INSTRUMENTATION AND CONTROL

TUTORIAL 3 – TRANSFER FUNCTION MANIPULATION

This tutorial is of interest to any student studying control system engineering.

On completion of this tutorial, you should be able to do the following.

- Explain a basic open loop system.
- Explain a basic closed loop system.
- Explain the use of negative feedback
- Manipulate transfer functions.
- Explain the use of velocity feedback.
- Explain the affect of disturbances to the output of a system.
- Explain the use of proportional and derivative feedback.
- Reduce complex systems to a single transfer function.

If you are not familiar with instrumentation used in control engineering, you should complete the tutorials on Instrumentation Systems.

In order to complete the theoretical part of this tutorial, you must be familiar with basic mechanical and electrical science.

You must also be familiar with the use of transfer functions and the Laplace Transform (see maths tutorials).

Contents

1. Introduction

2. Transfer Function

3. Open Loop Systems

4. Summing Devices

5. Basic Closed Loop Control

6. Closed Loop System Transfer Functions

7. Feedback Processing
7.1 Velocity Feedback
7.2 Disturbances
7.3 Eliminating the Affect of a Disturbance
7.4 Using Proportional and Differential Control In the Feedback Path

8. Simplifying Complex Systems

1. *Introduction*

The function of any control system is to automatically regulate the output and keep it at the desired value. The desired value is the input to the system. If the input is changed the output must respond and change to the new set value. If something happens to disturb the output without a change to the input, the output must return to the correct value. Here is a short list of some of the things we might be controlling.

The speed or angle of a motor (electric or hydraulic)
The speed or position of a linear actuator (e.g. in robotics, ship's stabilisers, aircraft control etcetera)
The temperature of an oven (e.g. heat treatment)
The pressure of a vessel (e.g. a steam boiler)
The quantity inside a container (e.g. metering contents in a vessel)
The flow of solids, liquids and gases (e.g. controlling the steam flow to a turbine)

There are some basic properties and terminology used with systems which we should examine next.

2. *Transfer Function*

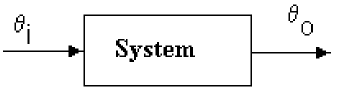

Figure 1

Any item in any system may be represented as a simple block with an input and output as shown. In general terms, the input is designated θ_i and the output θ_o.

The ratio of output over input is often shown as $G = \theta_o/\theta_i$. When the model is a differential equation the Laplace transform is used and this introduces the complex operator s. In this case G is called the ***Transfer Function*** and strictly we should write

$$\mathbf{G(s) = \frac{\theta_o(s)}{\theta_i(s)}}$$

If G is a simple ratio, it is still a Transfer function but if the model is not a simple ratio and cannot be transformed, it should not strictly be called a transfer function. You should study the tutorial on Laplace and Fourier transforms in the maths section in order to fully appreciate this tutorial.

3. *Open Loop Systems*

A system with no regulation is called an open loop system. For example a typical instrument system (see tutorials on instrumentation) is an open loop system with an input and output but no control action at all.

Let's take a d. c. servo motor as an example (see the tutorial on electric actuators). The speed of the servo motor depends on the voltage and current supplied to it. A typical system might use a potentiometer which you turn to an angle θ_i (the input) to produce a voltage V and this is amplified with a power amplifier producing electric power P that drives the motor at speed N (the output).

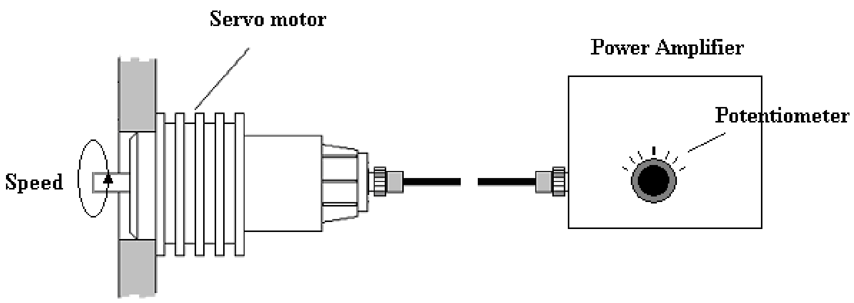

Figure 2

The block diagram looks like this.

Figure 3

The block diagram show that the signal path from input to output is a linear chain not forming any loop so this why it is called an ***Open Loop System.***

WORKED EXAMPLE No. 1

The speed of an electric motor is directly proportional to voltage such that N = 20V where V is in Volts and N in rev/min. The motor is controlled by a power supply which has an output voltage related to the position of the control knob by $V = 2\,\theta_i$ where V is in Volts and θ_i is in degrees.
Draw the block diagram and deduce the overall transfer function. Determine the output speed when the knob is set to 60^o.

SOLUTION

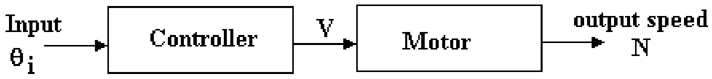

Figure 4

$G = N/\theta_i \quad = 2\theta_i \;\times 20 = 40$ rev/min per degree $N = 40 \times 60 = 2\,400$ rev/min

SELF ASSESSMENT EXERCISE No. 1

A simple control system consists of a potentiometer with a transfer function of 0.02V/mm in series with an amplifier with a gain of 12, in series with a V/I converter with a transfer function I = 0.5V where V is in volts and I in mA. The output current is amplified with a gain of 1200 and the output current supplied to an electro-magnetic torque arm which produces 3 Nm per Amp.

