SOLID MECHANICS

DYNAMICS

FRICTION BRAKES

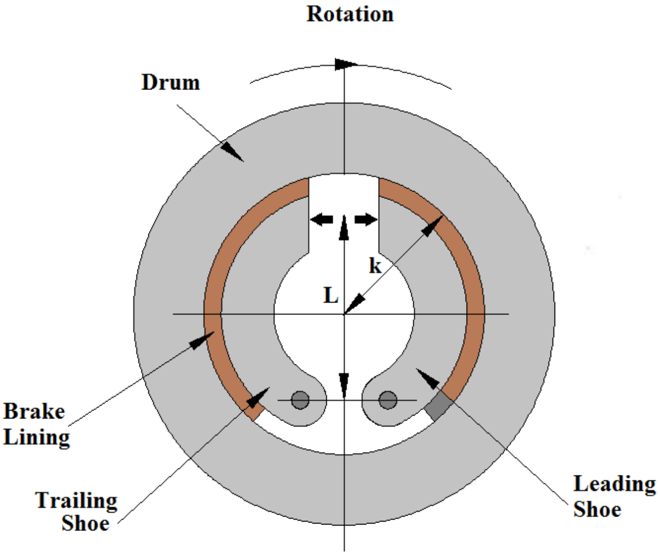
When you have completed this tutorial you should be able to:

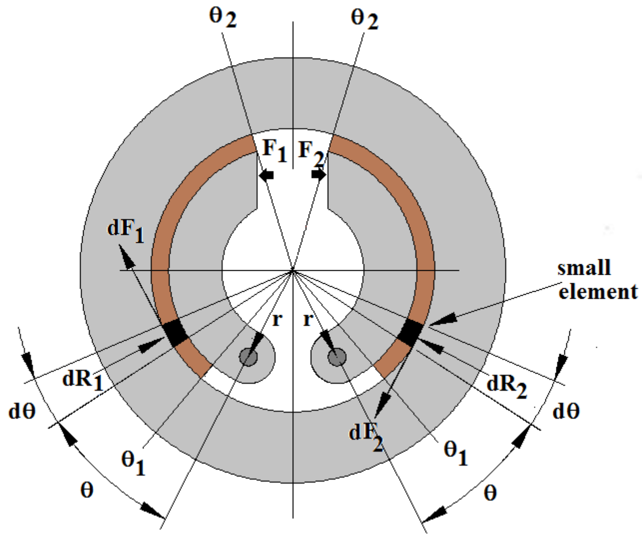
- Derive and apply the formula for friction on a brake drum
- Derive and apply the formula for friction on a brake shoe

Contents

1 *Shoe Brakes*

The diagram shows the basic design of the shoe brake. A force is applied mechanically at the top of the shoes making them expand and press against the inside of the drum.





Consider a small element of the lining shaded in black. Suffix 1 indicates the trailing edge and suffix 2 the leading edge. The normal reaction force between the friction lining and the drum is dR

The friction force between them is $dF = \mu\, dR$

r is the radius to the pivot

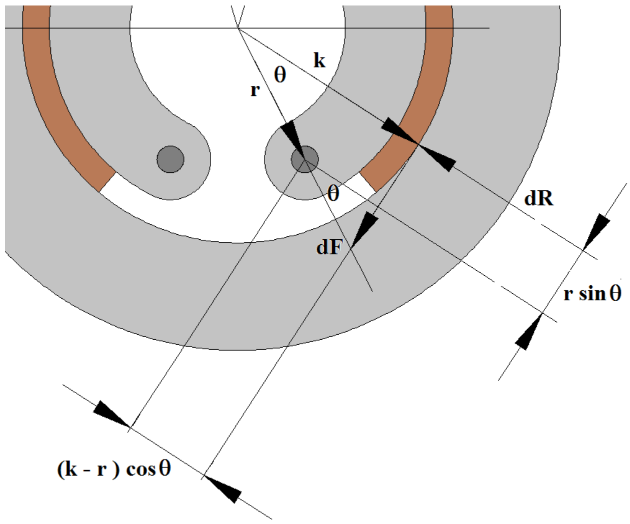
L is the radius from the pivot normal to and from the line of the applied force.

