## D209 Q4 2001

(a) In any beam subject to bending, what is the value of the shear force at the point of maximum bending moment?

(b) The diagram shows a beam consisting of a rectangular aluminium alloy section bonded to a steel T section. The beam is 1.5 m long and is simply supported at each end. It carries a uniform load of 3 kN per metre over half its length and 6 kN per metre over the other half. Determine the maximum stress in both the aluminium alloy section and the steel section. Young's modulus is 200 GPa for steel and 70 GPa for aluminium alloy.



1.125 m

0.75 m

Rı

Ν

 $R_2$ 

 $4.5 \,\mathrm{kN}$ 

R٦

0.75 m

(a) In most cases, the bending moment is a maximum when the shear force changes from positive to negative so the shear force might be expected to be zero but this is not true in all cases e.g. a cantilever with a point load at the end has constant shear force over its entire.

3 kN/m

0.75 m

R<sub>1</sub>

6 kN/m

0.75 m

(b) First we need the maximum bending moment. The distributed load of 3 kN/m over 0.75 m gives a total load of 2.25 kN. The distributed load of 6 kN/m over 0.75 m gives a total load of 4.5 kN. The equivalent point loads are as shown. Now find the reactions R<sub>1</sub> and R<sub>2</sub>.

 $R_2 \ge 1.5 = 2.25 \ge 0.375 + 4.5 \ge 1.125$   $R_2 = 5.906/1.5 = 3.9375 \ge 0.375 = 2.8125 \ge 0.375 = 2.8125 \ge 0.375 = 0.375$ 

To find the position of maximum bending we need to find where the shear force is zero. This is the quickest way.

At the left end SF = 2.8125 up At the right end SF = 3.9375 up to get back to zero. At the middle  $SF = 2.8125 - 3 \times 0.75 = 0.563$ 

By ratios B/A = 3.9375/0.563B = 6.994AB = 0.75 - A = 6.994A0.75 = 7.994 A A = 0.107 hence the zero point is 0.86 m from the left end.

 $M = (2.8125 \times 0.86) - (2.25 \times 0.482) - (6 \times 0.107 \times 0.107/2) = 1.3 \text{ kNm}$ Check with calc from right end.  $M = 3.9375 \times 0.643 - 6 \times 0.643 \times 0.643/2 = 1.3 \text{ kNm}$ 

Now consider the composite section. We must find the equivalent steel section. First change the width of the alloy to an equivalent width of steel.

 $B = 100 \text{ x } E_a/E_s = 100 \text{ x } 70/200 = 35 \text{ mm}$ 

Now find the bending moment at this point.

The section is equivalent to a symmetrical steel cross as shown. The second moment of area of a rectangle about the centre line is  $BD^{3}/12$  so we may find I by treating the shape as two rectangles 35 x 120 and 65 x 40

 $I_{NA} = 35 \times 120^3/12 + 65 \times 40^3/12 = 5.387 \times 10^6 \text{ mm}^4 = 5.387 \times 10^{-6} \text{ m}^4$ 

 $\sigma = My/I$  At y = 20 mm (the interface) the stress is  $\sigma = 1300 \times 0.02/5.387 \times 10^{-6} = 4.826$  MPa. This is the maximum stress in the steel section.

At y = 60 mm (the top) the stress is  $\sigma = 1300 \times 0.06/5.387 \times 10^{-6} = 14.48$  MPa. This is the maximum stress in the alloy.



35

35

100

40

40