Draw the block diagram and deduce the overall transfer function. (0.432 Nm/mm)

Determine the input position of the potentiometer in mm which produces a torque output of 60 Nm.
(138.9 mm)

Consider the example of the servo motor again. Suppose the motor drives a load and that the load suddenly increases. This would make the motor slow down as there would not be enough power to keep it at the original speed. We would now have an error between the speed selected with the potentiometer and the actual speed of the motor. To bring the speed back to the correct value, we have to turn up the power and do this automatically we need a closed loop system. Open loop systems are incapable of maintaining a correct output in all but the simplest cases.

4. *Summing Devices*

In order to regulate any control system we must determine the error between the output and the input. This is done with a summing device and the symbol for this is shown in the left diagram. These devices may be electrical (e.g. a simple differential amplifier), pneumatic (e.g. a differential pressure cell) or mechanical. We can put a Plus (+) or minus (-) sign in the symbol to show if it is adding or subtracting and the symbol can be used with more than one signal as shown in the right diagram. In modern digital systems it is a simple case of adding or subtracting the numbers stored in registers.

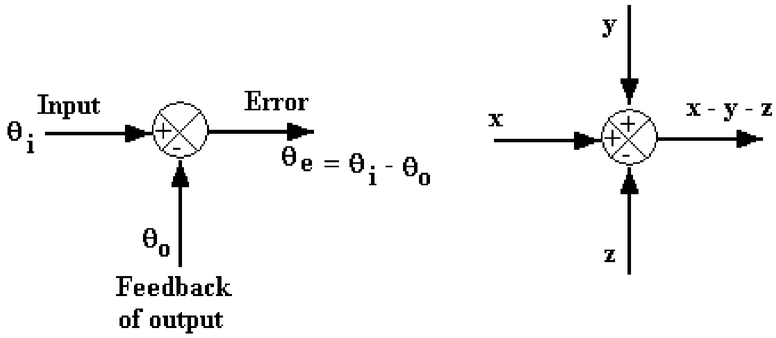

Figure 5

5. *Basic Closed Loop Control*

Consider the example of the servo motor again. This time suppose we wish to control the angle of the shaft θ_o. The input potentiometer produces a voltage V_i and the output potentiometer produces a voltage V_o to represent the angle of the shaft. If the two voltages are the same, the shaft is at the correct angle. If there is an error, the voltages are different. The differential amplifier acts as the summing device and produces a voltage V_e representing the error. The error is supplied to the power amplifier and power is sent to the motor to rotate it in the direction that corrects the angle. When the voltages are equal again, the error is zero and no power is supplied to the motor so it stops. Error in either direction can be corrected if the power amplifier is capable of producing positive and negative current.

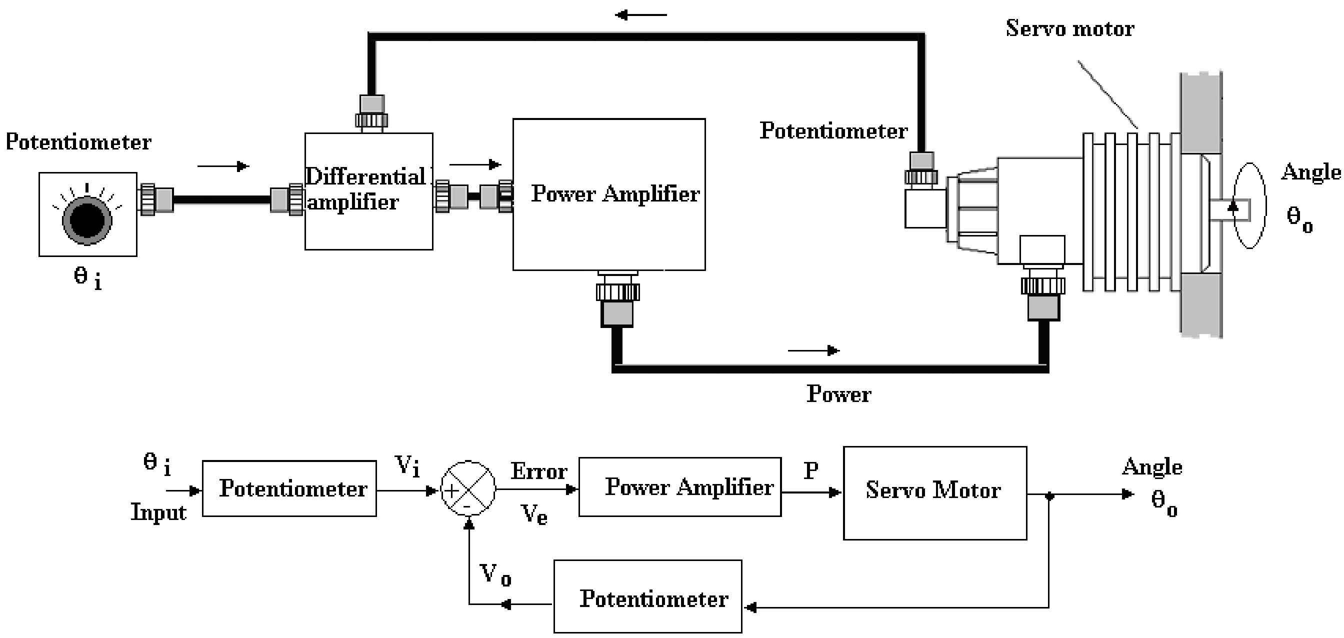

Figure 6

This description is somewhat over simplified and does not explain an actual working system. The motor would have difficulty staying at the correct angle if there is a load trying to turn it. Note how the voltage from the output potentiometer is fed back to the summing device so that the error is $V_e = V_i - V_o$. This is ***Negative Feedback*** and this is essential to make the system respond to the error.

