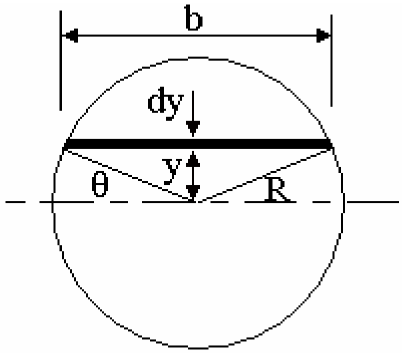
MECHANICS OF SOLIDS D209 SOLUTIONS 2004

1. (a) Derive from first principles, the second moment of area of a solid circular section about a diameter.



SOLUTION

The elementary strip has a second moment of area of $dI = by^2dy$ about the diameter.

$B = 2R\cos\theta \qquad y = R\sin\theta \qquad dy = R\cos\theta\, d\theta$

$dI = 2R\cos\theta\ (R\sin\theta)^2\ R\cos\theta\, d\theta = 2R^4\cos^2\theta\sin^2\theta$

$$I = 2R^4\int_0^{2\pi}\cos^2\theta\sin^2\theta$$

$$I = 2R^4\left[-\frac{\sin\theta\cos^3\theta}{4} + \frac{\cos\theta\sin\theta + \theta}{8}\right]_0^{2\pi}$$

$$I = 2R^4\left(\left[-\frac{\sin 2\pi\cos^3 2\pi}{4} + \frac{\cos 2\pi\sin 2\pi + 2\pi}{8}\right] - \left[-\frac{\sin 0\cos^3 0}{4} + \frac{\cos 0\sin 0 + 0}{8}\right]\right)$$

$$I = 2R^4\left(\left[-0 + \frac{0+2\pi}{8}\right] - \left[-0 + \frac{0+0}{8}\right]\right) = 2R^4\left[\frac{2\pi}{8}\right] = \frac{\pi R^4}{4} \text{ or } \frac{\pi D^4}{64}$$

The integration should be done by any method the student knows.

1 (b) A steel wire 3 mm diameter is wound onto a drum. Calculate the minimum diameter of the drum such that no permanent deformation (bending) occurs in the wire. (E = 200 GPa and the yield stress is 400 MPa)

SOLUTION

$M/I = E/R = \sigma/y$ Assuming the radius of curvature is the minimum radius then σ is the yield stress.

$R = Ey/\sigma \qquad y = 1.5 \times 10^{-3}$

$R = 200 \times 10^9 \times 1.5 \times 10^{-3}/400 \times 10^6 = 0.75$ m The minimum diameter is 1.5 m

1 (c) Calculate the torque required to turn the drum assuming no friction and no tension in the wire.

SOLUTION

The bending moment that produces the bending stress in the wire is assumed to be the torque required to turn the drum.

$M = EI/R$

$I = \pi \times (3 \times 10^{-3})^4/64 = 3.976 \times 10^{-12}$ R= 0.75m

$M = 200 \times 10^9 \times 3.976 \times 10^{-12} / 0.75 = 1.06$ Nm