k = radius of the friction lining and drum

w is the width of the lining

μ = the coefficient of friction between the lining and the drum

p is the mean effective pressure on the brake lining surface



The radial reaction forces on the small elements are:

$$dR = p\, w\, k\, d\theta$$

The friction forces are:

$$dF = \mu\, p\, w\, k\, d\theta$$

This produces a torque at the pin of:

$$dT = dF\,(k - r\cos\theta) = \mu\, p_1 w\, k\,(k - r\cos\theta)\, d\theta$$

The total Torque for the brake lining on the pin is:

$$T_B = \mu\, p\, w\, k \int_{\theta_1}^{\theta_2} (k - r\cos\theta)\, d\theta$$

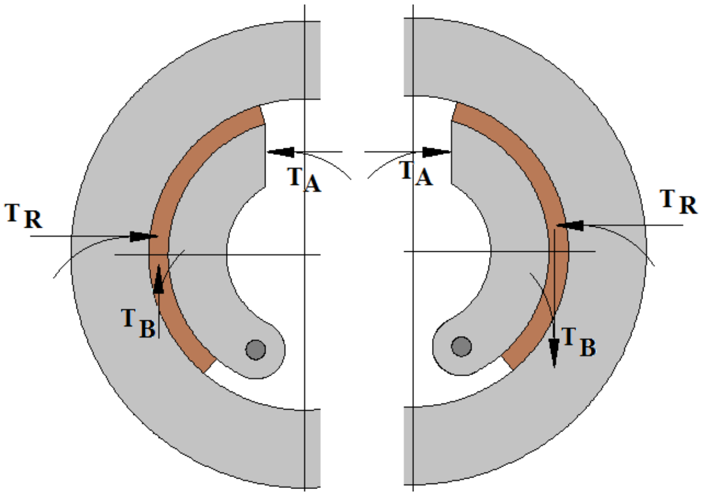
The torque radial force also produces a torque on the pivot pin of $dR\, r\sin\theta = p\, w\, k\, r\sin\theta\, d\theta$

The total torque on the pivot pin for one lining is:

$$T_R = p\, w\, k \int_{\theta_1}^{\theta_2} (r\sin\theta)\, d\theta$$

The applied torque at the actuator is $T_A = FL$

Next sum the toques at the pivot pin taking account of the direction.



For the leading shoe $T_A + T_B - T_R = 0$

$$F_2L + \mu p_2 wk\int_{\theta_1}^{\theta_2}(k - r\cos\theta)d\theta - p_2 wk\int_{\theta_1}^{\theta_2}(r\sin\theta)d\theta = 0$$

For the trailing shoe $T_A - T_B - T_R = 0$

$$F_1L - \mu p_1 wk\int_{\theta_1}^{\theta_2}(k - r\cos\theta)d\theta - p_1 wk\int_{\theta_1}^{\theta_2}(r\sin\theta)d\theta = 0$$

The above equations are evaluated to get the required results. It is assumed that the lining material is elastic in compression. This actually produces a pressure distribution that varies sinusoidally over the circumference of the lining but in the following we will simplify the problem by assuming it is constant.

The practical limit to the value of μ is reached on the leading shoe when the moment of the frictional drag together with that of the actuating force about the pivot becomes greater than due to the radial thrust. The shoe then locks to the drum and this is called ‘sprag.

