3 (a) Define the term "shear centre".
(b) Determine the position of the shear centre of each of the three thin walled sections shown. Each has a uniform thickness.

(a) The shear centre is that point through which the loads must act if there is to be no twisting, or torsion. The shear centre is always located on the axis of symmetry; therefore, if a member has two axes of symmetry, the shear centre will be the intersection of the two axes. Channels have a shear centre that is not located on the member.
(b) The ' $Z$ ' shape is symmetrical about the centroid in the x and y directions so the shear centre is at the centroid distance ' $a$ ' from the bottom and ' $a$ ' from the edges.
The ' $L$ ' shape is also symmetrical about two axes normal and perpendicular to a line drawn at $45^{\circ}$ so the shear centre will also be at the centroid.
The $U$ shape is the one requiring much work.
The centre of shear for the ' $U$ ' channel is to the left of the section as drawn. First calculate the position of the centroid. This must be on the horizontal centre line so we need to calculate the position from the vertical edge.

|  | Area | $\bar{z}$ | $\mathrm{~A} \overline{\mathrm{z}}$ |
| :--- | :--- | :--- | :--- |
| A | at | $\mathrm{a} / 2$ | $\mathrm{a}^{2} \mathrm{t} / 2$ |
| B | at | $\mathrm{a} / \mathrm{t}$ | $\mathrm{a}^{2} \mathrm{t} / 2$ |
| C | 2at | $\mathrm{t} / 2$ | $\mathrm{at}^{2}$ |
| Total | 4at |  | $\mathrm{a}^{2} \mathrm{t}+\mathrm{at}^{2}$ |

For the section $\bar{z}=\left(a^{2} t+a t^{2}\right) / 4 a t$ if $t$ is small the $t^{2}$ term may be ignored so
 $\bar{z}=a^{2} t / 4 a t=a / 4$
Next we calculate the shear stress in the section due to transverse shear force F $\tau=\frac{\mathrm{FA} \overline{\mathrm{y}}}{\mathrm{I}_{\mathrm{z}} \mathrm{b}} \mathrm{I}_{\mathrm{z}}$ is the second moment of area of the section about the z axis.
For the vertical section $I=t(2 a)^{3} / 12=(2 / 3) \mathrm{a}^{3} \mathrm{t}$
For the flanges we may approximate with $I=A \times a^{2}$ where $A=a t$ so $I=2 \times a^{3} t$
Adding we get $\mathrm{I}_{\mathrm{z}}=(2 / 3) \mathrm{a}^{3} \mathrm{t}+2 \mathrm{x} \mathrm{a}^{3} \mathrm{t}$

$$
\mathrm{I}_{\mathrm{z}}=\frac{8}{3} \mathrm{a}^{3} \mathrm{t} \quad \tau=\frac{\mathrm{FA} \overline{\mathrm{y}}}{\mathrm{I}_{\mathrm{z}} \mathrm{~b}}=\frac{3 \mathrm{FA} \overline{\mathrm{y}}}{8 \mathrm{a}^{3} \mathrm{tb}}
$$

## SHEAR DISTRIBUTION IN THE FLANGE

Area $\mathrm{A}=(\mathrm{a}-\mathrm{z}) \mathrm{t} \quad \overline{\mathrm{y}}=\mathrm{a} \quad \tau=\frac{3 \mathrm{FA} \overline{\mathrm{y}}}{8 \mathrm{a}^{3} \mathrm{tb}}$ and in this case $\mathrm{b}=\mathrm{t}$ the thickness of the flange.

$$
\tau=\frac{3 \mathrm{~F}(\mathrm{a}-\mathrm{z})}{8 \mathrm{a}^{2} \mathrm{t}} \text { and the shear flow is } \mathrm{q}=\tau \mathrm{t}=\frac{3 \mathrm{~F}(\mathrm{a}-\mathrm{z})}{8 \mathrm{a}^{2}}
$$

