D209 Q9 2001

(a) Write down Euler's equation for the critical load on a column.

(b) A vertical column is a tee section built in at the base and unrestrained at the end. The dimensions are shown.

(i) Where must the axial load be applied in order to avoid bending?

(ii) Calculate the length of the column if it buckles when the compressive stress reaches 400 MPa. Take E = 200 GPa.

(a) Euler's critical load is given by $F = n^2 \pi^2 \frac{EI}{L^2}$ n = mode E = modulus of elasticity I = second moment of area about axis of least resistance. L = length.

A column built in at one end and unconstrained at the other would have a mode n = 0.5

(b) (i) The load should be applied at the centre of area.
(b) (ii) E = 200 GPa compressive stress = 400 MPa

Cross sectional area A = $(30 \times 5) + (25 \times 5) = 275 \text{ mm}^2$ Axial force = stress x A = 400 x 275 = 110 kN

Now find the axis of least resistance by finding the second moment of area about the vertical and horizontal centre lines. Second Moment of Area about the vertical centre line. $I = (5 \times 35^3)/12 + (25 \times 5^3)/12 = 11.51 \times 10^3 \text{ mm}^4$

Second Moment of Area about horizontal centre line. First find the centroid using first moments of area and the base as the datum.

	Area	y	Αy
Flange	150	27.5	4125
Web	125	12.5	1562.5
Total	275		5687.5
T for an	-560	75/275 - 20	602 mm fr

 \overline{y} for section = 5687.5/275 = 20.682 mm from bottom (position of centroid)

Next find I about the bottom. Solve by finding I for outer rectangle and subtracting I for missing parts. $I_o = BD^3/3 = 30 \times 30^3/3 = 270 \times 10^3 \text{ mm}^4$ $I_m = 2 \times bd^3/3 = 130.2 \times 10^3 \text{ mm}^4$ $I = I_o - I_m = 139.8 \times 10^3 \text{ mm}^4$

Now find the second moment of area about the centroid using the parallel axis theorem. $I_{gg} = I - A \bar{y}^2 = 139.8 \times 10^3 - 275 \times 20.682^2 = 22.17 \times 10^3 \text{ mm}^4$ This is double the figure for the other axis so the strut will bend about the vertical centre line.

$$F = n^{2} \pi^{2} \frac{EI}{L^{2}} = 110 \times 10^{3} \quad (0.5)^{2} \pi^{2} \frac{200 \times 10^{9} \times 11.51 \times 10^{-9}}{L^{2}} = 110 \times 10^{3}$$
$$\frac{5680}{L^{2}} = 110 \times 10^{3}$$

 $L^2 = 0.0516$ L = 0.227 m

Slenderness ratio = L/k $k = \sqrt{(I/A)} = \sqrt{(11.51 \times 10^3 / 275)} = 6.47 \text{ mm}$ SR = 227/6.47 = 35 and this shows that it is not a strut but a column but to solve the buckling load for a column we would need to know the ultimate compressive stress of the material and apply the Rankine Gordon method.

