SOLID MECHANICS

TUTORIAL – FRICTION CLUTCHES

This work covers elements of the syllabus for the Edexcel module 21722P HNC/D Mechanical Principles OUTCOME 3.2

On completion of this short tutorial you should be able to do the following.

- Describe a conical and a flat plate clutch.
- Describe a multiplate clutch.
- Explain the constant wear theory.
- Explain the constant pressure theory.
- Solve problems involving power transmission with clutches.

It is assumed that the student is already familiar with the following concepts.

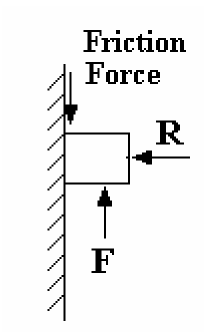
- Friction theory.
- Angular motion.
- Power transmission by a shaft.
- Basic integral calculus.

All these above may be found in the pre-requisite tutorials.

FRICTION CLUTCHES

1. INTRODUCTION

First let's revise the basics of dry friction.



Consider a small block sliding over a surface.

The force pressing the two surfaces together is R (the normal force). When the surfaces slip, the force F required to produce movement is equal and opposite of the friction force between the surfaces. F and R are related by Coulomb's Law of Friction.

$F = \mu R$ where μ is the coefficient of friction.

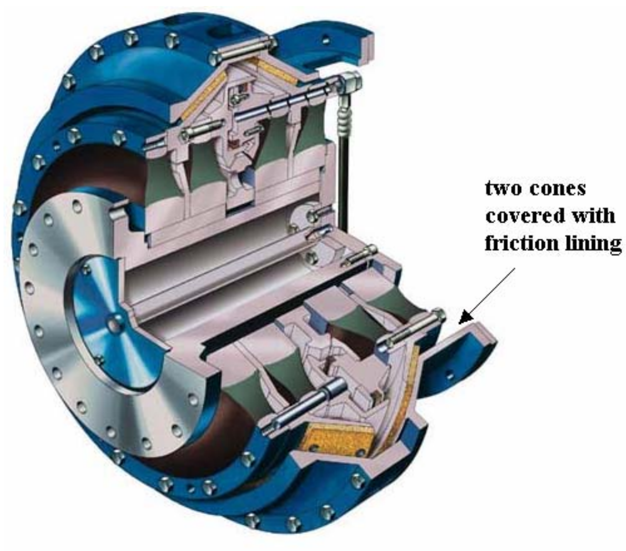
Figure 1

2. WEAR THEORY

Research has shown that the wear between two rubbing surfaces depends upon the pressure between the surfaces and the speed at which they rub.

There are two theories concerning the torque required to produce slip between the surfaces of a clutch. One theory assumes the pressure is even over the surface of contact in which case the wear is greater at the outside due to the greater velocity of rubbing. The other theory assumes that the wear is uniform in which case the pressure is not evenly distributed.

3. CONICAL CLUTCHES



The picture shows a typical conical clutch for larger power transmission applications. There are two cones covered in friction material and when they are forced apart they rub against the steel outer casings and lock them together thus engaging the two halves.

Figure 2

3.1 GEOMETRY

A conical clutch transmits rotation from one shaft to another through friction forces on the conical face. The cone has a half angle of β and the two halves are forced together with a force R.

