INSTRUMENTATION AND CONTROL

TUTORIAL 2 – ELECTRIC ACTUATORS

This is a standalone tutorial on electric motors and actuators. The tutorial is of interest to any student studying control systems and set at NQF Level 3.

On completion of this tutorial, you should be able to do the following.

➢ Describe the advantages and disadvantages of electric actuation.
➢ Describe how a. c. motors are used for actuation.
➢ Explain how d. c. motors are used for actuation.
➢ Explain the basic principles of stepper motors.
➢ Explain how linear motion is obtained.
➢ Explain in some detail the principles of various types of d. c. servo motors.

In order to complete the theoretical part of this tutorial, you must be familiar with basic electrical science with instrumentation used in control engineering

Contents

1. Introduction

Electrically actuated systems are very widely used in control systems because they are easy to interface with the control systems which are also electric and because electricity is easily available unlike fluid power which requires pumps and compressors. The advantages of electric systems are

> ➢ Electricity is easily routed to the actuators; cables are simpler than pipe work
> ➢ Electricity is easily controlled by electronic units
> ➢ Electricity is clean.
> ➢ Electrical faults are often easier to diagnose.

The disadvantages of electric actuators are

> ➢ Electrical equipment is more of a fire hazard than other systems unless made intrinsically safe, in which case it becomes expensive.
> ➢ Electric actuators have a poor torque - speed characteristic at low speed.
> ➢ Electric actuators are all basically rotary motion and complicated mechanisms are needed to convert rotation into other forms of motion.
> ➢ The power to weight ratio is inferior to hydraulic motors.

There are three types of motors used in control applications.

> ➢ A. C. motors.
> ➢ D. C. motors.
> ➢ Stepper motors.

2. A. C. Motors

A. C. motors are mainly used for producing large power outputs at a fixed speed.
Typically these are 1 420 or 2 900 rev/min. Such motors are controlled by switching them on and off.

Increasingly, speed control is being used with A. C. motors on applications such as pumps where it is found to be more economical to control the flow rate by changing speed rather than by opening or closing a pipe line valve. Speed control is achieved electronically by varying the frequency or by chopping the power supply.
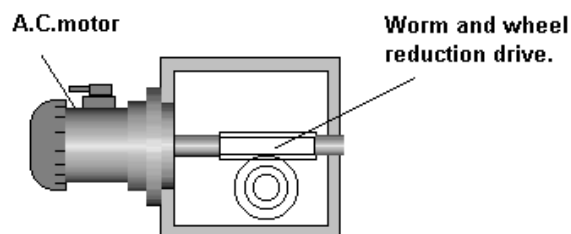

These motors are usually geared down in order to produce a greater torque and increase the control range. They may also have the rotation converted into linear movement by a lead screw mechanism.
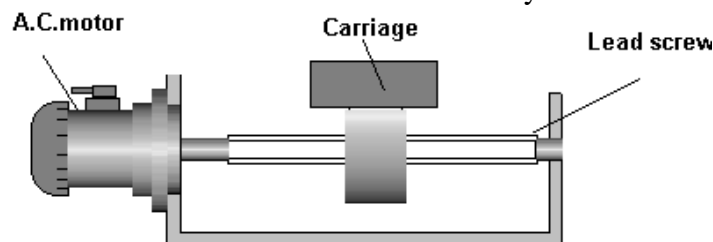
Lead screws are used to convert rotation into linear motion as shown. Rotation screws the carriage back and forth along the lead screw.

3.   D. C. Motors

Direct current motors are more widely used in control applications and they are usually referred to as *Servo Motors*. These are covered in detail later in the tutorial. The development of more powerful magnets is improving the power to weight ratio but they are still not as good as hydraulic motors in this respect. Servo motors usually have a transducer connected to them in order to measure the speed or angle of rotation. The diagram shows a typical arrangement.
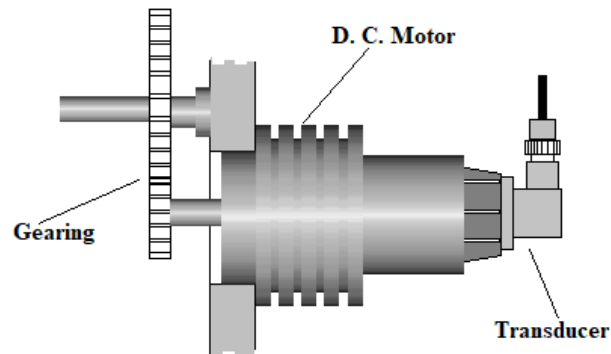
4.   Stepper Motors

Basically a stepper motor rotates a precise angle according to the number of pulses of electricity sent to it. Because there is confidence that the shaft rotates to the position requested, no transducer is needed to measure and check the position and so they are common on open loop systems.
There are 3 types of stepper motor in common use and these are

1. The Permanent Magnet Type.
*2. The Variable Reluctance Type.*
3. The Hybrid Type.

4.1   The Permanent Magnet Type

The rotor is a permanent magnet with a North and South poles as shown. Two pairs of poles are placed on the stator and energised to produce a pattern of
N - S - N - S (starting at the top). The rotor will take up a position in between the poles due to equal and opposite torques being exerted on it.
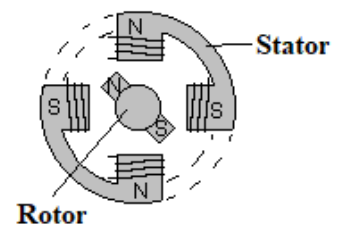
If the polarity of both pairs of poles are reversed the pattern will change to S - N - S - N and the rotor will flip 45° to a new position of balance. In order to obtain more steps, more pairs of poles are used but there are only two windings. Reversing the polarity of both windings moves the rotor on one step. Stepping is produced by simply reversing the polarity.
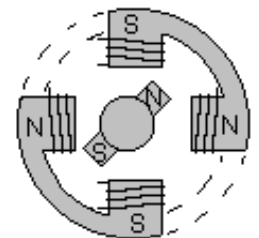
The rotor is held in position even when the poles are not energised. In order to obtain many steps, the poles are often stacked one behind the other and not in a single ring. The number of steps may also be increased by using a gear box on the output shaft.
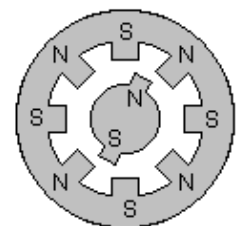
4.2.   *Variable Reluctance Type*