The torque acting on the drum is entirely due to the friction force since the radial forces are all normal.

$$dT_D = k\,dF = \mu\,p\,w\,k^2\,d\theta$$

$$T_D = \mu p w k^2\int_{\theta_1}^{\theta_2}d\theta = \mu p w k^2(\theta_2 - \theta_1)$$

Hence for both shoes:

$$T_D = \mu p_1 wk^2(\theta_2 - \theta_1) + \mu p_2 wk^2(\theta_2 - \theta_1)$$

$$T_D = \mu wk^2(\theta_2 - \theta_1)(p_1 + p_2)$$

WORKED EXAMPLE No. 1

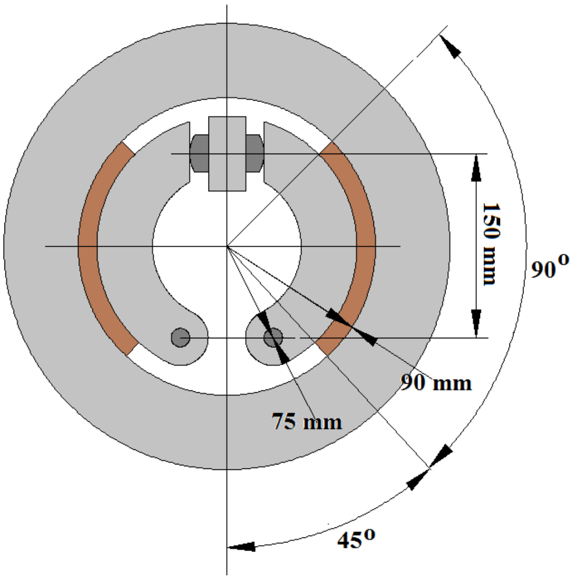
The diagram shows a shoe brake system and its dimensions (not drawn to scale). Both shoes may be assumed to have the same parameters including an equal actuator force of 250 N. The linings both occupy 90^o of the circumference and are 45 mm wide with a coefficient of friction is 0.45.

Determine:

i) the pressure on each lining

ii) the torque on the drum

SOLUTION



$F_1 = F_2 = 250$ N L = 150 mm w = 45 mm
k = 90 mm r = 75 mm $\theta_1 = 45^o = \pi/4$ radian
$\theta_2 = 135^o = 3\pi/4$ $\mu = 0.45$

Leading shoe:

$$F_2L + p_2\mu wk\int_{\theta_1}^{\theta_2}(k - r\cos\theta)d\theta - p_2wk\int_{\theta_1}^{\theta_2}(r\sin\theta)d\theta = 0$$

$$250 \times 150 + p_2 x\, 0.45 \times 45 \times 90\int_{\theta_1}^{\theta_2}(90 - 75\cos\theta)d\theta - p_2 \times 45 \times 90\int_{\theta_1}^{\theta_2}(75\sin\theta)d\theta = 0$$

$$37\,500 + 1822.5\int_{\theta_1}^{\theta_2}(90 - 75\cos\theta)d\theta - 4\,050p_2\int_{\theta_1}^{\theta_2}(75\sin\theta)d\theta = 0$$

$$37\,500 + 1\,822.5p_2[90\theta - 75\sin\theta]_{\pi/4}^{3\pi/4} + 4\,050p_2[75\cos\theta]_{\pi/4}^{3\pi/4} = 0$$

$$37\,500 + 1\,822.5\left[\left(90 \times \frac{3\pi}{4} - 75\sin\frac{3\pi}{4}\right) - \left(90 \times \frac{\pi}{4} - 75\sin\frac{\pi}{4}\right)\right]$$

$$+ 4\,050p_2\left[75\cos\frac{3\pi}{4} - 75\cos\frac{\pi}{4}\right] = 0$$

$$37\,500 + 1\,822.5[(212 - 53) - (70.7 - 53)] + 4\,050p_2[-53 - 53] = 0$$

$$37\,500 + 1\,822.5p_2[(212 - 53 - 70.7 + 53)] - 4\,050p_2[106] = 0$$

$$37\,500 + 257.5 \text{ x } 10^3 p_2 - 430 \times 10^3 p_2 = 0$$

$$37\,500 - 172.48 \text{ x } 10^3 p_2 = 0$$

$$p_2 = \frac{37\,500}{172.48 \text{ x } 10^3} = 0.217 \text{ N/mm}^2$$

Trailing shoe:

