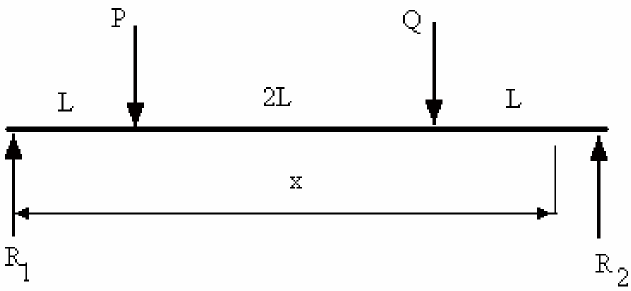
9. (a) Derive the expression for the deflection at the centre of the beam loaded and supported as shown in terms of P,Q,L and the flexural stiffness EI

(b) A rigid prop is used at the middle to prevent deflection. Derive an expression for the force in the prop.

There are several ways of solving this problem. The following used Macaulay full solution



REACTIONS

Moments about right end give

$R_1 \times 4L = P \times 3L + Q \times L$

$R_1 = (P \times 3L + Q \times L)/4L = (3P + Q)/4$

The bending moment at distance x is :

$M = R_1x - P[x - L] - Q[x - 3L]$

$EI\, d^2x/dy^2 = R_1x - P[x - L] - Q[x - 3L]$

$EI\, dy/dx = R_1x^2/2 - P[x - L]^2/2 - Q[x - 3L]^2/2 + A$

$EI\, y = R_1x^3/6 - P[x - L]^3/6 - Q[x - 3L]^3/6 + Ax + B$

The boundary conditions are y = 0 at x = 0 and at x = 4L

Put x = 0 $\quad EI\, y = 0 = 0 - 0 - 0 + 0 + B$ hence B = 0

Put x = 4L $\quad EI\, y = 0 = R_1(4L)^3/6 - P[3L]^3/6 - Q[3L]^3/6 + A(4L)$

$0 = R_1(4L)^3/6 - P[3L]^3/6 - Q[L]^3/6 + A(4L)$

$$A = \frac{-10.67L^3R_1 + 4.5PL^3 + 0.167QL^3}{4L} = -2.67L^2R_1 + 1.125PL^2 + 0.0417QL^2$$

$$EIy = \frac{R_1x^3}{6} - \frac{P[x-L]^3}{6} - \frac{Q[x-3L]^3}{6} + x\left(-2.67L^2R_1 + 1.125PL^2 + 0.0417QL^2\right)$$

put x = 2L (the middle position)

$$EIy = \frac{8R_1L^3}{6} - \frac{P[L]^3}{6} - \frac{Q[0]^3}{6} - 5.333L^3R_1 + 2.25PL^3 + 0.0835QL^3$$

Substitute for R_1

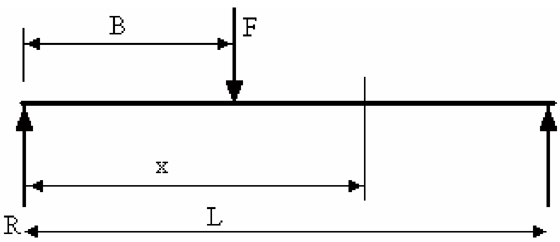
$$EIy = \frac{8L^3}{6}\left(\frac{3P+Q}{4}\right) - \frac{P[L]^3}{6} - 5.333L^3\left(\frac{3P+Q}{4}\right) + 2.25PL^3 + 0.0835QL^3$$

$$EIy = L^3\left[\frac{(3P+Q)}{3} - \frac{P}{6} - 1.333(3P+Q) + 2.25P + 0.0835Q\right]$$

$$EIy = L^3\left[P + \frac{Q}{3} - \frac{P}{6} - 4P - 1.333Q + 2.25P + 0.0835Q\right] = L^3\left[P + \frac{Q}{3} - \frac{P}{6} - 4P - 1.333Q + 2.25P + 0.0835Q\right]$$

$$EIy = L^3[-0.917P - 0.917Q] \qquad y = -0.917\frac{L^3[P+Q]}{EI}$$

An alternative solution is to know or derive the deflection for a single point load and then use the principle of superposition.



Using Macaulay it can be shown that the deflection at x is

$$yEI = \frac{F(L-B)x^3}{6L} - \frac{F(x-B)^3}{6} + \frac{Fx(L-B)^3}{6L} - \frac{FxL(L-B)}{6}$$ so long as x is larger than B.

Putting x =L/2 the deflection at the middle becomes.

$$yEI = \frac{F(L-B)L^2}{48} - \frac{F\left(\frac{L}{2}-B\right)^3}{6} + \frac{F(L-B)^3}{12} - \frac{FL^2(L-B)}{12}$$

For our question we need to substitute 4L for L , L for B and P for F

$$yEI = \frac{P(4L-L)L^2}{3} - \frac{P(2L-L)^3}{6} + \frac{P(4L-L)^3}{12} - \frac{16PL^2(4L-L)}{12}$$

$$yEI = PL^3 - \frac{PL^3}{6} + \frac{27PL^3}{12} - \frac{48PL^3}{12} = -\frac{11}{12}PL^3$$

The deflection at the middle due to P is $-11PL^3/12EI$

Because the load Q is the same distance from the other end, the equation for the deflection at the middle will be identical.

The deflection at the middle due to Q is $-11QL^3/12EI$

The total deflection is $-11PL^3/12EI - 11QL^3/12EI = -(11L^3/12EI)(P+Q) = -0.917(P+Q)/EI$

This is the same as found in the first method.

(b) If a prop is used to return the centre deflection back to zero then we use the principle of superposition again. A point load at the middle will produce a deflection of $FL^3/48EI$. (Standard formula that should be remembered or derived by putting B = L/2 above).

To eliminate the deflection it follows that $FL^3/48EI = (11L^3/12EI)(P+Q)$

$$F = 44\ (P+Q)$$