The rotor is constructed of soft iron with a number of teeth which are unequal in number to the number of poles on the stator. The stator has multiple poles which are energised by several separate phases. The diagram shows a system with three phases.
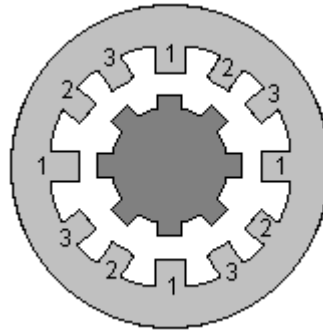
When a current is applied to the stator windings, the rotor aligns itself in the position of least magnetic reluctance. This position depends upon the number of phases energised.

The rotor retains very little magnetism so there is no holding torque when the current is removed. The number of steps is given by

$$N = \frac{SR}{S - R}$$

 S is the number of stator slots and R the number of rotor slots.

4.3   *Hybrid Motors*

Hybrid motors are a combination of the last two types. Each pole is divided into slots as shown. The rotor has two sets of slots, one behind the other with one set offset to the other by 1/2 slot pitch. The rotor is magnetised longitudinally. This produces a high resolution.
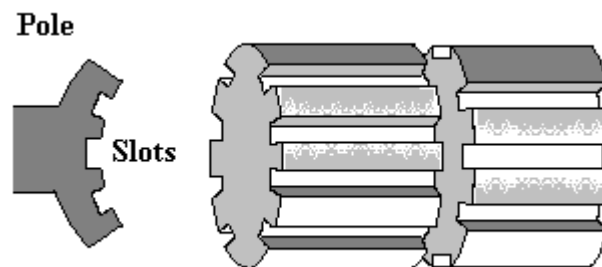
In general all stepper motors are controlled electronically.

5.   Linear Electric Actuators

In recent years, a range of linear electric actuators have been developed to perform functions similar to hydraulic and pneumatic cylinders. These are based on a motor driven lead screw. The motors may be AC or DC. The speed of the motor is re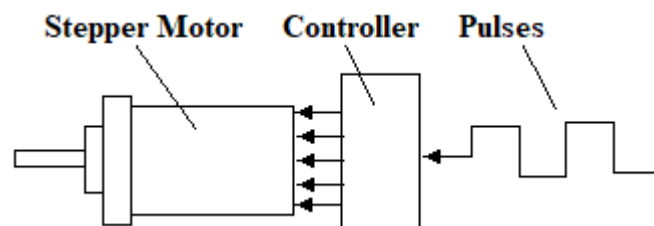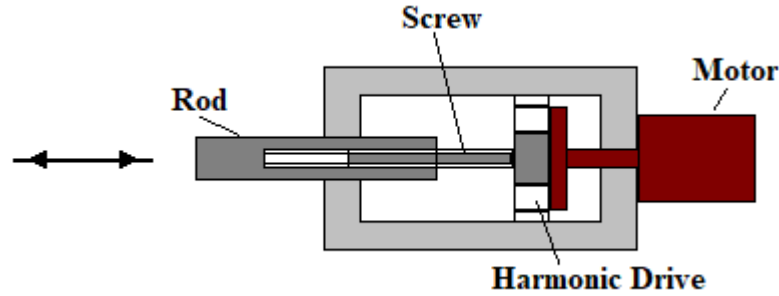duced with a compact gear box before driving the lead screw. Actuators have been developed with thrusts up to 15 kN and strokes up to 3 metres.

Now we will go on and examine the details of D.C. Motors.

6.   Detailed Analysis of D.C. Motors

The theory of electrical machines is based on two basic discoveries called the motor principle and the generator principle. In any given machine, the two go together.

6.1   The Motor Principle

A major discovery leading to the invention of the electric motor was that a conductor placed in a magnetic field experiences a force when current flows in it. Consider a conductor placed in a gap between the poles of a magnet. When current passes through the conductor, we have two magnetic fields, the circular lines around the conductor and the parallel lines between the poles. The lines of magnetism between the north and south poles would rather pass over the top of the conductor because both lines are in the same direction on top. The lines behave like elastic bands and force the conductor down. If the direction of either the current or the magnetic field is reversed, the force will act up.
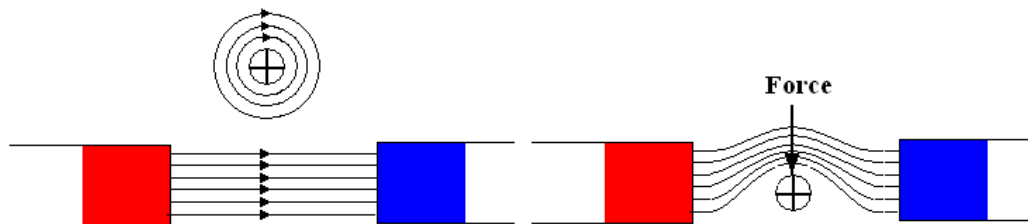
The force on the conductor is directly proportional to the current 'I' , the magnetic flux density 'B' and the length 'ℓ' of the conductor within the flux. This is the important equation for the force on a conductor.

$$\mathbf{F = B\,\ell I}$$

6.2   The Generator Principle

When a conductor moves at velocity v m/s through a flux of density B Tesla, an e. m. f is generated in the conductor such that $E = B\,\ell\,v$. This e. m. f opposes the flow of the applied current so a forward voltage is required to overcome it. This effect is produced in the conductors of motors as well as generators since the motor has moving conductors passing through a flux.