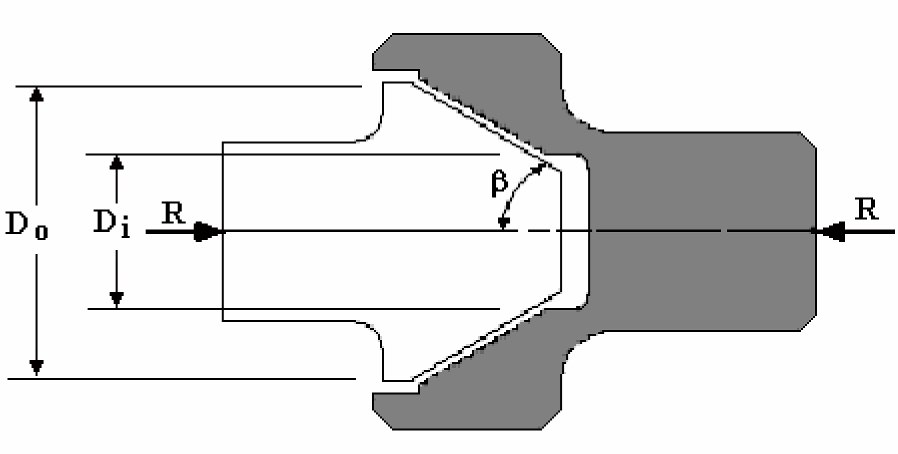


Figure 3

Consider an elementary ring on the face of the cone at radius r and radial width dr.

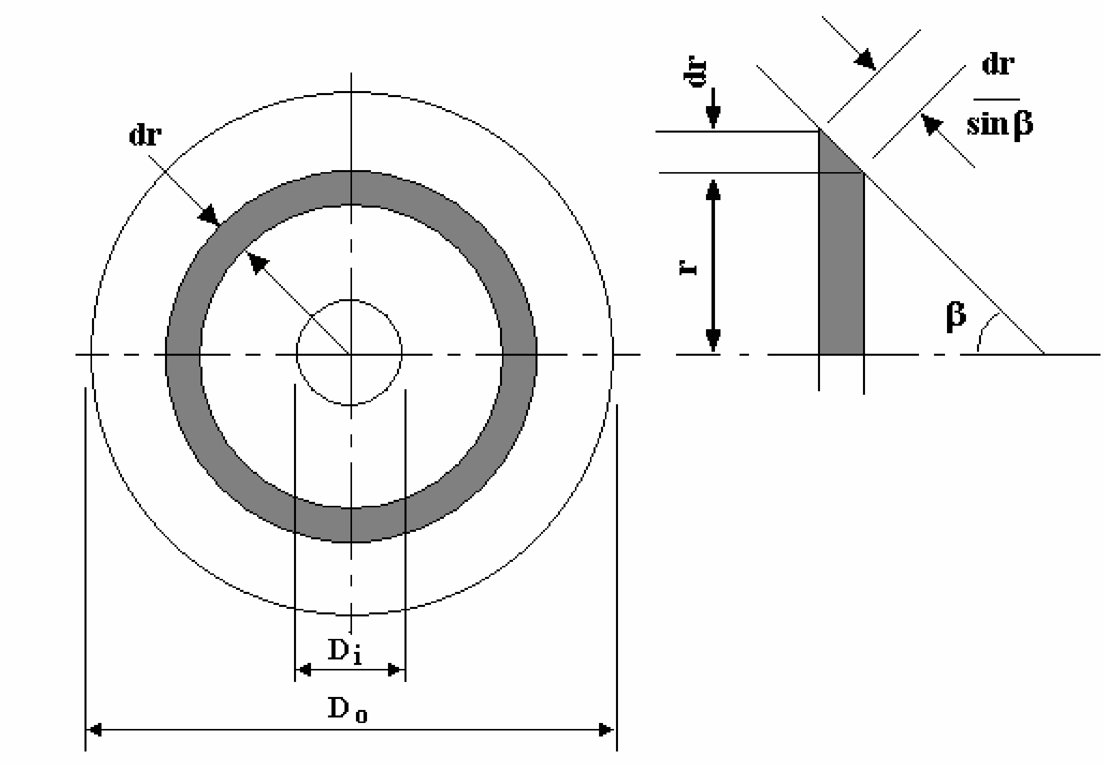


Figure 4

The length of the ring along the sloping surface is dr/sinβ. The area of the ring (dA) is approximately the circumference (2πr) times the width dr/sinβ.

$$dA = \frac{2\pi r dr}{\sin\beta} \text{.............(1)}$$

3.2 UNIFORM PRESSURE THEORY

The force pressing the surfaces together produces a uniform pressure between them of p N/m^2. The force normal to the surface is R' and the force on the element is dR'

$dR' = p\,dA$ substituting equation (1) for dA we have $dR' = \dfrac{2\pi\,prdr}{\sin\beta}$

The total force R' acting on the conical area is given by integrating.