The maximum shear stress and shear flow occurs when $\mathrm{z}=0$ and are:

$$
\tau_{\max }=\frac{3 \mathrm{~F}}{8 \mathrm{at}} \quad \mathrm{q}_{\max }=\frac{3 \mathrm{~F}}{8 \mathrm{a}}
$$

The minimum values are zero at $\mathrm{z}=\mathrm{a}$ In between the variation is linear.
The distribution in the bottom flange is the same but negative.
SHEAR DISTRIBUTION IN THE WEB
Next find the expression for the shear stress distribution in the vertical section. $\bar{y}$ is harder to find.

|  | Area | $\bar{y}$ | $A \bar{y}$ |
| :--- | :--- | :--- | :--- |
| A | at | $a$ | $a^{2} t$ |
| B | $(a-y) t$ | $(a+y) / 2$ | $\left(a^{2}-y^{2}\right) t / 2$ |

Total Area $=$ at $+(a-y) t=t(2 a-y)$
Total $1^{\text {st }}$ moment $=a^{2} t+\left(a^{2}-y^{2}\right) t / 2=\left(3 a^{2}-y^{2}\right) t / 2$
$\bar{y}$ for the shaded section is $\left(3 a^{2}-y^{2}\right) t / 2 \div t(2 a-y)=\left(3 a^{2}-y^{2}\right) / 2(2 a-y) \quad I_{z}=\frac{8}{3} a^{3} t$
$\tau=\frac{F A \bar{y}}{I_{z} t}=\frac{3 F t(2 a-y)\left(3 a^{2}-y^{2}\right)}{8 a^{3} t^{2} 2(2 a-y)}=\frac{3 F\left(3 a^{2}-y^{2}\right)}{16 a^{3} t} \quad q=\tau t=\frac{3 F\left(3 a^{2}-y^{2}\right)}{16 a^{3}}$
When $\mathrm{y}=0 \quad \tau=\frac{9 \mathrm{~F}}{16 \mathrm{at}} \quad \mathrm{q}=\frac{9 \mathrm{~F}}{16 \mathrm{a}} \quad$ When $\mathrm{y}=\mathrm{a} \quad \tau=\frac{3 \mathrm{~F}}{8 \mathrm{at}} \quad \mathrm{q}=\frac{3 \mathrm{~F}}{8 \mathrm{a}}$


The shear stress distribution is like this. The stress on the top and bottom flanges falls linearly to zero at the edges.

The forces in the flange are the area under the graphs.
$\mathrm{F}^{\prime}=\frac{3 \mathrm{~F}}{8 \mathrm{a}} \times \frac{\mathrm{a}}{2}=\frac{3 \mathrm{~F}}{16}$
These forces acts horizontally and are equal and opposite in direction in the top and bottom flanges.

Now we need the force in the web. This is twice the force in one half so

$$
\begin{aligned}
& \mathrm{F}^{\prime}=2 \int_{0}^{\mathrm{a}} \mathrm{q} d y=2 x \frac{3 \mathrm{~F}}{16 \mathrm{a}^{3}} \int_{0}^{2 a}\left(3 \mathrm{a}^{2}-y^{2}\right) d y=\frac{3 \mathrm{~F}}{8 \mathrm{a}^{3}}\left[3 \mathrm{a}^{2} y-\frac{y^{3}}{3}\right]_{0}^{\mathrm{a}}=\frac{3 \mathrm{~F}}{8 \mathrm{a}^{3}} x \frac{3 \mathrm{a}^{3}}{8} \\
& \mathrm{~F}^{\prime}=\mathrm{F}
\end{aligned}
$$

This is not totally unexpected since we have assumed very thin sections. The forces are like this.
Balancing moment about the centroid we have:
$\mathrm{F}\left(\mathrm{e}+\frac{\mathrm{a}}{4}\right)=\frac{3}{16} \mathrm{Fa}+\frac{3}{16} \mathrm{Fa}+\mathrm{F} \frac{\mathrm{a}}{4}$
$\left(e+\frac{a}{4}\right)=\frac{3 a}{8}+\frac{a}{4}$
$e=\frac{3 a}{8}$