The conductor is part of a coil rotating at speed N rev/s and $v = 2\pi NR$

6.3 Motor Analysis

Torque on a Loop

This principle may be used to produce a simple electric motor. Consider a single loop coil of diameter D placed in the magnetic field as shown. The current flows into one side and out of the other. This produces a downwards force on one side and an upwards force on the other at a radius D/2. This produces a torque on the coil of T = F D/2. This will make the loop rotate and but when it turns 90° the radius is zero and the torque is zero so it will stop.
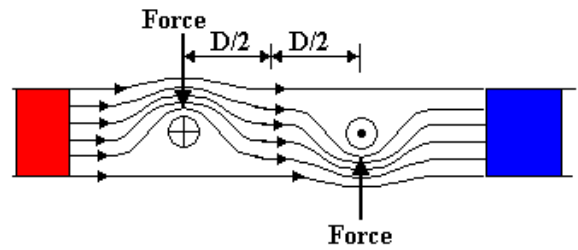
If we reverse the current as it passes the 90° position the torque will continue to make it rotate. Switching the direction of the current every half rotation will produce continuous rotation. This can be done with a split ring as shown.
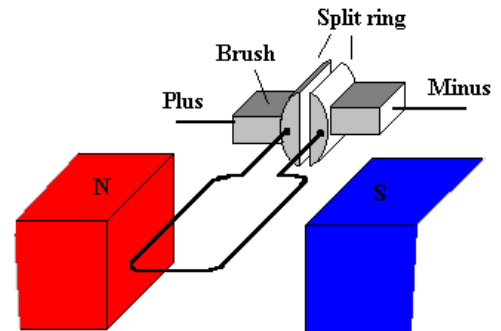
The design is further improved by using several loops and more magnets and switching the current to the loops within the flux. The split ring becomes a commutator with many segments. Each pair of segments is connected to a loop and each pair in turn becomes connected to the brushes as it rotates.

A practical D.C. motor has two main parts, the **Stator** and **Rotor**. Normally, many conductors are wound lengthwise in slots on the rotor as shown in the cross section. The magnets or poles are placed in the stator. When current is supplied to the loops, the magnetic field produced interacts with the field from the poles producing torque and rotation. In order to keeps the process going, the current must be switched to the loops moving into position under the poles. This may be done with a **Commutator** and **Brushes** but it can be done electronically.
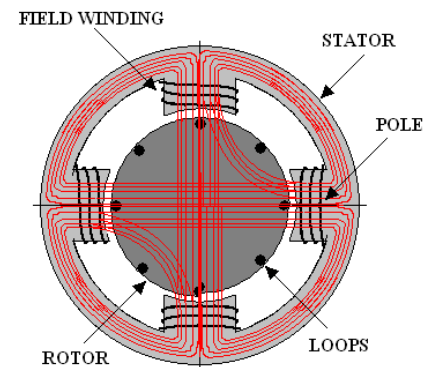
The pictures below show the main parts of a motor where the rotor is the armature.
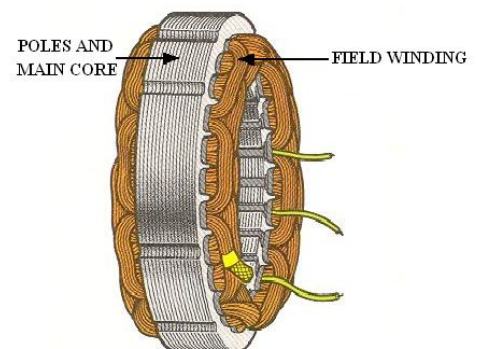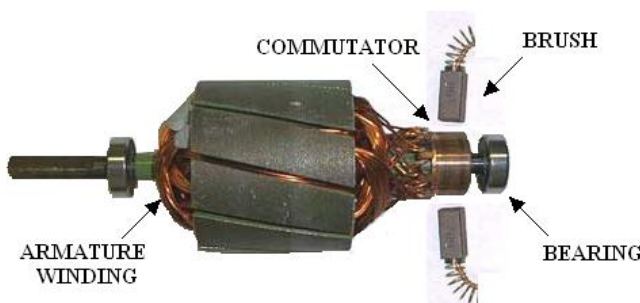
The magnetic poles may be permanent magnets but in more powerful motors they are electro-magnets with coils wound on the poles called the **Field Windings**. The stator and rotor form a magnetic circuit and the flux crosses the loops on the armature as shown. The strength of the magnetic flux can be changed by changing the current. The rotor in the diagram would be called the **Armature.**

 You can see the typical construction and explanation at this web location.
http://www.wisc-online.com/objects/index_tj.asp?objID=IAU9508

The next diagram shows a typical schematic representation of a D.C. motor. Current has to be supplied to the field windings to produce the magnetic flux. Current must be supplied to the armature to produce the torque. Note the brushes used to contact armature.

