SOLID MECHANICS
DYNAMICS
CAMS
SOLUTIONS TO SELF ASSESSMENT QUESTIONS

SELF ASSESSMENT EXERCISE No.1

1. A straight sided cam with a roller follower in line with the centre of rotation has the following dimensions:
   r_1 = 20 mm r_o = 8 mm   Total Lift = 5 mm Speed of revolution = 3 000 rev/min
   The angle of contact α is to be 70°.

Calculate the
   i) centre distance (7.6)
   ii) acceleration at the instant the lift starts. (2 763 m/s^2)
   iii) lift that occurs on the nose section. (4.1 mm)
   iv) acceleration at the point of transition from the straight edge to the nose. (3 223 m/s^2)

SOLUTION

$$\omega = 2\pi N = 2\pi \times 3000/60 = 100\pi \text{ rad/s}$$

Total Lift = 5 mm = d(1 – cos α) = d(1 – cos 70) = 0.658 d hence d = 7.6 mm

At the start of the lift θ = 0

$a = \omega^2(r_1 + r_o)(1 + 2\tan^2\theta) \sec\theta$

$a = \omega^2(r_1 + r_o)(1 + 2\tan^2 0) \sec 0 = (100\pi)^2(28)(1 + 0)\, 1 = 2\ 763\ 489\ mm/s^2$ or $2\ 763\ m/s^2$

At the transition from the edge to the nose

$$\theta = \tan^{-1}\left(\frac{d \sin\alpha}{r_1 + r_o}\right) = \tan^{-1}\left(\frac{7.6 \sin 70^o}{20 + 8}\right) = 14.31^o$$

$$x = (r_1 + r_o)(\sec\theta - 1) = 28(\sec 14.31^o - 1) = 0.896 \text{ mm}$$

It follows that the lift on the nose section is 5 – 0.896 = 4.1 mm

The acceleration at this point is

$$a = \omega^2(r_1 + r_o)(1 + 2\tan^2\theta) \sec\theta$$

$$a = (100\pi)^2(28)(1 + 2\tan^2 14.31^o) \sec 14.31^o = 3\ 223\ 119 \text{ mm/s}^2$$

2. A cam with a flat follower has a round nose and heel with circular arcs for the sides. The dimensions are as follows.

$r_1 = 15$ mm $r_2 = 5$ mm Centre distance d = 20 mm

Speed of revolution = 1 000 rev/min. The angle of contact α is to be 80°. Calculate the:

i) radius of the sides. (37.98 mm)
ii) lift on the sides. (4.57 mm)
iii) total lift. (10 mm)
iv) acceleration at commencement of lift. (252 m/s^2)
v) acceleration at the point of transition to the nose. (201.9 m/s^2 and -159.8 m/s^2)
vi) acceleration at the tip of the nose. (-219.3 m/s^2)

SOLUTION

$$\omega = 2\pi N = 2\pi \times 1000/60 = 33.33\pi \text{ rad/s}$$

$$R = \frac{r_1^2 - r_2^2 + d^2 - 2r_1 d\cos\alpha}{2(r_1 - r_2 - d\cos\alpha)} = R = \frac{15^2 - 5^2 + 20^2 - 2\times 15\times 20\cos 80^o}{2(15 - 5 - 20\cos 80^o)} = \frac{495.8}{13} = 38 \text{ mm}$$

$$\beta = \sin^{-1}\left(\frac{d\sin\alpha}{R - r_2}\right) = \sin^{-1}\left(\frac{20\sin 80^o}{38 - 5}\right) = 36.7^o$$

Lift on sides (A to B)

$$x = (R - r_1)(1 - \cos\theta) \text{ and } \theta = \beta = 36.7^o$$

$$x = (38 - 15)(1 - \cos 36.7^o) = 4.6 \text{ mm}$$

$$\text{Total Lift} = d - r_1 + r_2 = 20 - 15 + 5 = 10$$

Acceleration at commencement of lift (point A) θ = 0

$$a = \omega^2(R - r_1)\cos\theta = (33.33\pi)^2(38 - 15)\cos 0 = 252\ 223 \text{ mm/s}^2$$

Acceleration at transition to nose (point B) θ = β

$$a = \omega^2(R - r_1)\cos\theta$$

$$a = (33.33\pi)^2(38 - 15)\cos 36.7^o = 202\ 226 \text{ mm/s}^2$$

Acceleration just after the transition on the nose

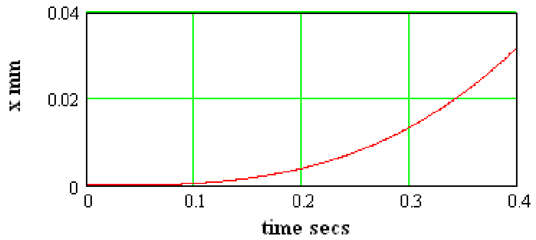
$$a = -d\,\omega^2\cos(\alpha - \theta) = -20\,(33.3\pi)^2\cos(80^o - 36.7^o) = -159\ 618 \text{ mm/s}^2$$