$$R' = \int_{\frac{D_i}{2}}^{\frac{D_o}{2}} \frac{2\pi\ prdr}{\sin\beta} = \frac{2\pi p}{\sin\beta}\int_{\frac{D_i}{2}}^{\frac{D_o}{2}} rdr = \frac{2\pi p}{\sin\beta}\left[\frac{r^2}{2}\right]_{\frac{D_i}{2}}^{\frac{D_o}{2}} = \frac{\pi p}{4\sin\beta}\left[D_o^2 - D_i^2\right]$$

$$p = \frac{4R'\sin\beta}{\pi\left[D_o^2 - D_i^2\right]} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

When the clutch slips, the friction force acting on the ring is μdR'. This force produces a small torque $dT = \mu\, rdR' = \mu\, 2p\,\pi\, r^2 \dfrac{dr}{\sin\beta}$

The total torque is obtained by integrating between the inside and the outside.

$$T = \frac{2\pi\, p\,\mu}{\sin\beta}\int_{\frac{D_i}{2}}^{\frac{D_o}{2}} r^2 dr = \frac{2\pi\, p\,\mu}{\sin\beta}\left[\frac{r^3}{3}\right]_{\frac{D_i}{2}}^{\frac{D_o}{2}} = \frac{\pi\, p\,\mu}{12\sin\beta}\left[D_o^3 - D_i^3\right]$$

Substitute equation (2) for p

$$T = \frac{\mu R'}{3}\frac{\left[D_o^3 - D_i^3\right]}{\left[D_o^2 - D_i^2\right]}$$

In this derivation, R' is the total force acting normal to the surface. If this is resolved to give the axial force R = R'sinβ and so

$$T = \frac{\mu R}{3\sin\beta}\frac{\left[D_o^3 - D_i^3\right]}{\left[D_o^2 - D_i^2\right]} \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

3.3 UNIFORM WEAR THEORY

Consider the elementary ring again. $dR' = pdA$

The velocity of any point is v m/s and the angular velocity is ω rad/s.

Uniform wear theory assumes that the wear is constant everywhere and it is directly proportional to pressure x velocity (when slipping). Wear $\propto$ p v

Since $v = \omega r$, then wear $\propto p\omega r$

For constant ω, wear $\propto$ pr   p $\propto$ wear/r

The wear is constant so it follows that $p = \text{constant}/r = c/r$

As before the normal force is $dR' = pdA$

Substitute equation (1) for dA

$$dR' = 2p\,\pi\,r\frac{dr}{\sin\beta} \quad \text{and substituting } p = \frac{c}{r}$$

$$dR' = 2c\pi\frac{dr}{\sin\beta}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Integrating between the inside and outside we get

$$R' = \frac{2c\pi}{\sin\beta}\int_{\frac{D_i}{2}}^{\frac{D_o}{2}} dr = \frac{2c\pi}{\sin\beta}\left[r\right]_{\frac{D_i}{2}}^{\frac{D_o}{2}} = \frac{c\pi}{\sin\beta}(D_o - D_i)$$

$$c = \frac{R'\sin\beta}{\pi(D_o - D_i)}\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

When the clutch slips, the friction force acting on the ring is $\mu dR'$

This force produces a small torque of dT.

$dT = \mu\, r dR'$ and substituting equation (4) for dR' we have $dT = \dfrac{2c\,\mu\,\pi\,r\,dr}{\sin\beta}$

Next we integrate.

$$T = \frac{2c\,\pi\,\mu}{\sin\beta}\int_{\frac{D_i}{2}}^{\frac{D_o}{2}} r dr = \frac{2c\,\pi\,\mu}{\sin\beta}\left[\frac{r^2}{2}\right]_{\frac{D_i}{2}}^{\frac{D_o}{2}} = \frac{c\,\pi\,\mu}{4\sin\beta}\left[D_o^2 - D_i^2\right]$$

Equation (5) was $c = \dfrac{R'\sin\beta}{\pi(D_o - D_i)}$ and substituting it in to the equation gives

$$T = \frac{\mu R'}{4}\frac{\left[D_o^2 - D_i^2\right]}{(D_o - D_i)} = \frac{\mu R'}{4}\frac{(D_o + D_i)(D_o - D_i)}{(D_o - D_i)}$$

$$T = \frac{\mu R'}{4}(D_o + D_i)$$

Again, resolving R' to give the axial force R we get:

$$T = \frac{\mu R}{4\sin\beta}\left(D_o + D_i\right)\text{...............}(6)$$

WORKED EXAMPLE No.1

A conical clutch has an included angle of 120°. The outer and inner diameters are 80 and 20 mm respectively. Calculate the force required to press the two halves together if it is to transmit 200W at 600 rev/min. The coefficient of friction is 0.3. Use both the uniform wear theory and the uniform pressure theory.