Terminology Used

I_a = armature Current
V = Supply voltage
E_a = EMF at the armature.
R_a = resistance of the armature coils.
$E_a = V - I_a R_a$
If the armature windings were perfect conductors then $E_a = V$
I_f = Field current
T = Torque
N = rotor speed in rev/s
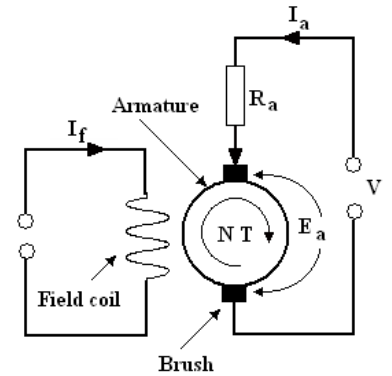
The conductors in a motor pass through the flux from the poles and so an e. m. f. is generated in them that opposes the flow of current from the source. The voltage applied at the terminals to make the motor rotate is V and this must be larger than E to overcome the resistance of the armature and brushes so it follows that

$$V = E_a + I_a R_a$$

If the conductor is part of a loop of radius D/2 (metres) then the torque produced is $T = F D/2$ (N m)

The force is $F = B \ell v$ and $B = \phi/A$ where ϕ is the flux in Weber and A the area of the flux path. Substitute for F and B and

$$T = \frac{\phi \lambda I D}{2A}$$

For a given motor the only variables are the flux and the armature current so the equation reduces to

$$T = k_1 I_a \phi \dots \dots (1)$$

The electric power consumed by the armature is $P = E_a I_a$

The mechanical power produced is $\quad P = \omega T = 2\pi N T$

Where ω is the speed in radians/s and N is the speed in rev/s. Note that $\omega = 2\pi N$

Equating mechanical and electric power we have

$$2\pi N T = I_a E_a \quad \text{or} \quad E_a = \frac{2\pi N T}{I_a} \dots \dots (2a)$$

Since

$$T = k_1 \phi I_a$$

We may substitute into (2a)

$$E_a = k_1 \phi 2\pi N T = K_2 \phi N \dots \dots (2)$$

Losses Due To Armature Resistance

The armature circuit has a resistance mainly due to the resistance of the copper windings and brushes. This is denoted R_a. The emf (E_a) of the armature is the terminal voltage V minus the resistance loss, hence:

$$E_a = V - I_a R_a \dots \dots (3)$$

Substitute (2) for E_a and

$$N = \frac{(V - I_a R_a)}{2\pi k_1 \phi} \dots \dots \dots (4)$$

Often the term $I_a R_a$ is small in comparison to V and a close approximation is

$$N = \frac{V}{k\phi}$$

These equations are the basis of the following work.

6.4   Constant Armature Current or Field Control

If I_a is kept constant it follows that

$$N = \frac{constant}{\phi} = \frac{constant}{I_f}$$

Since

$$T = k_1 \phi \, I_a$$

For constant I_a it follows that

$$T = constant \times \phi$$

Hence
$N = \frac{constant}{T}$
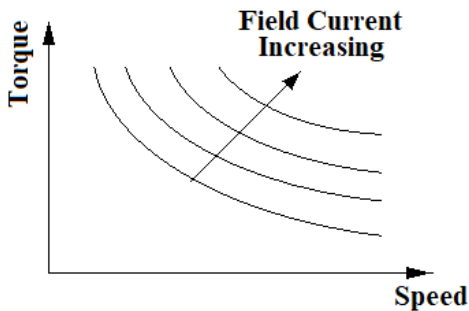
It follows that the torque and speed may be controlled by varying the field current. This has an advantage that a relatively large power may be controlled by a small field current and the power amplifier needed in the control circuit is relatively small. The diagram shows the relationship between torque and speed for constant field current.

6.5   Constant Field Current or Armature Control

In this case the field current is maintained constant or a permanent magnet is used to produce constant flux.

$$E_a = \frac{2\pi NT}{I_a} \quad \text{(equaion 2a)} \quad E_a \propto N\phi$$

If ϕ is constant then

$$E_a = 2\pi NK \dots \dots (5) \quad \text{and} \quad N = \frac{E_a}{2\pi K}$$

Substituting for N in equation (2a) gives $\;T = kI_a\;$ hence $\;T = k_t I_a$

$$I_a = \frac{T}{k_t} \dots \dots (6)$$

Equating (3) and (5) we have

$$E_a = V_a - I_a R_a \; = 2\pi NK = K_e \, N$$

Substitute (6) for I_a and

$$V_a - TR_a/k_t = N\,K_e$$

$$T = (V_a - Nk_e)k_t/R_a$$

$$\mathbf{T = C_1 V_a - C_2 N} \quad.........(7)$$
$$\mathbf{V_a = C_3 T + C_4 N}........ (8)$$

These are the equations commonly used to explain the steady state characteristic of a DC motor with armature control.
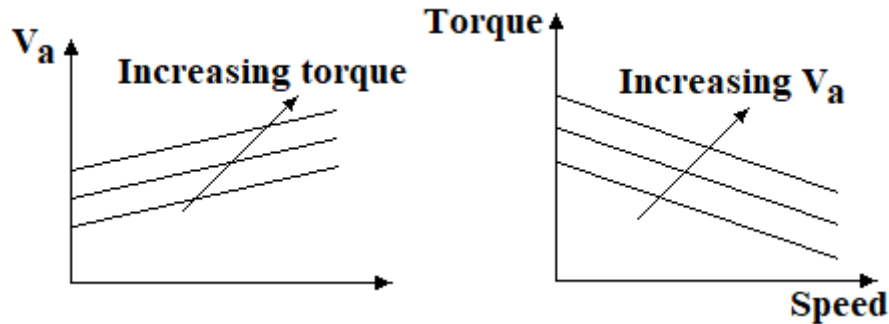
The diagrams above show equation 8 plotted for constant T and equation 7 for constant V_a.

6.6 Mixed Control

So far you have studied motors with separate field and armature windings and looked at the characteristics of these motors.

