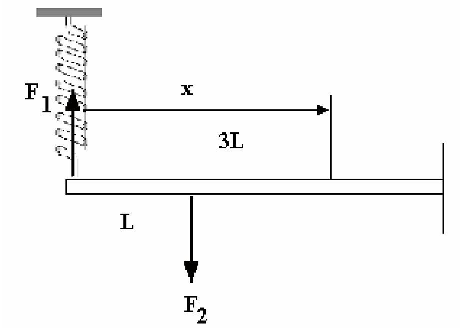
D209 Q3 2001

(a) Explain what is meant by a statically indeterminate beam and explain how to find the reactions.

b) A uniform horizontal cantilever is supported at the free end with a spring of stiffness k with zero force at zero deflection. A downward point load is applied at 1/3 of the length from the free end. Derive an expression for the deflection at the free end.




(a) The forces acting on a statically indeterminate beam cannot be resolved without some additional information. An example of this is a propped cantilever. The moments cannot be solved unless the deflection at the prop is known. In the case below, we must know the spring rate in order to relate the force to the deflection. The reactions cannot be found until the deflection has been found. When this is done, the forces may be resolved to find the reactions.

(b) Bending Moment at x $M = F_1 x - F_2[x-L]$

$$EI\frac{d^2y}{dx^2} = F_1 x - F_2[x - L] \qquad EI\frac{dy}{dx} = F_1\frac{x^2}{2} - \frac{F_2}{2}[x - L]^2 + A$$

At x = 3L the gradient is zero

$$0 = F_1\frac{(3L)^2}{2} - \frac{F_2}{2}[3L - L]^2 + A \qquad 0 = F_1\frac{9L^2}{2} - \frac{F_2}{2}4L^2 + A = L^2\left(4.5F_1 - 2F_2\right) + A$$

$$A = -L^2\left(4.5F_1 - 2F_2\right)$$

Integrate again

$$EIy = F_1\frac{x^3}{6} - \frac{F_2}{6}[x - L]^3 - L^2\left(4.5F_1 - 2F_2\right)x + B$$

At x = 3L y = 0

$$0 = F_1\frac{(3L)^3}{6} - \frac{F_2}{6}[3L - L]^3 - L^2\left(4.5F_1 - 2F_2\right)3L + B$$

$$0 = \frac{27L^3}{6}F_1 - \frac{8L^3}{6}F_2 - \left(4.5F_1 - 2F_2\right)3L^3 + B$$

$$B = -\frac{27L^3}{6}F_1 + \frac{8L^3}{6}F_2 + \left(4.5F_1 - 2F_2\right)3L^3$$

$$EIy = F_1\frac{x^3}{6} - \frac{F_2}{6}[x - L]^3 - L^2\left(4.5F_1 - 2F_2\right)x - \frac{27L^3}{6}F_1 + \frac{8L^3}{6}F_2 + \left(4.5F_1 - 2F_2\right)3L^3$$

At the free end x = 0 and negative brackets are zero.

$$EIy = -\frac{27L^3}{6}F_1 + \frac{8L^3}{6}F_2 + \left(4.5F_1 - 2F_2\right)3L^3 \qquad EIy = -\frac{27L^3}{6}F_1 + \frac{8L^3}{6}F_2 + \left(13.5F_1 - 6F_2\right)L^3$$

$$EIy = L^3\left[\left(13.5 - \frac{27}{6}\right)F_1 + \left(\frac{8}{6} - 6\right)F_2\right]$$

$$EIy = L^3\left[9F_1 - \frac{14}{3}F_2\right]$$

The force $F_1 = k\,y$ (spring )

$$\frac{EIy}{L^3} = 9\,k\,y - \frac{14}{3}F_2 \qquad \frac{EIy}{L^3} - 9\,k\,y = -\frac{14}{3}F_2$$

$$y\left(\frac{EI}{L^3} - 9\,k\right) = -\frac{14}{3}F_2 \qquad y = -\frac{\frac{14}{3}F_2}{\frac{EI}{L^3} - 9\,k}$$