$$F_1 L - p_1 \mu w k \int_{\theta_1}^{\theta_2} (k - r\cos\theta)d\theta - p_1 w k \int_{\theta_1}^{\theta_2} (r\sin\theta)d\theta = 0$$

$$37\,500 - 687.5 \text{ x } 10^3 p_1 = 0$$

$$p_1 = \frac{37\,500}{687.5 \times 10^3} = 0.055 \text{ N/mm}^2$$

The torque on the drum is:

$$T_D = \mu w k^2 (\theta_2 - \theta_1)(p_1 + p_2)$$

$$T_D = 0.45 \times 45 \times 90^2 \left(\frac{3\pi}{4} - \frac{\pi}{4}\right)(0.055 + 0.217)$$

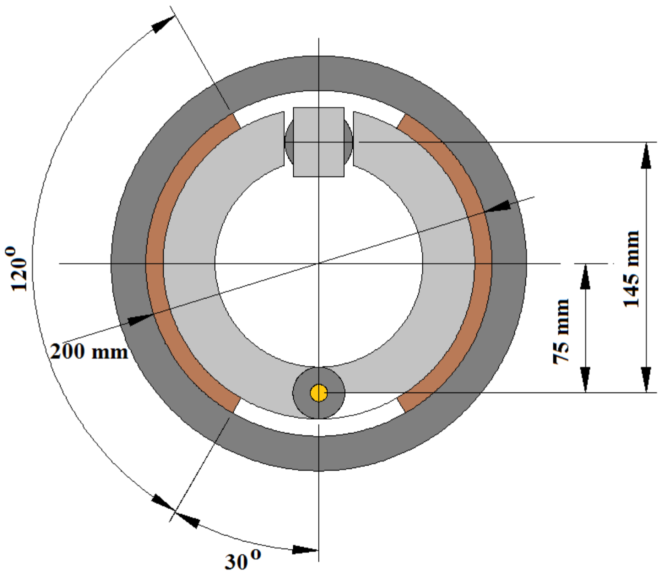
$$T_D = 70 \times 10^3 \text{ N mm or } 70 \text{ N m}$$

SELF ASSESSMENT EXERCISE No. 1

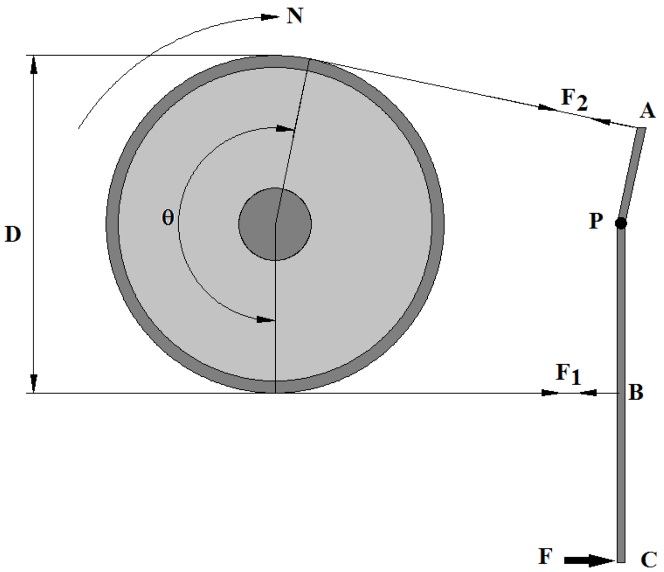
The diagram shows a shoe brake system and its dimensions. Both shoes may be assumed to have the same parameters including an equal actuator force of 200 N. The linings are 50 mm wide with a coefficient of friction is 0.4.

Determine:

i) the pressure on each lining
ii) the torque on the drum



2. *Band Brakes*



A typical band brake is illustrated. Basically these are belts rubbing on wheel. It was shown in tutorial 2 that the force in each end of the belt is related by the formula:

$$\mathbf{\frac{F_1}{F_2} = e^{\mu\theta} \text{ or } F_1 = e^{\mu\theta} F_2}$$

F_1 is larger force. In the layout shown P is the pivot point and F is the applied force.

