 N	558 N
260	321 N	582 N
280	377 N	683 N

$x_i$	Controlling Force $F = 5x_i + \frac{30\Delta R}{9}$	
	R = 92.5 mm $\Delta R = 0$	R = 167.5 mm $\Delta R = 75$ mm
35 mm	$x_T = 35$ mm F = 175 N	$x_T = 50$ mm F = 425 N
60 mm	$x_T = 60$ mm F = 300 N	$x_T = 125$ mm F = 550 N
85 mm	$x_T = 80$ mm F = 425 N	$x_T = 200$ mm F = 675 N

### SELF ASSESSMENT EXERCISE No.3

1. A Hartnell Governor as shown has the following data.

$A = 200$  mm,  $B = 100$  mm and  $R = 115$  mm.

The balls have a mass of 4 kg.

The stops are  $\pm 20$  mm from the position shown.

i. Calculate the spring rate to give an isochronous speed of 300 rev/in.

**Answer 31.6 N/mm**

ii. What is the initial compression required to make this speed occur at the mid position?

**Answer 8.7 mm**

2. The same governor as Q1 has a spring rate of 40 N/mm and the initial compression is 10 mm. Calculate the speeds at the upper and lower stops.

**Answer 158.1 rev/min and 334.8 rev/min**

3. Repeat Q2 with an initial compression of 100 mm

**Answer 479.4 rev/min and 563.3 rev/min**