(Note the sudden change in acceleration)

At the tip of the nose (θ = α)

$$a = -d\,\omega^2\cos(\alpha - \theta) = -20(33.33\pi)^2\cos(0^o) = -219\ 326 \text{ mm/s}^2$$

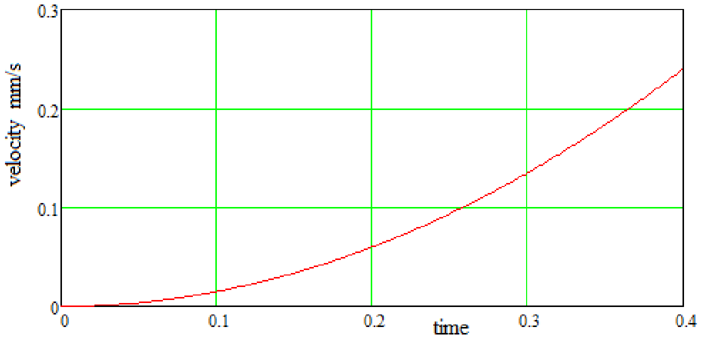
3. The diagram shows the time displacement profile of a cam.
   The profile follows the law $x = 0.5t^3$
   Produce the velocity – time graph and the acceleration – time graph.

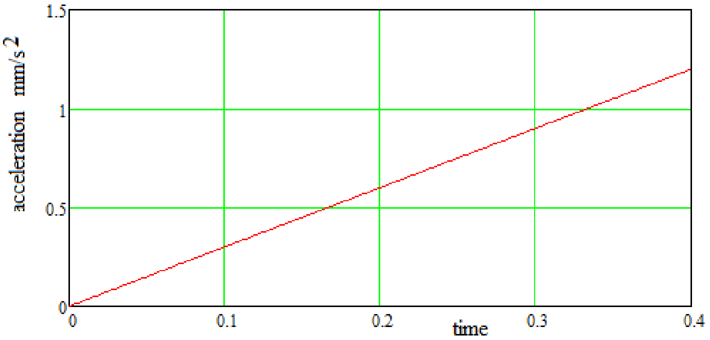


SOLUTION

$x = 0.5t^3 = t^3/2$
$v = dx/dt = 3t^2/2$
$a = dv/dt = 3t$

The plots are as shown





4. A cam must produce simple harmonic motion of the follower with amplitude 4 mm at a speed of 3000 rev/min. The follower has a mass of 30 g. Calculate the maximum acceleration and the minimum spring force needed to keep the follower in contact with the cam.
   (395 m/s^2 and 11.85 N)

SOLUTION

$$\omega = 2\pi N/60 = 314.16 \text{ rad/s}$$

$$x = 4 \sin(\omega t)$$

$$v = 4\,\omega \cos(\omega t)$$

$$a = -4\,\omega^2 \sin(\omega t)$$

Maximum acceleration is $-4\,(314.16)^2 \times 1 = 394\,784$ mm/s^2 or 394.784 m/s^2

Maximum force = M a = $0.03 \times 394.784 = 11.84$ N

5. The valves on an internal combustion engine must open for half of the revolution. The valves should be fully open for as much of this period as possible. Discuss the best form of cam and the limitations imposed by inertia on the shape.

To understand the camshaft, you need to understand: lift, duration, overlap, and timing. Lift is the maximum opening of the valve away from its seat.

Duration is the time or crank angle over which the valve is open. A longer duration means that more air can be drawn in and more exhaust gas pushed out.

Overlap refers to the period when the induction valve and exhaust valve are both open. The timing affects this and correct timing is important to performance.

The greater the valve lift (up to a limit) the greater the air flow which means there is more available for combustion so it is generally beneficial. If the lift is too large, the possibility of hitting the piston occurs and this limits the compression ratio. Greater lifts mean greater acceleration and inertia forces. This needs greater spring force in order to keep the cam in contact with the valve and prevent valve bounce.

The shape of the cam has to produce the opening period required by the engine designer but if the rise on the cam lobe is too sharp the valve may be bounced off the cam at the end of the rise.

In general the cam profile that is most commonly used is similar to that shown. Curved flanks and a circular nose seems to be normal meaning that the valve does not dwell in the open position for very long producing a lift diagram as shown.