The braking torque is

$$T = (F_1 - F_2)\frac{D}{2}$$

With these two equations problems can be solved. Substitute for F_1

$$T = (F_2 e^{\mu\theta} - F_2)\frac{D}{2} = F_2(e^{\mu\theta} - 1)\frac{D}{2}$$

$$\mathbf{F_2 = \frac{2T}{D(e^{\mu\theta} - 1)}}$$

WORKED EXAMPLE No. 2

The band brake illustrated previously has the following dimensions and parameters.

D = 200 mm   AP = 80 mm   BP = BC = 140 mm $\theta = 192^o$ $\mu = 0.25$

Calculate the force F required to produce a braking torque of 40 N m when the drum rotates clockwise and anti-clockwise.

SOLUTION

The braking torque is

$$T = 40 \text{ N m} = (F_1 - F_2)\frac{D}{2} = (F_1 - F_2) \times 0.1$$

$$(F_1 - F_2) = 400 \text{ N}$$

$$\theta = \frac{192}{180} \times \pi = 3.35 \text{ radian}$$

Clockwise

$$\frac{F_1}{F_2} = e^{\mu\theta} = e^{0.838} = 2.311$$

$$2.311F_2 - F_2 = 400$$

$$F_2 = \frac{400}{1.311} = 305.1 \text{ N} \quad F_1 = 705.1 \text{ N}$$

Moments about the pivot

$$(F \times 280) + (F_2 \times 80) = (F_1 \times 140)$$

$$280F + 80F_2 = 140(F_1)$$

$$280F = 140 \times 705.1 - 80 \times 305.1 = 74\ 306$$

$$F = 265.4 \text{ N}$$

Anti-Clockwise

In this case the greater force is F_2

$$F_1 = 305.1 \text{ N} \quad F_2 = 705.1 \text{ N}$$

Moments about the pivot

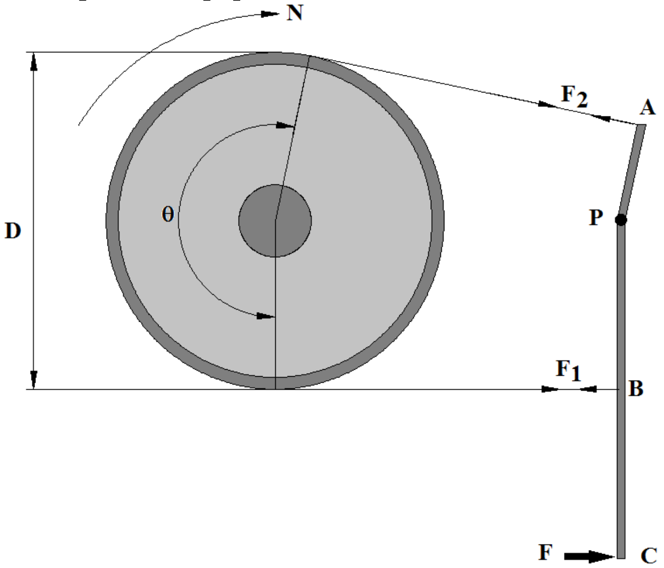
$$(F \times 280) + (F_2 \times 80) = (F_1 \times 140)$$

$$280F = 42\ 714 – 56\ 408 = -13\ 694 \qquad F = -48.9 \text{ N}$$

The applied force is smaller but in the other direction

SELF ASSESSMENT EXERCISE No. 2

This is question 6 from the sample exam paper.



The band brake illustrated has the following dimensions and parameters.

D = 120 mm
AP = 15 mm
BP = 50 mm
PC = 125 mm
$\theta = 210^o$
$\mu = 0.2$

i) State and prove the belt friction formula for a flat belt with negligible mass.

ii) Calculate the force F required to produce a braking torque of 36 N m when the drum rotates clockwise and anti-clockwise.