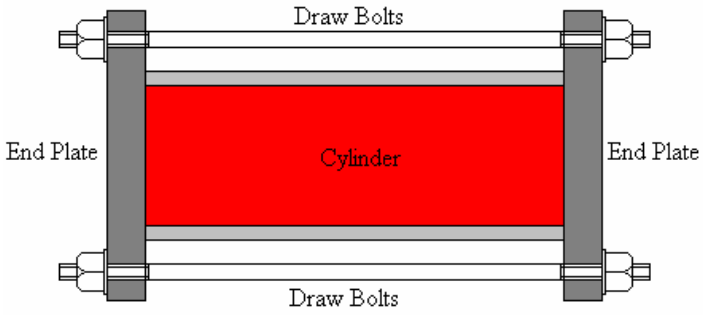
7 An open ended cylinder is subjected to an internal pressure.

(a) What is the longitudinal stress?

Answer – zero since no longitudinal force can be produced in the walls.

(b) Sketch a practical arrangement that allows an open ended cylinder to have internal pressure.



(c) Calculate the increase in the internal diameter of an open ended cylinder given the following data.

Outer Diameter	200 mm
Inner Diameter	140 mm
Internal Pressure	100 MPa
Young's Modulus	200 GPa
Poisson's ratio	0.3

Wall thickness = 30 mm D/t = 200/30 = 6.7 so it should be treated as a thick cylinder.

Lame's equations

$\sigma_R = a - \frac{b}{r^2}$ $\sigma_C = a + \frac{b}{r^2}$

The boundary conditions are

Inner surface r = 70 mm σ_R = - 100 MPa (compressive)

Outer surface r = 100 mm σ_R = 0 MPa (compressive)

Substituting into Lame's equation we have

σ_R = - 100 x 10^6 = a - b/r^2 = a - b/0.07^2

σ_R = 0 = a - b/r^2 = a - b/0.1^2 a = 100b

substitute

- 100 x 10^6 = 100b – 204.08b = -104.08b b = 960 x 10^3

a = 96.08 x 10^6

Now solve the circumferential stress. σ_c = a + b/r^2

Putting r = 0.1 σ_c = 192. MPa

Putting r = 0.07 σ_c = 292 MPa

There are two mutually perpendicular stresses at the inner surface of 292 MPa (tensile) and 100 MPa (compressive)

$$\varepsilon_c = \frac{1}{E}[\sigma_c - \nu\sigma_r] = \frac{1}{200}\left[292x10^6 - 0.3(-100 \times 10^6\right] = 1.611 \times 10^{-3}$$

Circumferential strain is the same as the diametral strain ε_c =Δd /d

Change in diameter = Δd = ε_c d = 1.611 x 10^{-3} x 140 = 0.225 mm