Q. 6 A steel column with the cross-section shown is 3.5 m long and is built in at both ends.
(a) Determine the point where the axial load should be applied in order to maximise the strength.
(b) Using the appropriate Euler equation, determine the load at which it will buckle. Take E $=200 \mathrm{GPa}$.


Note - The question states that this is a column and columns by definition to not buckle but fail by crushing unless there is a bending moment added. In order to solve this correctly we would need to be given the crushing stress. Euler's formula for buckling applies only to slender struts and it appears that the question should be solved by treating as such.
(a) To maximise the strength the load should be applied at the centroid so that no bending occurs. To find this point we need to take first moments of area about the top edge and the right edge.

Divide into two parts A and B.
First solve the first moments about the axis $\mathrm{x}^{\prime}$. $\mathrm{y}^{\prime}$ is the distance to the centroid of each part.

|  | Area | $\bar{y}$ | $A \bar{y}$ |
| :--- | :--- | :--- | :--- |
| Section A | 1500 | 0 | 0 |
| Section B | 1350 | 50 | 67500 |
| Totals | 2850 |  | 67500 |

The overall $\bar{y}$ is $67500 / 2850=23.68 \mathrm{~mm}$
Now repeat about the $y^{\prime}$ axis. $x^{\prime}$ is the distance to the centroid of each part.

|  | Area | $\overline{\mathrm{x}}$ | $\mathrm{A} \overline{\mathrm{x}}$ |
| :--- | :---: | :---: | :---: |
| Section A | 1350 | 75 | 101250 |
| Section B | 1500 | 0 | 0 |
| Totals | 2850 |  | 101250 |



Section B $1500 \quad 0 \quad 0$
Totals $2850 \quad 101250$
The overall $\overline{\mathrm{x}}$ is $101250 / 2850=35.53 \mathrm{~mm}$
The centroid is 28.68 mm from the top edge and 43.03 mm from the left edge.
(b) The strut will buckle about the weakest axis. This is the one with minimum I. To find this we first need the product second moment $\mathrm{I}_{\mathrm{xy}}$

This is zero for any symmetrical shape so for part A and $\mathrm{B}_{\mathrm{x}^{\prime} \mathrm{y}^{\prime}}=0$ about their centroids.
About the centroid of the whole shape $\mathrm{I}_{\mathrm{xy}}=\mathrm{A} \overline{\mathrm{y}} \overline{\mathrm{x}}$ (from the parallel axes theorem)

$$
\mathrm{I}_{\mathrm{xy}}=\mathrm{A} \overline{\mathrm{y}} \overline{\mathrm{x}}=2850 \times 23.68 \times 35.53=2.398 \times 10^{6} \mathrm{~mm}^{4}
$$

This may be positive or negative but it doesn't matter in the following work.
We must calculate the minimum I. The formula is:
$\mathrm{I}_{\max / \min }=\frac{\mathrm{I}_{\mathrm{x}}+\mathrm{I}_{\mathrm{y}}}{2} \pm \sqrt{\frac{\left(\mathrm{I}_{\mathrm{x}}-\mathrm{I}_{\mathrm{y}}\right)^{2}}{2}+\mathrm{I}_{\mathrm{xy}}^{2}}$

First calculate $I_{x}$ by taking second moments about the bottom edge.

$\mathrm{I}_{\text {base }}=150 \times 100^{3} / 3-135 \times 90^{3} / 3=17.195 \times 10^{6} \mathrm{~mm}^{4}$
$\mathrm{I}_{\mathrm{x}}=17.195 \times 10^{6}-2850 \times 71.32^{2}=2.698 \times 10^{6} \mathrm{~mm}^{4}$
Now find $\mathrm{I}_{\mathrm{y}}$ by repeating about the right edge.
$I_{\text {edge }}=100 \times 150^{3} / 3-90 \times 135^{3} / 3=38.689 \times 10^{6} \mathrm{~mm}^{4}$
$\mathrm{I}_{\mathrm{y}}=38.689 \times 10^{6}-2850 \times 106.97^{2}=6.078 \times 10^{6} \mathrm{~mm}^{4}$
$\mathrm{I}_{\max / \min }=\frac{\mathrm{I}_{\mathrm{x}}+\mathrm{I}_{\mathrm{y}}}{2} \pm \sqrt{\frac{\left(\mathrm{I}_{\mathrm{x}}-\mathrm{I}_{\mathrm{y}}\right)^{2}}{2}+\mathrm{I}_{\mathrm{xy}}^{2}}$
$\mathrm{I}_{\text {max } / \min }=\frac{6.078 \times 10^{6}+2.698 \times 10^{6}}{2} \pm \sqrt{\left(\frac{6.078 \times 10^{6}-2.698 \times 10^{6}}{2}\right)^{2}+\left(2.398 \times 10^{6}\right)^{2}}$
$\mathrm{I}_{\text {max } / \text { min }}=4.388 \times 10^{6} \pm \sqrt{2.856 \times 10^{12}+\left(5.75 \times 10^{12}\right)}$
$\mathrm{I}_{\max / \min }=4.388 \times 10^{6} \pm 2.933 \times 10^{6}$
$\mathrm{I}_{\text {min }}=1.454 \times 10^{6} \mathrm{~mm}^{4}$
Euler's buckling formulae is $F=\frac{n^{2} \pi^{2} E I}{L^{2}} n$ is the buckling mode which for a strut built in at both ends is 2 .
$\mathrm{F}=\frac{\mathrm{n}^{2} \pi^{2} \mathrm{EI}}{\mathrm{L}^{2}}=\frac{2^{2} \times \pi^{2} \times 200 \times 10^{9} \times 1.455 \times 10^{-6}}{3.5^{2}}=937.8 \mathrm{kN}$

