D209 Q8 2001

(i) A timber beam has a section 50 mm wide and 75 mm deep. Calculate the maximum bending stress in it when the bending moment is 4 000 N m.

(ii) The beam is reinforced with steel plates 50 mm wide and 10 mm thick firmly bonded to the top and bottom layers. Calculate the maximum bending stress in the timber and steel when the bending moment is 4000 N m. E = 200 GPa for steel and 10 GPa for timber.

(i) For the wood on its own we have a neutral axis through the centroid and this is at the middle.

M = 4 kNm $I_{gg} = BD^3/12 = 50 \times 75^3/12 = 1.758 \times 10^6$ mm^4 $=1.758 \times 10^{-6}$ m^4

$\sigma = My/I$ $y = D/2 = 37.5$ mm $\sigma = 4000 \times 0.0375/1.758 \times 10^{-6} = 85.33$ MPa

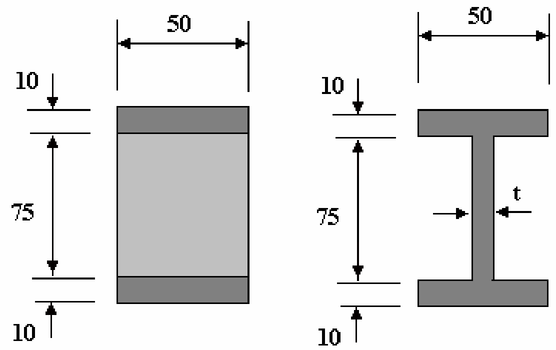
(ii) The bending equation states $M/I = E/R = \sigma/y$ so $y/R = \sigma/E$

At the interface, y and R must be the same for both materials so it follows that $\sigma_t = \sigma_s E_t/E_s$ where t refers to timber and s to steel.

If the beam was made entirely out of steel, it would have to have the same flexural stiffness (EI) as the composite beam while retaining the same vertical dimension for the interface.

Since $I = B\int y^3\,dy$ it is necessary to maintain the same y values so B must be changed in the ratio of the values of E.

The width of an equivalent steel web must be $t = 50 \times E_t / E_s = 50 \times 10/200 = 2.5$ mm



Now calculate the I value for the equivalent beam. $I = 50 \times 95^3/12 - 47.5 \times 75^3/12 = 1.9025 \times 10^{-6}$ m^4

The stress at y = 37.5 mm $\sigma = My/I = 4000 \times 0.0375/1.9025 \times 10^{-6} = 78.845$ MPa
The stress in the timber at this level will be different because of the different E value.
$\sigma_t = \sigma_s E_t/E_s = 3.942$ MPa

The stress at y = 47.5 mm will be the stress at the edge of the steel.
$\sigma_s = My/I = 4000 \times 0.0475/1.9025 \times 10^{-6} = 99.87$ MPa