We will now study the characteristics of various motor configurations of the type mainly used on large D.C. Machines. In the previous work it was shown that

$$\mathbf{V = E_a + I_a\,R_a}$$

$$\mathbf{E = k_1\phi 2\pi N = kN\phi}$$

$$\mathbf{T = K_1\,\phi\,I_a}$$

E_a is the back e. m. f on the armature, ϕ is the flux per pole, T is the torque and N the speed (rev/s).

The Mechanical Power output of the motor is less than the Electrical Power at the terminals because of losses. The efficiency of the motor is defined as

$$\eta = \frac{\text{Mechanical Power}}{\text{Electrical Power}}$$

Mechanical Power = $2\pi NT$ and Electrical Power is terminal Volts × Amperes.

6.7 Series Motor
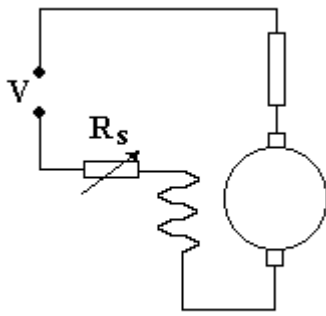
Series Motor

In this case, the field winding is in series with the armature. The same current flows through the armature and the field winding.

Equating mechanical and electrical powers we have

$$P = 2\pi NT = E_a I_a$$

Rearranging we have

$$T = \frac{E_a I_a}{2\pi N}$$

If the electric power is constant, $E_a I_a$ are constant so

$$T = \frac{E_a I_a}{2\pi N} = \frac{\text{Constant}}{N}$$

In this case for constant electrical power the relationship between torque and speed is inversely proportional as shown.
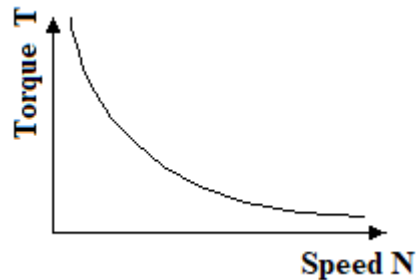
The torque - speed characteristic shows that at low torque (no load conditions) the motor is liable to over speed and become damaged.

At low speed there is a high torque (starting torque) which is ideal for servo applications.

WORKED EXAMPLE No. 1

A series wound motor produces 30 kW of mechanical power and is connected to a 250 V supply. The motor runs at 800 rev/min when under load. The load is reduced to 200 N m and a resistance of 0.5 Ω is connected in series with the motor. Assuming no energy losses, calculate the speed.

SOLUTION
Initial conditions

$$\text{Motor Curent} = I = \frac{\text{Power}}{\text{Volts}} = \frac{30\ 000}{250} = 120\ \text{A}$$

$$\text{Mechanicl Power} = \frac{2\pi NT}{60} = 30\ 000\ \text{A}$$

$$T = \frac{30\ 000 \times 60}{2\pi \times 800} = 358\ \text{Nm} = K_1 \phi I_a$$

Since flux is proportional to current and the field current is the armature current then

$$T = 358 = K_1 \phi I_a = K I_a^2 = K \times 120^2 \quad K = 0.02486$$

Final conditions using the same value for K

$$T = 200 = 0.02486 I_a^2 \quad I_a = \sqrt{\frac{200}{0.02486}} = 89.7\ \text{A}$$

Input Power

$$P = VI_a = 250 \times 89.7 = 22\ 423\ \text{W}$$

Loss in 0.5Ω resistance is

$$I_a^2 R = 89.7^2 \times 0.5 = 4\ 023\ \text{W}$$

Useful Power

$$22\ 423 - 4\ 023 = 18\ 400\ \text{W} = \frac{2\pi NT}{60}$$

$$N = \frac{18\ 400 \times 60}{2\pi N} = \frac{18\ 400 \times 60}{2\pi \times 200} = 878.5\ \text{rev/min}$$

SELF ASSESSMENT EXERCISE No. 1

1. A series motor is connected to a 500 V supply. The motor power output is 10 kW at 1 200 rev/min. The efficiency of the motor is 92 %.

   Determine the current and torque. (21.74 A and 79.58 N m)

2. A series motor has an armature resistance of 0.5 ohms and a field resistance of 1 ohm. It is connected to a 300 V supply and draws 50 Amperes from it. The motor runs at 1000 rev/min. The flux per pole is 0.03 Wb. The current is dropped to 25 Amperes and the flux per pole drops to 0.02 Wb. Determine the speed of the motor under these conditions. (1 750rev/min)

6.8   Shunt Motor
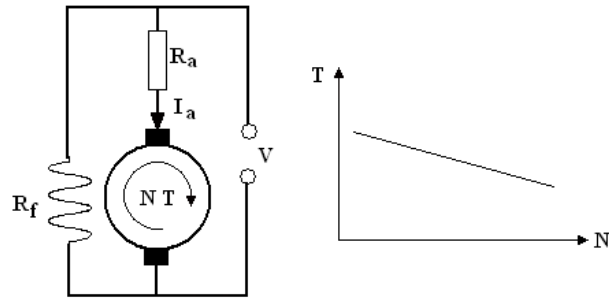
SHUNT MOTOR

In this case the field winding is connected across the supply. Since the field current is constant, the characteristics are those of an armature controlled motor covered earlier.

$$E_a = V - I_a R_a \qquad T = k_1 \phi I_a \qquad I_a = \frac{T}{k_1 \phi} \qquad E_a = V - \frac{T R_a}{k_1 \phi} = kN\phi$$

ϕ is constant so everything is a constant except T and N so for constant electrical power this reduces to

$$T = C_1 - C_2 N$$

This shows that at zero speed the starting torque is C_1 and as speed increases, the torque drops off. The ideal Torque - Speed characteristic is as shown. In reality the line is curved down due to other effects not considered.

WORKED EXAMPLE No. 2

Part 1.   A D.C. motor is shunt wound and is supplied with 500 V. The armature resistance is 1.0 Ω and the field winding has a resistance of 500 Ω. When running with a torque of 100 Nm, the motor takes 21 Amps. Determine the speed of the motor.

