DYNAMICS – MOMENT OF INERTIA

SOLUTIONS TO SELF ASSESSMENT EXERCISE No.1

1. A cylinder has a mass of 1 kg, outer radius of 0.05 m and radius of gyration 0.03 m. It is allowed to roll down an inclined plane until it has changed its height by 0.6 m. Assuming it rolls with no energy loss, calculate its linear and angular velocity at this point.

SOLUTION

PE lost = mgz = 1 x 9.81 x 0.6 = 5.886 J KE gained = $mv^2/2 + I\,\omega^2/2$
$\omega = v/R = v/0.05 = 20v$ KE gained = $mv^2/2 + I\,(20v)^2/2$
$I = mk^2 = 1 \times 0.03^2 = 0.0009$ kg m^2
KE gained = 5.886 = $(1)\,v^2/2 + 0.0009\,(20v)^2/2$
$5.886 = 0.5\,v^2 + 0.18\,v^2$ $5.886 = 0.68\,v^2$
$v^2 = 8.656$ v = 2.942 m/s $\omega = 20v = 58.884$ rad/s

2. A cylinder has a mass of 3 kg, outer radius of 0.2 m and radius of gyration 0.15 m. It is allowed to roll down an inclined plane until it has changed its height by 2 m. Assuming it rolls with no energy loss, calculate its linear and angular velocity at this point.

SOLUTION

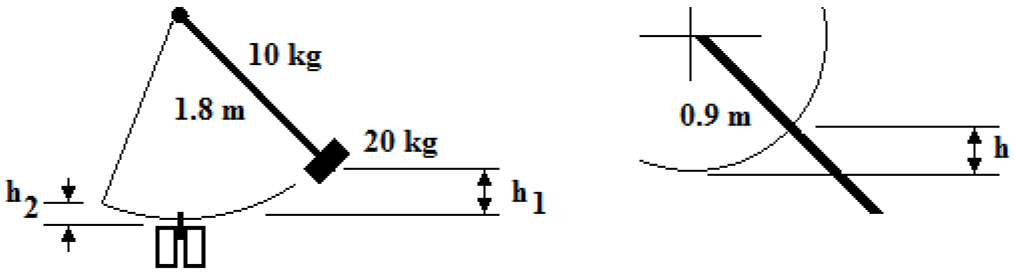
PE lost = mgz = 3 x 9.81 x 2 = 58.86 J KE gained = $mv^2/2 + I\,\omega^2/2$
$\omega = v/R = v/0.2 = 5v$ KE gained = $mv^2/2 + I\,(5v)^2/2$
$I = mk^2 = 3 \times 0.15^2 = 0.0675$ kg m^2
KE gained = 58.86 = $(3)\,v^2/2 + 0.0675\,(5v)^2/2$
$58.86 = 1.5\,v^2 + 0.84375\,v^2$ $58.86 = 2.34375\,v^2$
$v^2 = 25.11$ v = 5.01 m/s $\omega = 5v = 25.06$ rad/s

SOLUTIONS TO SELF ASSESSMENT EXERCISE No. 2

Q1. A pendulum similar to that shown on figure 9 has a mass of 20 kg. It is fixed on the end of a rod of mass 10 kg and 1.8 m long. The pendulum is raised through 45° and allowed to swing down and strike the specimen. After impact it swings up 20° on the other side. Determine the following.

i. The velocity just before impact

ii. The energy absorbed by the impact.



SOLUTION

$h_1 = 1.8 - 1.8 \cos 45^o = 0.527$ m

$h_2 = 1.8 – 1.8 \cos 20^o = 0.109$ m

The initial potential energy of the hammer = mg h_1 = 20 x 9.81 x 0.527 = 103. 4 J

The centre of the rod falls by half this height

The initial potential energy of the rod is 10 x 9.81 x 0.527/2 = 25.849 J

Total = 129.25 J

The Kinetic energy at the lowest point will be the same if none is lost so equate them.

$KE = m_1v^2/2 + I\omega^2/2$ ω = v/radius = v/1.8

$KE = m_1v^2/2 + Iv^2/(1.8^2 \times 2) = v^2 (m/2 + 0.154I)$

Radius of gyration k = 0.577 x 1.8 =1.039 $I = m_2 k^2 = 10 \times 1.039^2 = 10.79$ kg m^2

$129.25 = v^2 (20/2 + 10.79 \times 1.54) = 11.67$

$v^2 = 129.25/11.67$

v = 3.33 m/s

The potential energy at the end

For the hammer PE = $m_1 g h_2$ = 20 x 9.81 x 0.109 = 21.39 J

For the rod the height is half this PE = $m_2 g h_2/2$ = 10 x 9.81 x 0.109/2 = 5.35 J

Total = 26.74 J

Energy lost in the impact = 129.25 – 26.74 = 102.5 J

Q2 A lift has a mass of 1200 kg. It is raised by a rope passing around a winding drum and a counterbalance mass of 1000 kg hangs down on the other end. The drum has a radius of 1 m and a moment of inertia of 500 kg m^2.