6. *Closed Loop System Transfer Functions*

The most basic block diagram for a closed loop system is shown below. The main block is an open loop system with a transfer function G_{ol}. This relates the error and the output so that $G_{ol} = \theta_o/\theta_e$.

The transfer function for the closed loop system is $G_{c.l}$. This relates the input θ_i and output θ_o.

The error is obtained by comparing the output value with the input value in the signal summing device. This produces the result $\theta_e = \theta_i - \theta_o$ and because θ_o is subtracted, this idea is called ***Negative Feed Back***. The block diagram shows that the signal passes around a closed loop hence the name ***Closed Loop System.***

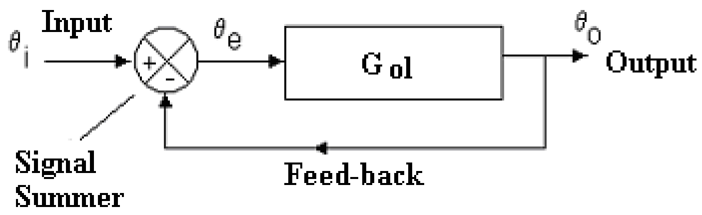

Figure 7

The system shown is said to have ***Unity Feed Back*** as there is no processing in the feed back path. The following result will be used many times in later tutorials on system analysis.

$$G(s) = \frac{\theta_o}{\theta_e}(s) \quad \text{substitute } \theta_e = \theta_i - \theta_o$$

$$G(s) = \frac{\theta_o}{\theta_i - \theta_o} \text{ divide the top and bottom by } \theta_o$$

$$\frac{1}{\frac{\theta_i}{\theta_o} - 1} \quad \text{rearrange} \quad \frac{\theta_i}{\theta_o} - 1 = \frac{1}{G(s)}$$

$$\frac{\theta_i}{\theta_o} = \frac{1}{G(s)} + 1 = \frac{1 + G(s)}{G(s)} \quad \text{invert} \quad \frac{\theta_o}{\theta_i} = \frac{G(s)}{1 + G(s)} = \frac{1}{\frac{1}{G(s)} + 1}$$

The transfer function for the closed system is hence

$$\frac{\theta_i}{\theta_o} = \frac{1}{\frac{1}{G(s)} + 1}$$

This is the transfer function for closed loop unity feed back system. In practice, at the very least, we have a transducer to measure the output so we should show a block in the path representing the transducer. In addition we may put other signal conditioners and processors in the path such as an amplifier or attenuator.

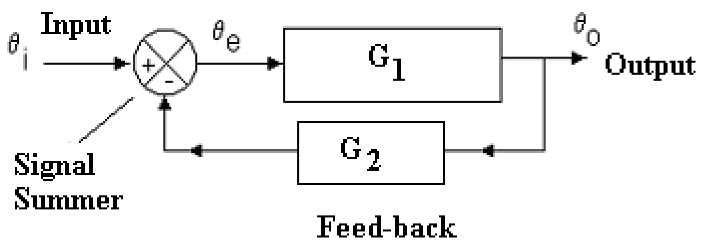

Figure 8

G_1 is the OPEN LOOP transfer function and is in the forward path. G_2 is in the feed back path.

The open loop transfer function is related to G_1 and G_2 as follows.

$$G_1 = \frac{\theta_o}{\theta_e} \qquad \theta_e = \theta_i - G_2\theta_o$$

$$G_1 = \frac{\theta_o}{\theta_i - G_2\theta_o}$$

$$(G_1\theta_i - G_1G_2\theta_o) = \theta_o$$

$$G_1\theta_i = G_1G_2\theta_o + \theta_o$$

$$G_1\theta_i = \theta_o(1 + G_1G_2)$$

The transfer function for the closed system is hence

$$G = \frac{\theta_o}{\theta_i} = \frac{G_1}{1 + G_1G_2}$$

WORKED EXAMPLE No. 2

The transfer function for a hydraulic system comprising of a hydraulic valve and actuator is

$$G(s) = \frac{1}{Ts}$$

Write down the closed loop transfer function.

SOLUTION

$$G(s)(\text{closed Loop}) = \frac{1}{\frac{1}{G(s)} + 1} = \frac{1}{Ts + 1}$$

WORKED EXAMPLE No. 3

The transfer function for a hydraulic system comprising of a hydraulic valve and actuator with inertia attached is

$$G(s) = \frac{1}{T^2s^2 + 2\delta Ts}$$

Write down the closed loop transfer function.