SOLUTION

The field is connected across the supply. The current taken by the field winding is

$$I_f = \frac{500}{500} = 1 \text{ A}$$

The current taken by the armature is hence 21 - 1 = 20 amps.

$$E_a = V - I_a R_a = 500 - 20 \times 1 = 480 \text{ V}$$

Equate electric power and mechanical power

$$E_a I_a = 480 \times 20 = 9\,600 \text{ W}$$

Assuming the conversion process is 100% the Mechanical power = 9 600

$$2\pi NT = 9\,600 \quad N = \frac{9\,600}{2\pi T} = \frac{9\,600}{2\pi \times 100} = 15.27 \text{ rev/s or } 916.6 \text{ rev/min}$$

Part 2.   The field current remains unchanged and the torque is increased to 120 Nm. Determine the new speed.

SOLUTION

$$T = k_1\phi I_a \text{ but the flux is constant so } T = KI_a$$

From the first set of data we have

$$100 = K \times 20 \quad \text{so } K = 5$$

For the second set of data

$$T = 120 = KI_a = 5\,I_a \quad I_a = \frac{120}{5} = 24\text{ A}$$

$$E = 500 - 24 \times 1 = 476\text{ V}$$

$$\text{Electric Power} = 476 \times 24 = 11\,424\text{ Watts}$$

Equating to mechanical power

$$11\,424 = 2\pi NT = 2\pi N \times 120$$

$$N = \frac{11\,424}{2\pi \times 120} = 15.15\text{ rev/s or } 909\text{ rev/min}$$

6.9   *Compound Motor*
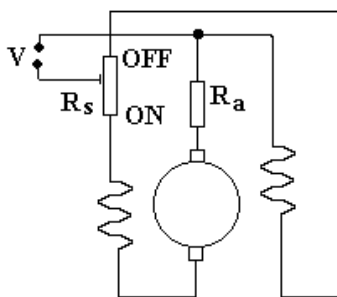
Compound Motor

The compound motor is a cross between the other two with both a parallel and series field winding. For constant electric power, the Torque - speed characteristic is between that of the other two.

SELF ASSESSMENT EXERCISE No. 2

1. A 250 V, 30 kW series motor runs at 800 rev/min on full load. A resistance of 0.5 Ω is connected in series with the motor and an output torque of 200 Nm is produced. Calculate the speed at this condition. It may be assumed that there are no energy losses and that the field flux is directly proportional to current.

    (Ans. 878.5 rev/min)

2. The armature of a 200 V d.c. motor has a resistance of 0.4 Ω. The no load armature current is 2.0 A. When a torque is applied, the armature current increases to 50 A, and the speed is 1 200 rev/min. Find the no load speed stating any assumptions made.

    (Ans. 33 200 rev/min)

3   A 230 V d.c. motor has a separate field coil with a constant current of 2.2 A. The armature resistance is 0.15 Ω. The motor was tested with no load applied to the shaft. The armature current was found to vary with armature voltage as follows.

Current	4.9 A	4.5 A	4.0 A	3.6 A
Volts	230	180	130	80

    The speed at 230 V was 1150 rev/min. Determine the speeds at the other voltages.

    (Ans. 890, 650, 400 rev/min)

4. The armature resistance of a 250 V d.c. shunt wound motor is 0.7 Ω and the armature current is 2.0 A when operating under no load conditions. A torque is then applied to the shaft and the armature current rises to 60 A and the speed falls to 1 000 rev/min. Determine the no load speed.

    (Ans. 1 200 rev/min)

5. Sketch the Speed - Torque characteristic for a series motor and shunt motor. Explain why the series motor is more suited to traction and winching operations.

6.10 D.C. Servo Motors Manufacturers Approach

Smaller servo motors are used for robotic applications, that is, the control of position and speed of a shaft. They may use field control or armature control or both.
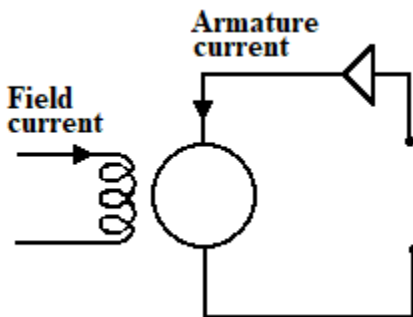
Field Control
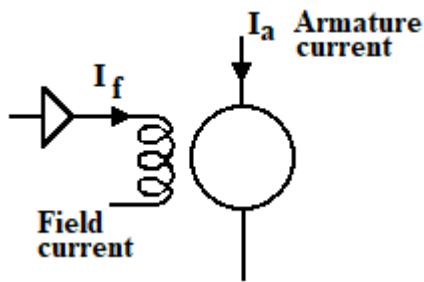
The armature current I_a is maintained constant and the field current I_f is supplied through a power amplifier and controls the torque. The torque is unaffected by the speed. The relationship between torque and current is

$$T = kI_f$$

Armature Control

This is quite common since smaller servo motors use permanent magnets. With the development of more powerful permanent magnets, DC servo motors are improving their power to weight ratio but are still not as good as hydraulic motors in this respect.

Manufacturers of such motors present the steady state characteristics based on equations 7 and 8. However some practical aspects must be brought in. First, there is a permanent loss of torque due to friction and a current is needed to overcome friction torque before any useful torque is produced. This is expressed as T_f in catalogues.

There is also a loss of torque due to damping which is directly proportional to the speed of the motor. This torque is T_d and is found by

$$\mathbf{T_d = k_d\ N} \quad \text{where N is in 1 000 rev/min.}$$

The other important constants quoted for such motors are the Torque constant k_t and the e.m.f. constant k_e. Torque is normally quoted in N cm which is not a recommended SI unit and the shaft speeds are quoted in 1000 rev/min.

