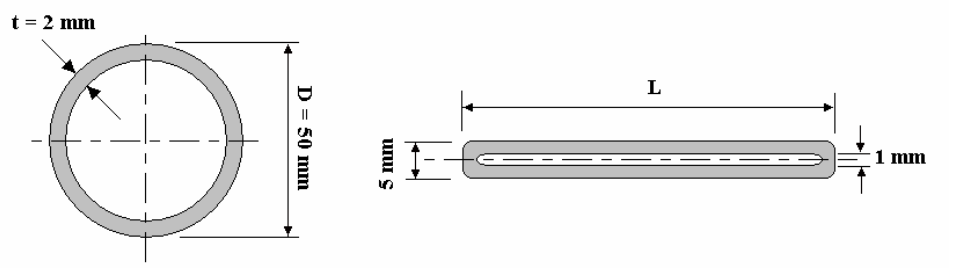
5. A thin walled cylinder is 50 mm outer diameter and has a wall 2 mm thick. It is squashed into the flat shape shown. Ignoring stress concentration, explain how the following properties are affected by the change. (The dimensions are added as part of the solution)

(a) The torsional strength.
(b) The torsional stiffness.
(c) The bending strength.
(d) The bending stiffness.



Bending equation
$\frac{M}{I}=\frac{E}{R}=\frac{\sigma}{y}$

Torsion Equation $\frac{T}{J}=\frac{G\theta}{L}=\frac{\tau}{r}$

Cylinder $I_{xx}=I_{yy}=\frac{\pi(D^4-d^4)}{64}=\frac{\pi(50^4-46^4)}{64}=87000\ mm^4$ $J=2I=174000\ mm^4$

Flat shape The approximate dimensions of the hollow rectangular sections are found as follows.
The mean length of the centre line in the wall $\pi D_m = \pi$ x 48 = 150.8 mm. Treating the shape as a rectangle, $150.8 = 2L_m + 6$ $L_m = 72.4$ mm.
Outer dimensions are L = 74.4 mm H = 5 mm
Inner dimensions are l = 70.4 mm and h = 1 mm
Using $I = (L H^3 - l h^3)/12$
$I_{xx} = \{74.4 \times 5^3 - 70.4 \times 1^3\}/12 = 769\ mm^4$
$I_{yy} = \{5 \times 74.4^3 - 1 \times 70.4^3\}/12 = 142520\ mm^4$

(a) TORSIONAL STRENGTH
The shape with the greatest torsional strength is the one with the minimum stress τ.
<u>Cylinder</u>
For any given torque $\tau = Tr/J$ r = 25 $J/r = 174000/25 = 6960\ mm^3$ $\tau = T/6960$

Thin Rectangle
Without proof $\tau = T/2At$ where A is the cross sectional area based on the dimensions of the centre line of the wall. A = (72.4) x (3) = 214.2 mm^2. T/(2 x 217.2 x 2) = T/869 and hence this shape has the largest stress so it is the weaker section.
Check cylinder with same formula $A = \pi \times 48^2/4 = 1809.6\ mm^2$. $\tau = T/(2x1809.6x2) = T/7238$ (close)

(b) Torsional stiffness = $T/\theta = GJ/L$. It follows that the shape with the greatest value of J will have the greater stiffness for a given modulus G and length L. Instinctively, the thin rectangle would twist easier than the cylinder. For a thin solid rectangle J is usually given as $LH^3/3 = 74.4 \times 5^3/3 = 3100\ mm^4$ so this would be much weaker.
(c) Bending Strength stress = $\sigma = My/I$ so the greatest strength is the shape with the smallest value of y/I or greatest value of I/y = z where z is normally referred to as the section modulus.

Cylinder
y = 25 mm $z_{xx} = z_{yy} = 87000/25 = 3480\ mm^3$
Thin Rectangle
About the x x axis y = 2.5 mm so $z_{xx} = 769/2.5 = 308\ mm^3$
About the y y axis y = 37.2 mm so $z_{yy} = 142520/37.2 = 3831\ mm^3$

The cylinder has the greater bending strength about the x x axis but the smaller about the y y axis.

(d) Bending stiffness is EI so the shape with the greatest value of I has the greater bending stiffness.

About the axis x x I is greater for the cylinder and about the y y axis it is greater for the flat shape.