SOLUTION

$$G(s)(\text{closed Loop}) = \frac{1}{\frac{1}{G(s)} + 1} = \frac{1}{T^2s^2 + 2\delta Ts + 1}$$

SELF ASSESSMENT EXERCISE No. 2

1. Find the closed loop transfer function for the system shown below.

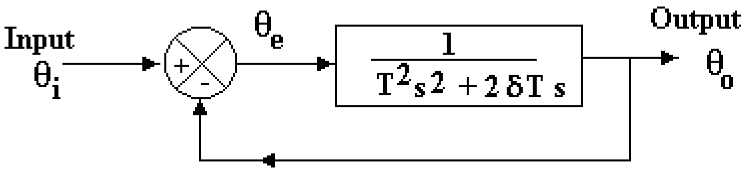

Figure 9

2. Find the closed loop transfer function for the system shown below.

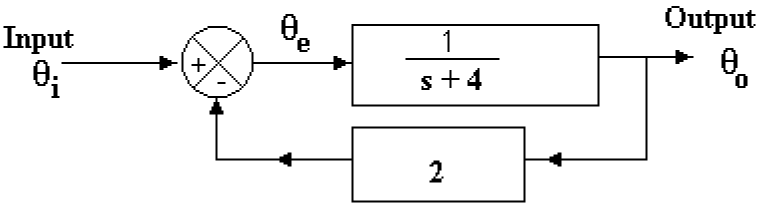

Figure 10

3. Find the closed loop transfer function for the system shown below.

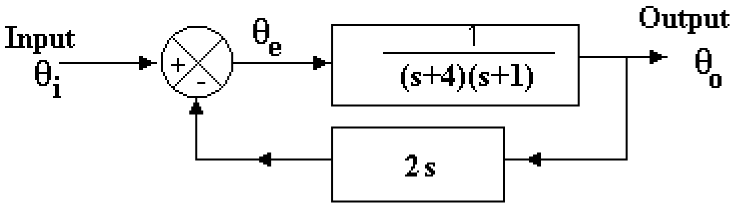

Figure 11

7. Feedback Processing

7.1 Velocity Feedback

Velocity feedback is widely used to stabilise a system which tends to oscillate. This is not the same as derivative control covered later. If the output of a system is motion x_o then the rate of change dx_o/dt is a true velocity but the idea can be used for any output. Consider the block diagram below.

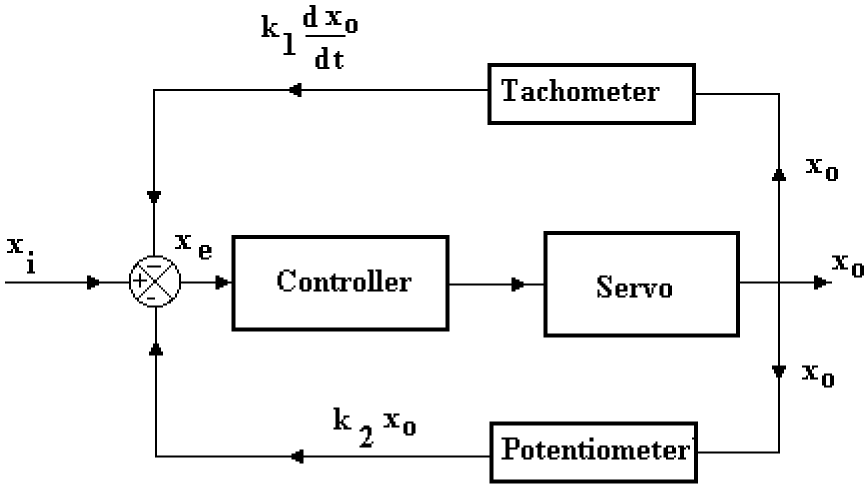

Figure 12

A typical electrical servo system uses position sensing and velocity feedback. The velocity signal is derived from a tacho-generator or some other suitable speed measuring device. The resulting signal is compared with the input and the output as shown.

The error signal is

$$x_e = x_i - k_2 x_o - k_1 \frac{dx_o}{dt}$$

When a sudden (step) change is made, the error is a maximum and so the output changes very rapidly. The velocity feedback is hence greatest at the start. The effect of the feedback is to reduce the error in a manner directly proportional to the velocity. When the output is static the feedback is zero and so no error results from it. The feedback has the same affect as damping and if the complete analysis is made, we would see that control over k_1 gives control over damping. This is useful in stabilising an oscillatory system.

WORKED EXAMPLE No. 4

The diagram shows a closed loop system with velocity and negative feed-back. The transfer functions for the system is

$$G(s) == \frac{k}{T^2s^2 + 2\delta Ts}$$

Derive the closed loop transfer function.

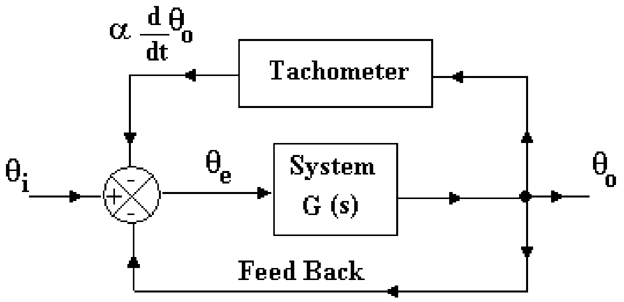

Figure 13

SOLUTION

The open loop transfer function is

$$G(s) = \frac{k}{T^2s^2 + 2\delta Ts}$$

$$\theta_e = \theta_i - \theta_o - \alpha s\theta_o = \theta_i - \theta_o(\alpha s + 1)$$

$$\theta_o = G\theta_e = G\{\theta_i - \theta_o(\alpha s + 1)\}$$

$$\theta_o = G\theta_i - G\theta_o(\alpha s + 1)$$

$$\theta_o + G\theta_o(\alpha s + 1) = G\theta_i$$

$$\theta_o\{1 + G(\alpha s + 1)\} = G\theta_i$$

$$\frac{\theta_o}{\theta_i} = \frac{G}{1 + G(\alpha s + 1)} = \frac{1}{\frac{1}{G} + (\alpha s + 1)}$$