The current required to operate such a motor is given by the equation

$$\mathbf{I = \frac{T_L + T_f + T_d}{k_t}}$$

T_L is the load torque. The useful torque from the motor is

$$\mathbf{T_L = k_t I - T_f - T_d}$$

The voltage required at the terminals is

$$V = \frac{Nk_e}{1\ 000} + I_a R_a$$

WORKED EXAMPLE No. 3

1. Using the manufacturers data sheet, determine the terminal voltage and current required to produce a torque of 75 N cm at 2 000 rev/min.

SOLUTION

From the data sheet the GR12C has the following constants.

k_t = 10.8 N cm per Amp.

k_e = 11.3 V per 1 000 rev/min

k_d = 1.16 N cm per 1 000 rev/min

T_f = 4.2 N cm

Armature resistance = 0.95 Ohms.

Voltage at 3000 rpm is 44.5 V

$$T_d = \frac{Nk_d}{1\,000} - \frac{2\,000 \times 1.16}{1\,000} = 2.32 \text{ N cm}$$

$$I = \frac{T_L + T_f + T_d}{k_t} = \frac{75 + 4.2 + 2.32}{10.8} = 7.55 \text{ A}$$

$$V = \frac{Nk_e}{1\,000} + I_a R_a = \frac{2\,000 \times 11.3}{1\,000} + 7.55 \times 0.95 = 29.8 \text{ V}$$

DATA TABLES FOR MOTORS

Motor Constants		GR12C	GR12CH	GR16C	GR16CH	GR19CH
Torque	K_t Ncm/Amp	10.8	17.0	23.7	37.3	24.0
EMF	K_e V/krpm	11.3	17.8	24.8	39.0	25.0
Damping	K_d Ncm/krpm	1.16	1.95	3.57	6.44	7.76
Friction Torque	T_f Ncm	4.2	4.2	7.7	7.7	9.8
Terminal Resistance @ 5A	R_m Ohm	0.95	0.95	0.95	0.95	0.65
Rotor Moment of Inertia	J kg.cm^2	1.2	1.2	5.93	5.93	12.71

SELF ASSESSMENT EXERCISE No. 3

Using the manufacturers data sheet determine the voltage and current needed to run the motor GR16C at 3 000 rev/min with a load torque of 40 N cm. (Answer 76.7 V)

7.   *Starting Large D.C. Motors*
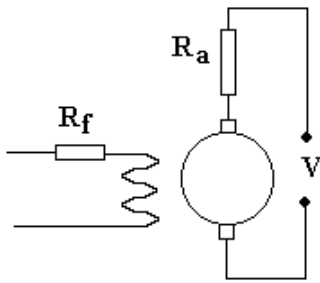
Consider a basic D. C Motor. The terminal voltage is V. The back e.m.f on the armature is E. The armature resistance is R_a. The field resistance is R_f. The flux per pole is Φ. N is the motor speed.

From earlier work it was shown that $V = E + I_a R_a$ $E = K_1 N\Phi$ $T = K_2 \Phi I_f$

When the motor is started, the speed is zero so there is no back e.m.f. It follows that $V = I_a R_a$
The starting current without protection would be V/R_a and this would be very large. In addition to this, there will be a load with inertia connected to the motor and a large current is needed to provide the torque.


WORKED EXAMPLE No. 4

A motor with a separate field coil has a terminal voltage V = 400 V and an armature resistance of 0.2Ω. Calculate the starting current in the armature.

SOLUTION

$$I_a = \frac{V}{R_a} = \frac{4\ 000}{0.2} = 2\ 000\ A$$

In order to limit the current, it is normal to insert a variable resistance in series with the armature which is gradually reduced as the motor speeds up and then latched in place in the zero resistance position. In the event of an interruption to the power, the starting resistance is unlatched and springs back to the starting position. We should consider how the starting resistance is used with different motor field configurations. The three types or configurations are shown below.

Series Motor   Shunt Motor   Compound Motor
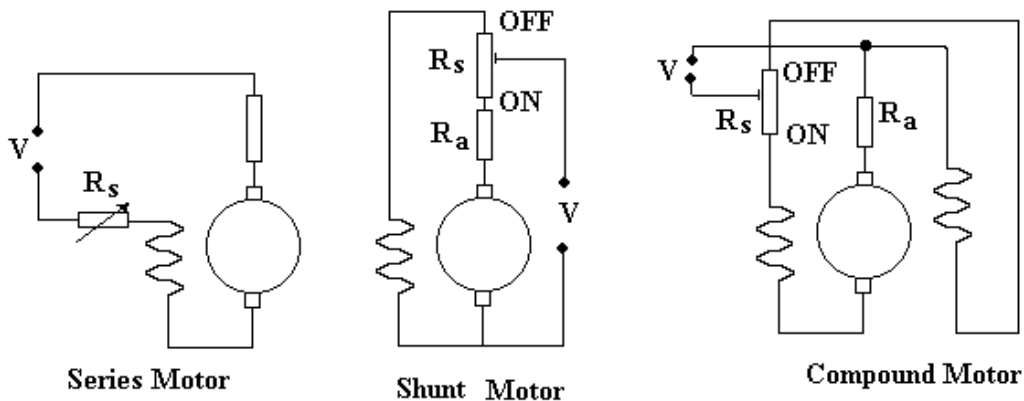
Series Motor
In the case of a series wound motor, the starting resistance is placed in series as shown.

Shunt Motor
In the case of a shunt wound motor, the starting resistance is placed as shown so that the field is initially connected to the supply and the armature is in series with it. As the motor speeds up the field resistance is gradually increased and the resistance in series with the armature is reduced.