SOLUTION

Identify the variables and constants.
N = 600 rev/min $\beta = 120/2 = 60^o$. Do = 0.08 m $D_i = 0.02$ m $\mu = 0.3$
Power P = 200 W

UNIFORM PRESSURE

$$T = \frac{60P}{2\pi\ N} = \frac{60 \text{ x } 200}{2\pi\, 600} = 3.1831 \text{ Nm}$$

$$T = \frac{\mu R}{3\sin\beta}\frac{\left[D_o^3 - D_i^3\right]}{\left[D_o^2 - D_i^2\right]} = \frac{0.3R}{3\sin 60^o}\frac{\left[0.08^3 - 0.02^3\right]}{\left[0.08^2 - 0.02^2\right]} = 3.1831$$

$$R = \frac{3.1831 \text{ x } 3 \text{ x } \sin(60^o) \text{ x } \left[0.08^2 - 0.02^2\right]}{0.3 \text{ x } \left[0.08^3 - 0.02^3\right]} = 328.172 \text{ N}$$

UNIFORM WEAR

$$T = \frac{\mu R}{4\sin\beta}\left(D_o + D_i\right)$$

$$R = \frac{T \text{ x } 4 \sin(\beta i}{\mu\left(D_o + D_i\right)} = \frac{3.1832 \text{ x } 4 \text{ x } \sin(60)}{0.3 \text{ x } (0.08 + 0.02)} = 367.553 \text{ N}$$

SELF ASSESSMENT EXERCISE No.1

1. The following data is for a conical clutch.

Inside diameter	30 mm
Outside diameter	110 mm
Coefficient of friction	0.23
Axial force	800 N.
Included angle	80^o.
Speed	1000 rev/min

Calculate the torque and power that can be transmitted without slipping using

a) The uniform pressure theory. (11.11 Nm and 1163 W)
b) The uniform wear theory. (10.02 Nm and 1049 W)

2. The following data is for a conical clutch.

Inside diameter	20 mm
Outside diameter	120 mm
Coefficient of friction	0.3
Included angle	100^o.
Speed	3000 rev/min

Calculate the axial force needed to allow the transmission 800 watts without slipping using

a) The uniform pressure theory. (204.1 N)
b) The uniform wear theory. (238.8)

4. FLAT CLUTCH PLATES

The diagram shows a basic flat clutch. A disc with friction material is pressed against a second disc thus engaging them by friction and making both discs rotate together.

Figure 5

A flat clutch is a special case of a conical clutch with an included angle of 180°. It may be idealised like this.

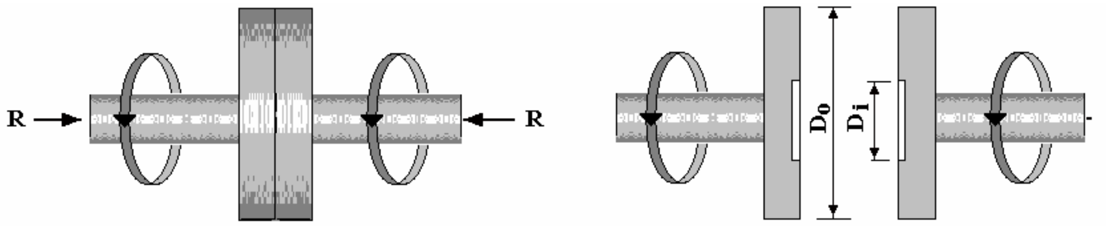


Figure 6

Consider a rotating shaft with a disc at the end that presses up against another so that rotation is transmitted from one to the other by friction.