Now substitute

$$G = \frac{k}{T^2s^2 + 2\delta Ts}$$

$$\frac{\theta_o}{\theta_i} = \frac{1}{\frac{1}{\frac{k}{T^2s^2 + 2\delta Ts}} + (\alpha s + 1)} = \frac{1}{\frac{T^2s^2 + 2\delta Ts}{k} + (\alpha s + 1)}$$

$$\frac{\theta_o}{\theta_i} = \frac{k}{T^2s^2 + 2\delta Ts + k(\alpha s + 1)} = \frac{k}{T^2s^2 + 2\delta Ts + k\alpha s + k}$$

$$\frac{\theta_o}{\theta_i} = \frac{k}{T^2s^2 + s(2\delta T + k\alpha) + k}$$

This is the closed loop transfer function

The term with s is the effective damping term and as can be seen this is affected by the value of $k\alpha$.

SELF ASSESSMENT EXERCISE No. 3

The diagram shows how the arm of a robot is controlled using a controller and motor with position and velocity feed-back. Determine the closed loop transfer function for the system.

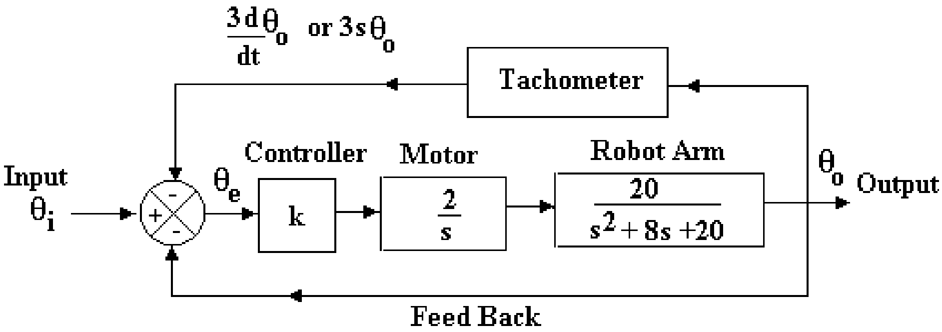

Figure 14

7.2 Disturbances

The affect of a disturbance on the output may be idealised on a block diagram as follows.

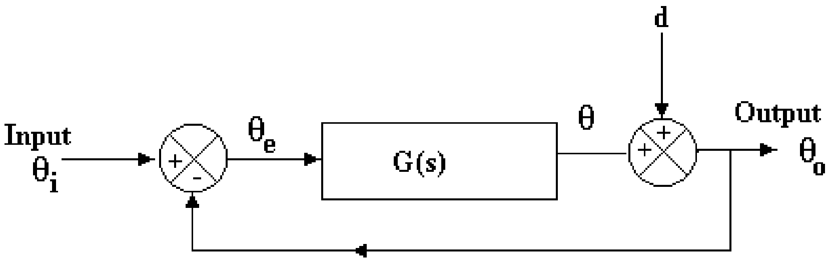

Figure 15

The disturbance d is added to the output θ to produce a new output θ_o. G is the forward path transfer function.

$$\theta_e = \theta_i - \theta_o \qquad \theta_o = \theta + d \qquad \theta = G\theta_e \qquad \theta_o = G\theta_e + d$$

$$\theta_o = G(\theta_i - \theta_o) + d = G\theta_i - G\theta_o + d$$

$$\theta_o + G\theta_o = G\theta_i + d \qquad \theta_o(1+G) = G\theta_i + d \qquad \theta_o = \frac{G\theta_i + d}{(1+G)}$$

WORKED EXAMPLE No. 5

A simple closed loop system consists of an amplifier with a gain of 10. For an input of 4 mA, show that the effect of a disturbance added to the output of magnitude i) 0 and ii) 2

SOLUTION

i) $d = 0$, $G = 10$ $\theta_i = 4$

$$\theta_o = \frac{G\theta_i + d}{(1+G)} = \frac{10 \times 4 + 0}{(1+10)} = \frac{40}{11}$$

i) $d = 2$, $G = 10$ $\theta_i = 4$

$$\theta_o = \frac{G\theta_i + d}{(1+G)} = \frac{10 \times 4 + 2}{(1+10)} = \frac{42}{11}$$

This shows that a disturbance of 2 produces an output error of 2/11.

7.3 Eliminating the Affect of a Disturbance

A special feed-back path is used to reduce or eliminate the affect of a disturbance added to the output. The idealised system is shown below.