Compound Motor
Compound motor has a starting resistance as shown.

8.   *The Effect of Inertia and Inductance*

Most of the work covered so far has been concerned with the steady state analysis of motors. When sudden changes are made to the speed or torque, it takes time for the system to respond because of the time dependant effects such as inertia, damping and inductance. A motor may have damping torque which is directly proportional to speed. This was given previously as:

$$T_d = K_d \times \frac{N}{1\,000}$$

N is rev/min and T is in N cm.

If we use S. I. units the damping torque would be given by $T_d = K_d\,\omega$ where ω is rad/s and T is Nm

ω is the rate of change of angle per second and may be expressed in calculus form as $d\theta/dt$. Hence:

$$T_d = K_d\,d\theta/dt.$$

If the motor is accelerated, torque is needed to overcome the inertia T_i and this is directly proportional to the angular acceleration. $T_i = I\alpha$

α is in rad/s^2 and I is the moment of inertia in kg m^2. (Many manufacturers use the symbol J for moment of inertia)

The acceleration is the second derivative of angle with respect to time so we may write

$$T_i = I\frac{d^2\theta}{dt^2}$$

When a motor is producing acceleration the total torque acting on it is

$$T = T_L + T_f + k_d\frac{d\theta}{dt} + I\frac{d^2\theta}{dt^2}$$

This results in a mechanical time constant τ_m which governs how quickly a motor will accelerate. This is defined by manufacturers as the time taken to accelerate the motor up to 63% of the required speed with no load on it. This will be analysed and defined in detail in another tutorial. Now consider that the terminal voltage of a servo motor was defined earlier as $V = E_a + I_aR_a$

If the current is changing, the inductance of the armature winding also produces a voltage (back emf) given by L di/dt where L is the inductance in Henries and i the transient current.

The total terminal voltage is then

$$V = L\frac{di}{dt} + E_a + I_aR_a$$

This gives rise to an electrical time constant defined as

$$\tau_e = \frac{L}{R_{ae}}$$

This will be explained in another tutorial.

If the motor uses field control, the electrical time constant is based on the field winding inductance and resistance.

WORKED EXAMPLE No. 5

A d.c. servo motor must accelerate from rest to 30 rad/s in 3 seconds. The rotating mass has a moment of inertia of 5 kg m^2. The load torque is 20 N m and the friction torque is 1.2 Nm. The damping coefficient is 0.05 Nm s/radian. The torque constant is 120 Nm per ampere.

Determine the required current just before the acceleration ends.

SOLUTION

$$T_L = 20 \text{ Nm}$$
$$T_f = 1.2 \text{ Nm}$$

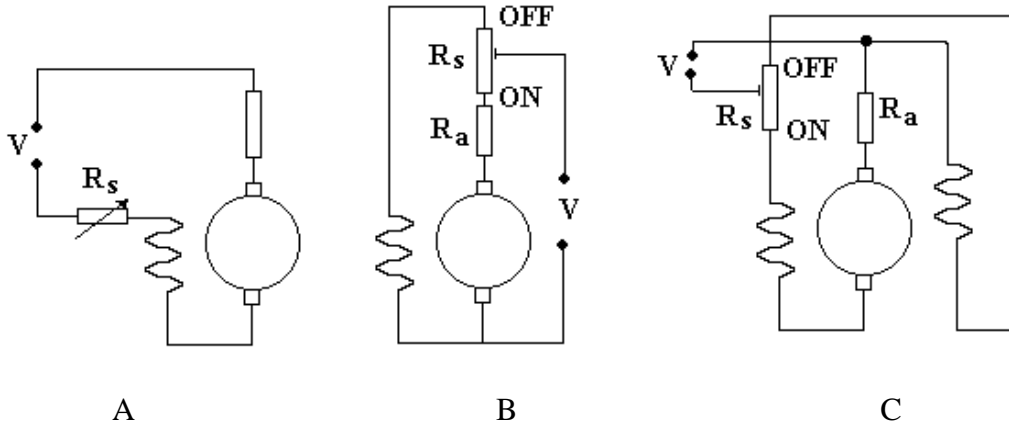$\omega = 30$ rad/s $\qquad T_d = K_d \omega = 0.05 \times 30 = 1.5$ Nm

$\alpha = (30 - 0)/3 = 10$ rad/s$^2 \qquad T_i = I\alpha = 5 \times 10 = 50$ Nm

$T = T_L + T_f + T_d + T_i = 20 + 1.2 + 1.5 + 50 = 72.7$ Nm

$I = 72.7/120 = 0.606$ Amperes

SELF ASSESSMENT EXERCISE No. 4

1.   Identify which of the diagrams shows a series, shunt and compound wound motor.

A   B   C

2.   A DC servo motor has the following constants.

k_e = 7.1 V per 1 000 rev/min
k_t = 6.7 N cm /A
k_d = 0.78 N cm per 1 000 rev/min
T_f = 3.2 N cm
R_a = 0.85 Ohm

Calculate the current, terminal voltage and electric power when the motor speed is 3 000 rev/min and the load torque is 45 N cm. (27.71 V, 7.543 A and 209 W)

3.   A 500 V shunt wound d.c. motor has an armature resistance of 0.5Ω and a field coil resistance of 250Ω. Calculate the total current taken from the source at start up. What value resistance must be placed in series with the armature to reduce the starting current to 50 A? (9.917 Ω)

4.   A d. c. servo motor must accelerate from rest to 15 rad/s in 2 seconds. The rotating mass has a moment of inertia of 2 kg m^2. The load torque is 12 N m and the friction torque is 1.5 N m. The damping coefficient is 0.08 N m s/radian. The torque constant is 90 Nm per ampere.

Determine the required current just before acceleration ends. (0.33 A)