2. A shaft has a diameter of 50.05 mm and is an interference fit with a sleeve 60 mm outer diameter, 50 mm inner diameter and 60 mm long. Calculate the force needed to slide the sleeve on the shaft if the coefficient of friction is 0.25 . The elastic properties for both parts are the same with $\mathrm{E}=200 \mathrm{GPa}$ and Poisson's ratio $=0.3$


Consider the problem as two thick cylinders. The outer cylinder has a pressure p acting all over the inner surface. The inner cylinder has the same pressure p acting all over the outer surface.

Apply Lame's equation to the outer cylinder.
$\sigma_{\mathrm{R}}=\mathrm{a}-\frac{\mathrm{b}}{\mathrm{r}^{2}} \quad \sigma_{\mathrm{C}}=\mathrm{a}+\frac{\mathrm{b}}{\mathrm{r}^{2}}$
At radius $R_{0}$ the radial stress is zero. $0=a-\frac{b}{R_{o}^{2}} \quad a=\frac{b}{R_{o}^{2}}$
At radius $R$ the radial stress is compressive and equal to $p$
$-\mathrm{p}=\mathrm{a}-\frac{\mathrm{b}}{\mathrm{R}^{2}} \quad-\mathrm{p}=\frac{\mathrm{b}}{\mathrm{R}_{\mathrm{o}}^{2}}-\frac{\mathrm{b}}{\mathrm{R}^{2}}=\mathrm{b}\left(\frac{\mathrm{R}^{2}-\mathrm{R}_{\mathrm{o}}^{2}}{\mathrm{R}_{\mathrm{o}}^{2} \mathrm{R}^{2}}\right) \quad \mathrm{b}=-\mathrm{p} \frac{\mathrm{R}_{\mathrm{o}}^{2} \mathrm{R}^{2}}{\mathrm{R}^{2}-R_{o}^{2}}$
$a=-p \frac{R^{2}}{R^{2}-R_{o}^{2}} \quad \sigma_{C}=a+\frac{b}{r^{2}}=-p \frac{R^{2}}{R^{2}-R_{o}^{2}}-p \frac{R_{0}^{2} R^{2}}{\left(R^{2}-R_{o}^{2}\right) r^{2}}$
At radius $R$ the circumferential stress is
$\sigma_{C}=-p \frac{R^{2}}{R^{2}-R_{o}^{2}}-p \frac{R_{0}^{2} R^{2}}{\left(R^{2}-R_{o}^{2}\right) R^{2}}=\frac{-p}{R^{2}-R_{o}^{2}}\left(R^{2}+R_{o}^{2}\right)$
At radius $R$ the radial stress is $-p$ The circumferential strain is
$\varepsilon_{\mathrm{c}}=\frac{1}{\mathrm{E}}\left(\sigma_{\mathrm{c}}-v \sigma_{\mathrm{R}}\right)=\frac{1}{\mathrm{E}}\left[\left(\frac{-\mathrm{p}}{\mathrm{R}^{2}-\mathrm{R}_{\mathrm{o}}^{2}}\left(\mathrm{R}^{2}+\mathrm{R}_{\mathrm{o}}^{2}\right)\right)+v \mathrm{p}\right] \quad \varepsilon_{\mathrm{c}}=\frac{\mathrm{p}}{\mathrm{E}}\left(\frac{\mathrm{R}^{2}+\mathrm{R}_{\mathrm{o}}^{2}}{\mathrm{R}_{\mathrm{o}}^{2}-\mathrm{R}^{2}}+v\right)$
Now repeat the process for the inner cylinder.

$$
\varepsilon_{c}=\frac{p}{E}\left(\frac{R^{2}+R_{i}^{2}}{R_{i}^{2}-R^{2}}+v\right) \text { but if } R_{i}=0 \text { then } \varepsilon_{c}=\frac{p}{E}\left(\frac{R^{2}}{-R^{2}}+v\right)=\frac{p}{E}(-1+v)
$$

The circumferential strain is the same as the ratio $\Delta R / R \Delta R=\varepsilon_{c} R$
For the outer cylinder $\Delta R=\frac{p R}{E}\left(\frac{R^{2}+R_{o}^{2}}{R_{o}^{2}-R^{2}}+v\right)$ For the shaft $\Delta R=\frac{p R}{E}(-1+v)$
Add the two and we get the interference $\delta \delta=\frac{\mathrm{pR}}{\mathrm{E}}\left(\frac{\mathrm{R}^{2}+\mathrm{R}_{\mathrm{o}}^{2}}{\mathrm{R}_{\mathrm{o}}^{2}-\mathrm{R}^{2}}+v\right)+\frac{\mathrm{pR}}{\mathrm{E}}(-1+v)$
$\delta=\frac{\mathrm{pR}}{\mathrm{E}}\left[\frac{\mathrm{R}^{2}+\mathrm{R}_{\mathrm{o}}^{2}}{\mathrm{R}_{\mathrm{o}}^{2}-\mathrm{R}^{2}}+v-1+v\right] \quad \delta=\frac{\mathrm{pR}}{\mathrm{E}}\left[\frac{\mathrm{R}^{2}+\mathrm{R}_{\mathrm{o}}^{2}}{\mathrm{R}_{\mathrm{o}}^{2}-\mathrm{R}^{2}}+2 v-1\right]$
Rearrange to make p the subject $\mathrm{p}=\frac{\delta}{\frac{\mathrm{R}}{\mathrm{E}}\left[\frac{\mathrm{R}^{2}+\mathrm{R}_{\mathrm{o}}^{2}}{\mathrm{R}_{\mathrm{o}}^{2}-\mathrm{R}^{2}}+2 v-1\right]}$
$\mathrm{R}_{\mathrm{o}}=30 \mathrm{~mm} \quad \mathrm{R}=25 \mathrm{~mm} \quad \delta=0.025 \mathrm{~mm} \quad \mathrm{E}=200 \mathrm{GPa} \quad v=0.3 \mathrm{~L}=0.06 \mu=0.25$
Evaluate and $\mathrm{p}=38.87 \mathrm{MPa}$
The force normal to the interface is pA where $\mathrm{A}=2 \pi \mathrm{RL}=2 \pi \times 0.025 \times 0.06=9.425 \times 10^{-3} \mathrm{~m}^{2}$
Normal Force $=\mathrm{N}=\mathrm{pA}=366.3 \mathrm{kN}$
Friction force $=\mu \mathrm{N}=91.58 \mathrm{kN}$