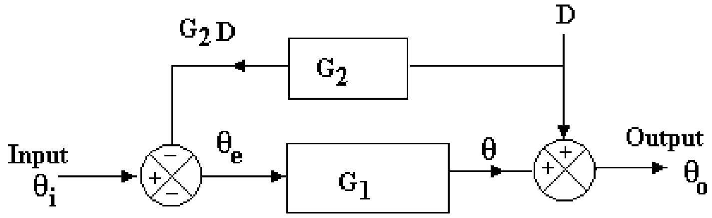

Figure 16

The disturbance (D) is processed through a transfer function G_2 and added to the input. G_1 is the forward path transfer function. G_2 is the feed-back path transfer function.

$$\theta = G_1\theta_e = G_1(\theta_i - G_2D)$$

$$\theta_o = \theta + D$$

$$\theta_o = G_1(\theta_i - G_2D) + D$$

$$\theta_o = G_1\theta_i - G_1G_2D + D$$

From the last line it can be seen that if $G_1G_2D = D$ then $\theta_o = G_1\theta_i$

The affect of the disturbance is completely removed when $G_1 = 1/G_2$

WORKED EXAMPLE No. 6

For the system described above, the forward path transfer function is

$$G_1(s) = \frac{4}{s+1}$$

Determine the transfer function for the feedback path which eliminates the affect of a disturbance.

SOLUTION

$$G_2(s) = \frac{1}{G_1(s)} = \frac{s+1}{4}$$

SELF ASSESSMENT EXERCISE No. 4

1. A simple closed loop system consists of two amplifiers in series one with a gain of 3 and one with a gain of 2. For an input of 6 mA, determine the output when a disturbance added to the output of magnitude i) 0 and ii) 3

2. The forward path transfer function for a controlled system is

$$G_1(s) = \frac{2}{3s^2+1}$$

Determine the transfer function for the feedback path which eliminates the affect of a disturbance.

7.4 Using Proportional and Differential Control In the Feedback Path

Let us consider that the feed-back transfer function is a P + D transfer function (Proportional plus differential).

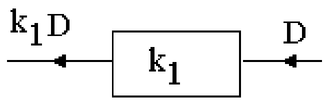

Figure 17 – Proportional

For a proportional system, output is directly proportional to input. In effect it is an amplifier or attenuator and k_1 is a simple ratio.

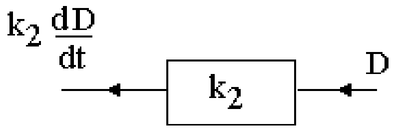

Figure 18 - Differential

For a differential block, the output is directly proportional to the rate of change of the input with time. A tachometer is an example of this. In Laplace form the output is k_2 sD. The units of k_2 must be seconds. We may write $k_2 = T\, k_1$ where T is called the ***Derivative Time***.

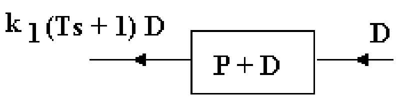

Figure 19 Proportional Plus Differential

For P + D it follows that the transfer function is $k_1(Ts + 1)$

WORKED EXAMPLE No. 7

Find the value of derivative time T and the constant which will eliminate the disturbance D in the system below. The transfer function G_1 is

$$G_1 = \frac{5}{s+2}$$

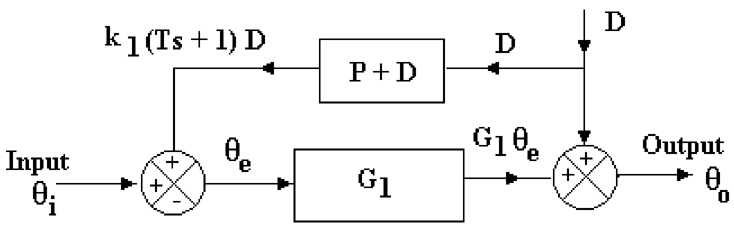

Figure 20

SOLUTION

$$G_2 = \frac{1}{G_1} = \frac{s+2}{5} = k_1(Ts+1) \qquad \frac{1}{5}(s+2) = k_1(Ts+1)$$

$$\frac{2}{5}\left(\frac{s+2}{2}\right) = 0.4(0.5s+1) = k_1(Ts+1)$$

Hence by comparison $k_1 = 0.4$ and $T = 0.5$

SELF ASSESSMENT EXERCISE No. 5

1. The forward path transfer function of a controlled system is

$$G(s) = \frac{6}{s + 3}$$

In order to eliminate disturbances added to the output, a P + D feedback path is used. Find the value of the derivative time and the constant required.
(0.333 sec and 0.5)

2. The forward path transfer function of a controlled system is

$$G(s) = \frac{10}{2s + 5}$$

In order to eliminate disturbances added to the output, a P + D feedback path is used. Find the value of the derivative time and the constant required.
(0.4 s and 0.5)

3. The forward path transfer function of a controlled system is

$$G(s) = \frac{8}{5s + 10}$$

In order to eliminate disturbances added to the output, a P + D feedback path is used. Find the value of the derivative time and the constant required.
(0.5 s and 1.25)

8. *Simplifying Complex Systems*

The symbol H is often used for feed back transfer functions but we can use any appropriate symbol to help us simplify complex circuits. The diagrams below show the stages in reducing a block diagram to one block with one transfer function.