During operation, the lift is accelerated upwards from rest at a rate of 2 m/s^2 for 2 seconds. The lift then rises at constant velocity for another 10 seconds and then the drive torque is removed from the drum shaft and the lift coasts to a halt.

Determine the following.

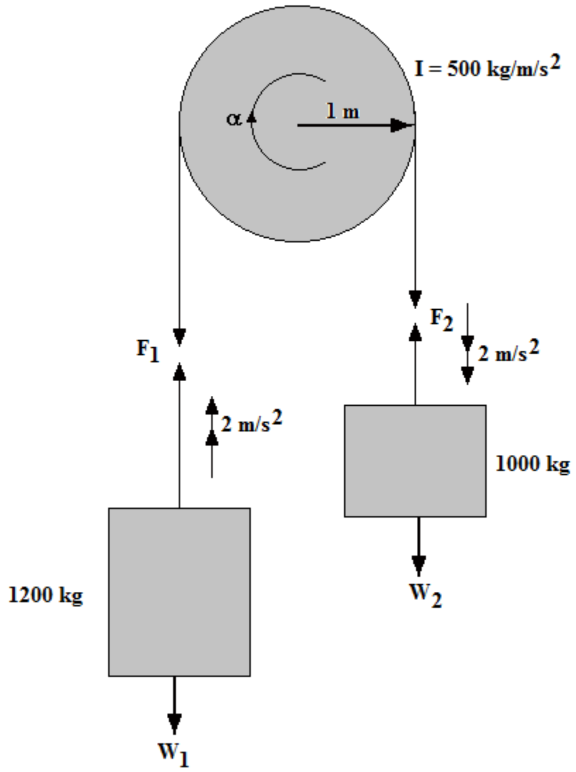
i. The maximum and minimum force in rope during this period.

ii. The torque applied to the drum during the acceleration period.

iii. The rate of deceleration.

iv. The distance moved by the lift during the journey.

SOLUTION



The key to this solution is to bear in mind that the acceleration of the two masses is always opposite in direction

$a_1 = 2\ m/s^2 \quad a_2 = -a_1 = -2\ m/s^2 \quad R = 1\ m$

$M_1 = 1200\ kg \quad M_2 = 1000\ kg$

Assuming clockwise is positive

$\alpha = a_1/R$

Balance forces on left

$F_1 = W_1 + M_1 a_1 = M_1 g + M_1 a_1$

$F_1 = 1200(9.81 + 2) = 14172\ N$

Balance forces on right

$F_2 = W_2 + M_2 a_2 = M_2 g + M_2 a_2$

$F_2 = 1000(9.81 - 2) = 7810\ N$

Balance the torque on the drum

$T = I\alpha + R(F_1 - F_2) = Ia/R + R(F_1 - F_2)$

$T = 500 \times 2/1 + 1(14712 - 7810) =$ 7362 N m

When the drum is stationary $F_1 = W_1 = 9810\ N$

The maximum force in the rope on the left is 14.172 KN and the minimum is 9.81 kN

When coasting the system decelerates T = 0 $F_1 = M_1(g + a_1) \quad F_2 = M_2(g + a_2)$

$T = 0 = Ia_1/R + R(F_1 - F_2) = 500\ a_1/1 + 1[M_1(g + a_1) - M_2(g + a_2)]$

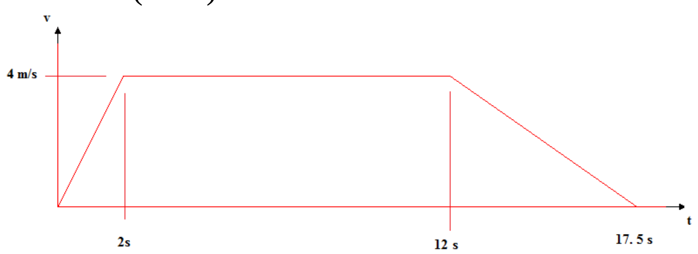
$0 = 500\ a_1 + 1200(g + a_1) - 1000(g + a_2)$

$0 = 500\ a_1 + 1200\ g + 1200\ a_1 - 1000\ g - 1000\ a_2$ Now change $a_2 = -a_1$

$0 = 500\ a_1 + 1200\ g + 1200\ a_1 - 1000\ g + 1000\ a_1$

$0 = 200\ g + 2700\ a_1 \qquad a = -200g/2700 =$ $-0.727\ m/s^2$

Time taken to decelerate to rest. at = (0 – 4) = -0.727t t = 5.5 s



Distance = Area under graph = (4 x 2)/2 + (10 x 4) + (4 x 5.5)/2 = 55 m

3. An experiment is performed to find the moment of inertia of a flywheel as follows. A mass is attached by a string to the axle which has a radius of 37.5 mm. The mass is adjusted until its weight is just sufficient to overcome frictional resistance and rotate the flywheel without acceleration. This mass is 2 kg. Another 2.5 kg is added and the mass is allowed to fall under the action of gravity and measurements show that it takes 5 s to fall 1.5 m. Determine the following.

   i. The angular acceleration.

   ii. The moment of inertia of the flywheel.