This is the special case of the cone clutch when β = 90° and sinβ = 1. This produces the results:

4.1 UNIFORM PRESSURE THEORY

$$T = \frac{\mu R\left[D_o^3 - D_i^3\right]}{3\left[D_o^2 - D_i^2\right]} \quad \text{per surface of contact}$$

4.2 UNIFORM WEAR THEORY

$$T = \frac{\mu R}{4}\left(D_o + D_i\right) \quad \text{per surface of contact}$$

4.3 MULTI-PLATE CLUTCHES

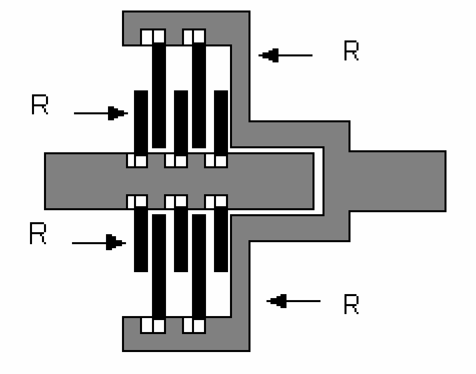


Figure 7

These are constructed with one set of plates attached to the inner shaft and the other plates to the outer case. The plates are forced together with a mechanism. On the diagram, there are five surfaces in contact and this allows a greater torque to be transmitted before slip occurs. If there are n surfaces of contact then the maximum torque is increased n times.

Values of pressure p vary from 350 kPa to 2800 kPa depending on the material. The coefficient of friction is typically 0.25 for dry materials and 0.05 when immersed in oil.

WORKED EXAMPLE No.2

The following data is for a multiplayer clutch.

Number of Contact surfaces.	5
Speed rev/min	2000
Outside diameter mm	150
Inside diameter mm	80
Coefficient of friction	0.25

Axial force R is 600 N

- ***Calculate the maximum power that the clutch can transmit without slipping based on constant wear theory.***
- ***Calculate the maximum power that the clutch can transmit without slipping based on constant pressure theory.***

SOLUTION

Identify the following

$n = 5$

$N = 2000$ rev/min

$D_o = 0.15$ m

$D_i = 0.08$ m

$\mu = 0.25$

$R = 600$ N

Uniform Pressure

$$T = \frac{\mu R}{3}\frac{\left(D_o^3 - D_i^3\right)}{\left(D_o - D_i\right)}n = \frac{0.25 \text{ x } 600}{3}\frac{\left(0.15^3 - 0.08^3\right)}{\left(0.15 - 0.08\right)} \text{x } 5 = 44.457 \text{ Nm}$$

$$\text{Power} = \frac{2\pi \text{ NT}}{60} = \frac{2\pi \text{ x } 2000 \text{ x } 44.457}{60} = 9311 \text{ Watts}$$

Uniform Wear

$$T = \frac{\mu R}{4}\left(D_o + D_i\right)n = \frac{0.25 \text{ x } 600}{4}\left(0.15 + 0.08\right) \text{x } 5 = 43.125 \text{ Nm}$$

$$\text{Power} = \frac{2\pi \text{ NT}}{60} = \frac{2\pi \text{ x } 2000 \text{ x } 43.125}{60} = 9032 \text{ Watts}$$

SELF ASSESSMENT EXERCISE No.2

1. A multi-plate clutch must transmit 20 kW of power without slipping at 4000 rev/min. The coefficient of friction is 0.28. The inner and outer diameters are 80 and 160 mm respectively. The axial force applied to the plates is 460 N. Determine the number of plates required using:

   i. The uniform pressure theory. (5.958 round up to 6)
   ii. The uniform wear theory. (6.178 round up to 7)

2. A multi-plate clutch must has three contact surfaces and transmits power at 1500 rev/min. The coefficient of friction is 0.4. The inner and outer diameters are 30 and 150 mm respectively. The axial force applied to the plates is 400 N. Calculate the torque and power that can be transmitted without slipping using:

   i. The uniform pressure theory. (24.8 Nm and 3896 Watts)
   ii. The uniform wear theory. (21.6 Nm and 3393 W)