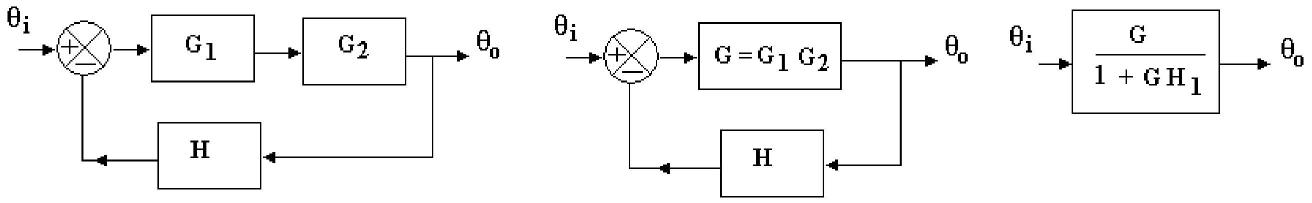

Figure 21

Here is a more complex one.

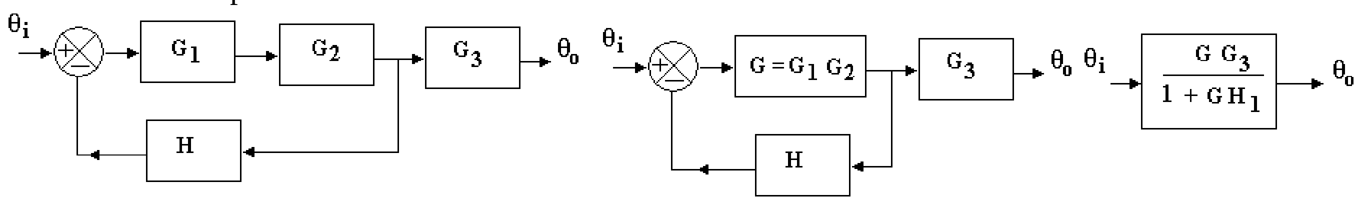

Figure 22

Here is an even more complex one.

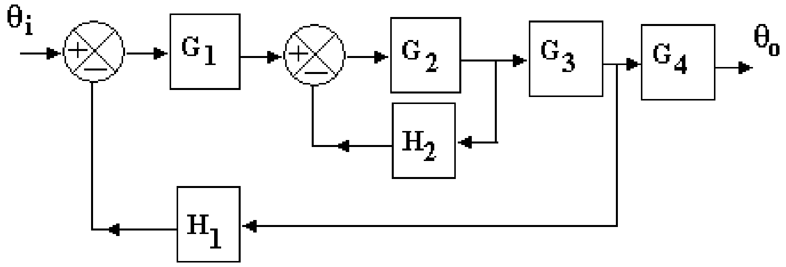

Figure 23

The technique now is to find the transfer function for the inner loop and work outwards as follows.

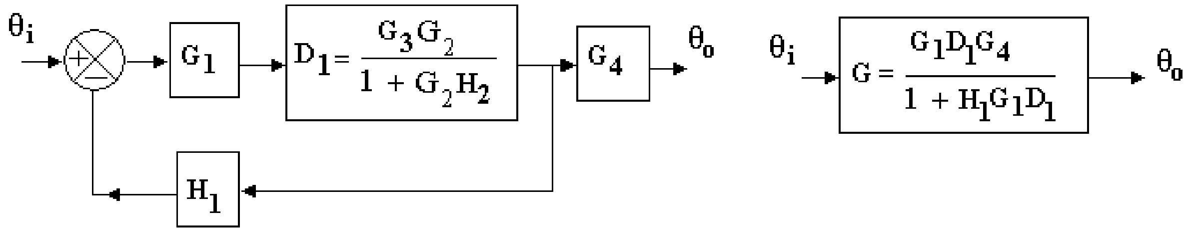

Figure 24

WORKED EXAMPLE No. 8

Derive the transfer function for the system below.

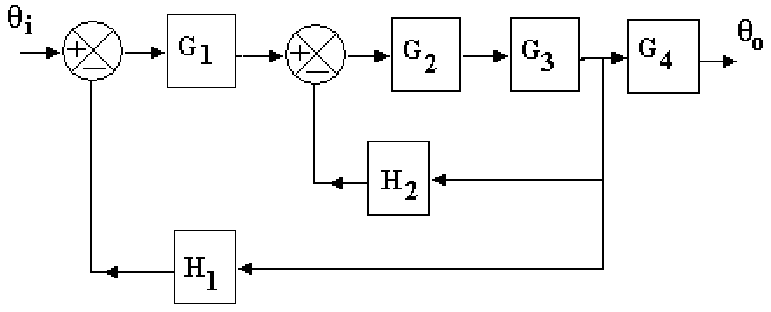

Figure 25

$$G_1 = \frac{3}{s} \quad G_2 = \frac{1}{4s+5} \quad G_3 = 4 \quad G_4 = \frac{1}{s} \quad H_1 = 5 \quad H_2 = 0.5$$