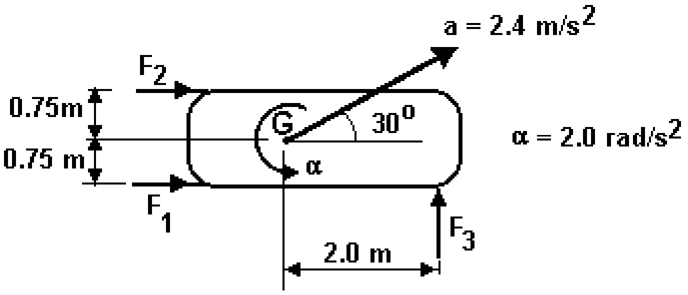
   SOLUTION

   Friction torque = T_f = mgr = 2 x 9.81 x 0.0375 = 0.73575 N m
   From the equations for velocity, distance and time we know distance $s = at^2/2$
   $1.5 = a(5)^2/2$ =   a = 0.12 m/s^2 α = a/r = 0.12/0.0375 = 3.2 rad/s^2

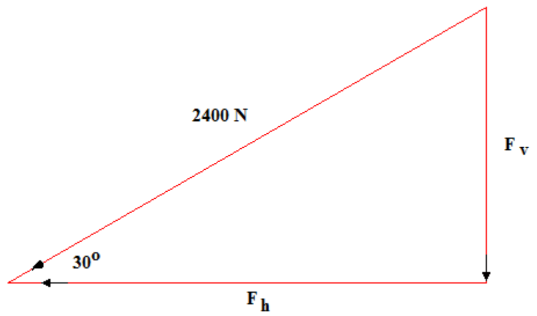
   Force in string = m(g – a) = 4.5(9.81 – 0.12) = 43.605 N
   Torque = Fr = 43.605 x 0.0375 = 1.6352 N m
   $T = T_f + I\alpha$ 1.6352 - 0.73575 = 3.2 I   I = 0.281 kg m^2

4. Figure 13 shows a schematic of a space module orbiting in a gravity free zone. Rockets are attached at points and exert forces F_1, F_2 and F_3 as shown all acting in the same plane as the centre of mass G. The forces produce a linear acceleration a and angular acceleration α at G. The module has a mass of 1000 kg and moment of inertia 800 kg m^2 about G. Determine the three forces F_1, F_2 and F_3.

   (1573 N, 505 N and 1200 N)



SOLUTION
The inertia force F_i = M a = 1000 x 2.4 = 2400 N
Resolve vertically and horizontally



$F_v = 2400 \sin 30^o = 1200$ N
$F_h = 2400 \cos 30^o = 2078.5$ N
Balance forces
$F_1 + F_2 = 2078.5$ N $F_1 = 2078.5 - F_2$
$F_3 = 1200$ N
Balance the torques (positive Clockwise)
$F_2 \times 0.75 + I\alpha = F_3 \times 2 + F_1 \times 0.75$
Substitute

$0.75F_2 + 1600 = 2400 + 0.75(2078.5 - F_2) = 2400 + 1558.9 - 0.75F_2$
$1.5F_2 = 2400 + 1558.9 - 1600 = 2358.9$
$F_2 = 2358.9/1.5 = 1573$ N
$F_1 = 2078.5 – 1573 = 505.5$ N

5. A drum which is 300 mm diameter has a moment of inertia of 200 kg m^2. It revolves at 2 rev/s. A braking torque of 285.8 N m is applied to it producing uniform deceleration. Determine the following.

i. The initial kinetic energy stored in the drum

ii. The angular deceleration of the drum.

iii. The time taken to for the drum to stop.

SOLUTION

$\omega = 2\pi N = 2\pi(2) = 4\pi$ rad/s
KE = $I\omega^2/2 = 200(4\pi)^2/2 = 15791$ J
$\alpha = T/I = 285.8/200 = 1.428$ rad/s^2
$\alpha = \Delta\omega/t$ $\quad$ $t = \Delta\omega/\alpha = 4\pi/1.428 = 8.8$ s