SOLUTION

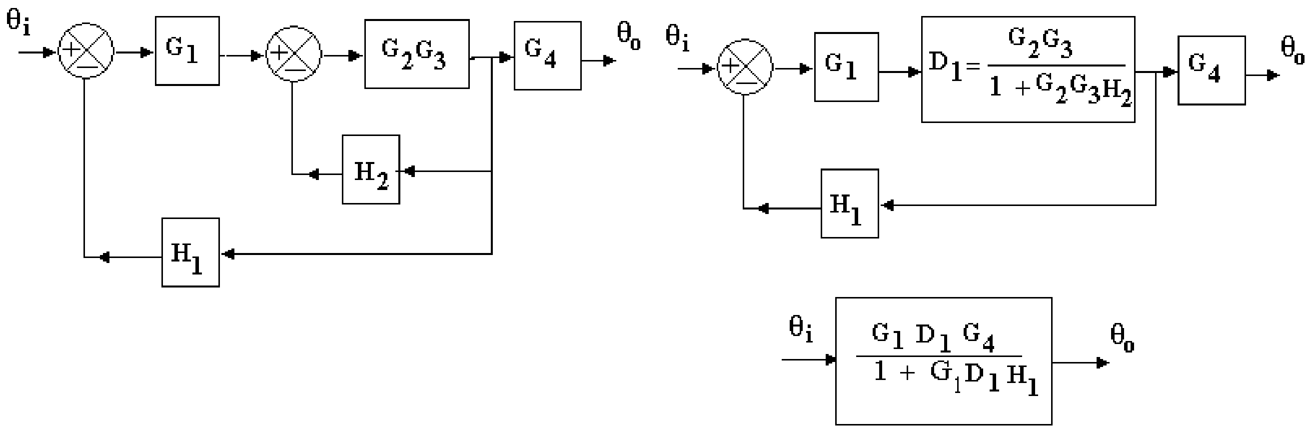

Figure 26

Put in the data and

$$D_1 = \frac{\frac{4}{4s+5}}{1 + \frac{4 \times 0.5}{4s+5}} = \frac{4}{(4s+5)\left(1 + \frac{2}{4s+5}\right)} = \frac{4}{(4s+7)}$$

$$G = \frac{\left(\frac{3}{s}\right)\left(\frac{1}{s}\right)D_1}{1 + \left(\frac{3}{s}\right)(5)(D_1)} = \frac{\left(\frac{3}{s^2}\right)\left(\frac{4}{4s+7}\right)}{1 + \left(\frac{15}{s}\right)\left(\frac{4}{4s+7}\right)} = \frac{\frac{12}{s^2(4s+7)}}{1 + \frac{60}{s(4s+7)}} = \frac{12}{s^2(4s+7) + 60s^2\left(\frac{4s+7}{4s+7}\right)}$$

$$G = \frac{12}{s^2(4s+7) + 60s^2} = \frac{12}{4s^2 + 7s^2 + 60s^2} = \frac{12}{4s^2 + 67s^2}$$

SELF ASSESSMENT EXERCISE No. 6

Derive the overall transfer functions for the systems below.

1.

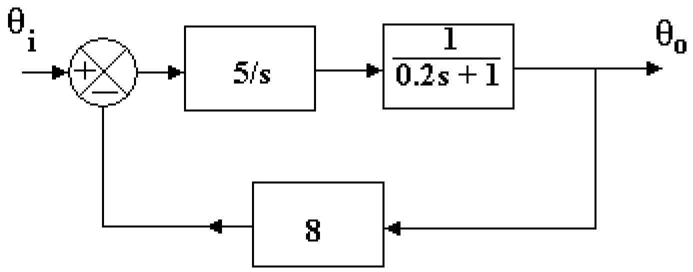

Figure 27

$5/(0.2s^2 + s + 40)$

2.

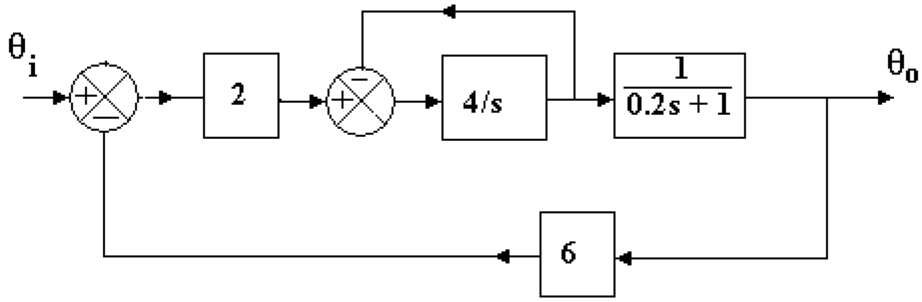

Figure 28

$8/(0.2s^2 + 1.8s + 52)$

3.

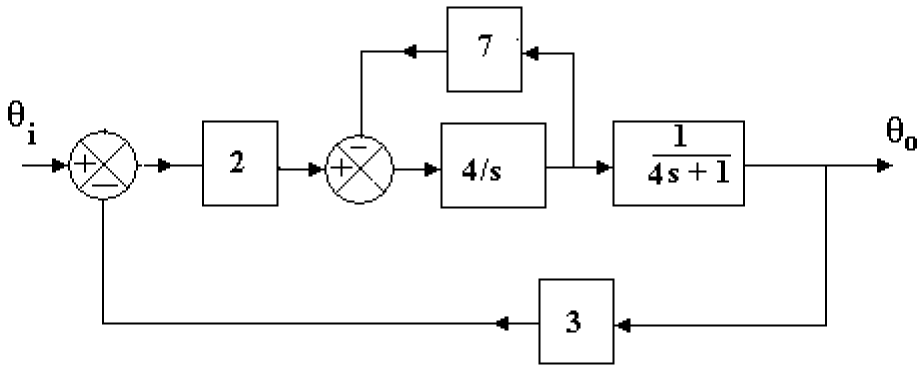

Figure 29

$8/(4s^2 + 113s + 52)$